Interlocution among problem solvers collaborating online: a case study with prospective teachers

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Abstract

We present case studies of the discursive interactions of teams Brazilian prospective teachers to understand participation and growth in mathematical thinking through their construction of mathematical ideas and reasoning as they collaboratively solve challenging, open-ended combinatorial problems online. The findings suggest that participants' discursive interactions fall within four interlocution properties, that interpretive and negotiatory interlocution support the development of mathematical thinking, and that deliberate attention to these properties in instructional practice may yield benefits for learners. Identifying and analyzing these two properties the researcher can obtain more information regarding prospective teachers' development of professional knowledge, particularly, epistemological, didactic and mediation.

Key words

virtual environment; VMT-Chat; interactions; Mathematical reasoning.

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Introduction

Communicative interactions are implicit in thinking and increased understanding. Mathematics education researchers have theorized close links between communication and thinking (SFARD, 2008) and between mathematical discourse and collaborative group or social cognition (MARTIN, TOWERS *et al.*, 2006; POWELL, 2006; STAHL, 2009a). The work of Toulmin (1969) and Walton (1992; 2007) have been employed to understand, respectively, formal and informal mathematical argumentation (ABERDEIN, 2006; INGLIS, MEIIA-RAMOS *et al.*, 2007; RASMUSSEN e STEPHAN, 2008; STEPHAN e RASMUSSEN, 2002; WEBER, MAHER *et al.*, 2008). These investigations have concerned mathematics students in presential, classroom environments and, except for Weber, Maher, Powell, and Stohl Lee (2008), examined argumentation among advanced mathematical thinkers. Collectively, these studies have yielded insights into how such students develop mathematical arguments. Nevertheless, little research exists on how unsophisticated mathematical thinkers such as average-performing high school students or how prospective mathematics teachers engage in thoughtful mathematical discourse, particularly in online communicative environment, collaborating to solve challenging, open-ended problems. Building on previous work (BAIRRAL, POWELL *et al.*, 2007; POWELL e LAI, 2009), this research report explores lessons that researchers and practitioners can gather from an inquiry into the interlocution of students and of teachers working collaboratively in small groups when engaged in "talking" and "listening" to each other in an online communication environment. We use the term interlocution to denote discursive practices of actors in conversational exchanges. Questions that motivate this research included the following: In online collaborative environments, what discursive practices do interlocutors employ as they interact to understand and resolve mathematical tasks? How do these interactions influence the development of their mathematical ideas and reasoning?

Conceptual framework

Our conceptual framework for collecting and analyzing data rests on two theoretical ideas: links between communication and thinking and interlocution. In offline as well as online environments, users express objects, relations, and other ideas graphically as text and as inscriptions, which are special instances of the more general semiotic category of signs. A sign is a human product—an utterance, gesture, or mark—by which a thought, command, or wish is expressed. As Sfard notes, "in semiotics every linguistic expression, as well as every action, thought or feeling, counts as a sign" (SFARD, 2000). A sign expresses something and, therefore, is meaningful and as such communicative, at the very least, to its producer and, perhaps, to others. Some signs are ephemeral such as unrecorded speech and gestures, while others like drawings and monuments persist. Whether ephemeral or persistent, a sign's meaning is not static; its denotation and connotation are likely to shift over time in the course of its discursive use.

As a discursive entity, a sign is a linguistic unit that can be said to contain two, associated components. Saussure (1983) proposes that a sign is the unification of the phonic substance that we know as a "word" or *signifier* and the conceptual material to which it refers or *signified*. He conceptualizes the linguistic sign (say, the written formation) as representing both the set of noises (the pronunciation or sound image) one utters for it and the meaning (the concept or idea) one attributes to it. Examples of the written formation of a linguistic sign are "chair" and "$\cos^2 x$", each with its associated, socially constructed meanings. Saussure observes further that a linguistic sign is arbitrary, meaning that both components are arbitrary. The signifier is arbitrary since there is no inherent link between the formation and pronunciation of a word or mathematical symbol and what it indexes. *A monkey* is called *o macaco* in Portuguese and *le singe* in French, and further in English the animal is denoted "monkey" and not "telephone" or anything else. The arbitrariness of the signified can be understood in the sense that not every linguistic community chooses to make salient by assigning a formation and a sound image to some aspect of the experiential world, a piece of social or perceptual reality. Consider, for example, the signifieds *cursor*, *mauve*, and *zero*. They index ideas that not all linguistic communities choose to lexicalize or represent.

Signs can be considered to represent ideas. However, Sfard (2000) argues that a sign is constitutive rather than strictly representational since meaning is not only presented in the sign but also comes into existence through it. Specifically, she states that mathematical discourse and its objects are *mutually constitutive*: It is the discursive activity, including its continuous production of symbols, that creates the need for mathematical objects; and these mathematical objects (or rather the object-mediated use of symbols) that, in turn, influence the discourse and push it into new directions (p. 47, original emphasis).

This theoretical stance on the mutually constitutive nature of meaning and sign provides a foundation for analysis of the discursive emergence of mathematical ideas, reasoning, and heuristics. On the one hand, signs can represent encoded meanings that based on previous discursive interactions interlocutors can grasp as they decode them. On the other hand, through moment-to-moment discursive interactions, interlocutors can create signs and, during communicative actions, achieve shared meanings of the signs. In this sense, the sameness of meaning for interlocutors that allows for successful communication is not something pre-existing but rather an achievement of communicative acts. Such accomplishment may compel interlocutors to bring into existence signs to further their discourse.

Mathematical signs—objects, relations, symbols, and so on—are components of mathematical discourse and are intertwined in constituting mathematical meanings. Signs exist in many different forms, and inscriptions or written signs are but one. They are produced for personal or public consumption and for an admixture of purposes: to discover, construct, investigate, or communicate ideas. As mathematicians and other mathematics education researchers also emphasize (DÖRFLER, 2000; LESH e LEHRER, 2000; SPEISER, WALTER *et al.*, 2002; SPEISER, WALTER *et al.*, 2003), building and discussing inscriptions are essential to building and communicating mathematical and scientific concepts. Lehrer, Schauble, Carpenter, and Penner

(2000) illustrate how learners work "in a world of inscriptions, so that, over time, the natural and inscribed worlds become mutually articulated" and the importance of a "shared history of inscription" (p. 357). In mathematics, the invention, application, and modification of appropriate symbols to express and extend ideas are constitutive activities in the history of mathematics (STRUIK, 1948/1967).

For researchers in mathematics education and in computer-supported collaborative learning, the arbitrariness of signifieds is a more significant point about Saussure's observation concerning the arbitrariness of signs. The reason is that the conceptual material that a person (or a small group of people) lexicalizes, for example, with pencil and paper, with text in a chat window, or with drawn objects on a shared, digital workspace indicates to what that user attends, her insight into material reality that is external or internal to her mind. The inscriptions of individuals working online in a small-group or team provide observers, who must interpret meanings constituted in the inscriptions, evidence of individual and collective thinking. The small group's inscriptions present ideas it chooses to attend and lexicalize or symbolize. By analyzing the unfolding and use of inscriptions, researchers can understand how participants constitute their mathematical ideas, reasoning, and heuristics, the meanings they attribute to their inscriptions, and how their inscriptions influence emergent meanings. As Speiser, Walter and Maher (2003) underscore, what counts as mathematical in analyzing inscriptions is not the inscription itself, which are "tools or artifacts, but rather how the students have chosen to *work*" (p. 22, original emphasis) with their inscriptions.

Our inquiry into interlocution emerges from an ongoing, longitudinal investigation into the discursive, interactive development of mathematical ideas and reasoning by individual students as they work collaboratively in small teams. Davis's (1996) theory of pedagogical listening contains three different modes, not necessarily mutually exclusive: evaluative, interpretive, and hermeneutic. Evaluative listening occurs when a teacher maintains a "detached, evaluative stance" (p. 52). In contrast, a teacher listening interpretively endeavors "to get at what learners are thinking...to open up spaces for re-presentation and revision of ideas—to *access* subjective sense rather than to merely *assess* what has been learned" (pp. 52-53, original emphasis). Finally, Davis posits hermeneutic listening as "more negotiatory, engaging, and messy, involving the hearer and the heard in a shared project...an imaginative participation in the formation and the transformation of experience through an ongoing interrogation of the taken-for-granted and the prejudices that frame perception and actions" (p. 53). He further describes that, in this mode of listening, a teacher participates in "the unfolding of possibilities *through collective action*" (p. 53, original emphasis). Martin (2001) provides evidence for how the listening patterns of a teacher can occasion opportunities for students "to construct and modify their own images in response to her interventions" (p. 251).

Both Martin (2001) and Davis (1996) inquire into teacher listening and its consequent impact on the growth of student understanding. Our study broadens this scope of inquiry as well as applies and extends Davis's categories to analyze not just listening but rather discursive practices of learners in conversational exchanges in an online environment. Building on Davis's theory (1996), we employ the conceptual category of interlocution to denote the discursive practices of actors in conversational exchanges, as developed by Powell (2003), to investigate the interlocution of students in presential situations to guide our inquiry into how online participants' discursive exchanges structure their interactions and contribute to growth in their mathematical understanding. The concept of interlocution has these four interactive properties:

- *Evaluative*: an interlocutor maintains a non-participatory and evaluative stance, judging statements of his or her conversational partner as either right or wrong, good or bad, useful or not.
- *Informative*: an interlocutor requests or announces factual data to satisfy a doubt, a question, or a curiosity (without evidence of judgment).
- *Interpretive*: an interlocutor endeavors to tease out what his or her conversational partner is thinking, wanting to say, expressing, and

meaning; an interlocutor engages an interlocutor to think aloud as if to discover his or her own thinking.

- *Negotiatory*: an interlocutor engages and negotiates with his or her conversational partner; the interlocutors are involved in a shared project; each participates in the formation and the transformation of experience through an ongoing questioning of the state of affairs that frames their perception and actions.

It is worth noting that in presential situations, when students are engaged in this category of interlocution, they are open to what Powell (2006) terms "socially emergent cognition." That is, participants engaged in negotiatory interlocution have the potential to develop jointly mathematical ideas and reasoning that are not the *a priori* insight of a interlocutor but rather emerge in the discourse of the interlocutors and later is reflective of each interlocutor's understanding.

The four properties of interlocution are neither hierarchical nor mutually exclusive; a unit of meaningful interaction may have more than one interlocutory property. Interlocution as a conceptual category in our research enables us to track the participation, changes in understanding, and autonomy of learners in their construction of mathematical ideas and reasoning. Moreover, as researchers, we have found that tracking interlocution allow us to analyze how conversational partners exchange meanings, ideas, and concepts, which, for us, are indicators of collaboration on the given mathematical task.

The tasks that we use provide opportunities for decision-making, which becomes an invitation for collaboration. Similar to characteristics of rich mathematical tasks that Lo and Gaddis (2010) identify, characteristics of tasks that we use include the following:

1. Accessible
2. Connect with prior mathematical knowledge
3. Encourage connections among different mathematical ideas
4. Admit multiple ways to solve and solutions
5. Expandable and applicable to important mathematical ideas

When research participants engage with tasks with these characteristics, particularly, in virtual environments, the data generated tend to allow researchers to analyze in detail the development of mathematical ideas and reasoning through participants' inscriptions and interlocution.

We believe that technological mediation—through ICT—has a crucial influence on the comprehensive development of domains of Professional Content Knowledge (PCK). In this study we consider three intertwined domains in the PCK of the prospective teachers: epistemological, mediation and didactic. These domains are summarized in the following table.

Epistemological	Mediation	Didactic
•Base mathematics teaching on the mental powers of learners (Gattegno, 1987). •Discussing objects and relations among them.	•Motivation regarding ICT. •The use of different representations (writing, inscriptions, etc.) to form and exchange mathematical ideas and reasoning.	•Posing and solving problems. •Challenging tasks to promote communication and collaborative work.

We build on this to track properties of interlocution, which allow researchers to investigate different ideas and reasoning that emerge in specific sequences of chat interactions.

Research context and data source

The data come from an ongoing research project, *e*Math now in its sixth year, a collaborative project among researchers in United States and Brazil[1], conducted among prospective teachers and among students in public and private secondary schools in working- and middle-class, suburban and urban districts. Overall, our longitudinal study aims to contribute basic scientific understanding of cognitive behaviors as well as pedagogical conditions for which mathematics learning occurs as a process of sense making in online environments. For this case study, data come from Brazilian prospective

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[1] In Brazil, the research project was supported by CNPq and FAPERJ.

teachers from Juiz de Fora in Minas Gerais, who engaged combinatorial tasks in an online communication portal—Virtual Math Teams (VMT)—developed with support from the National Science Foundations in the USA by researchers at Drexel University (STAHL, 2009b). The participants only communicated among themselves in a VMT chat room, working in seven teams of four and had no prior experience with the type of task proposed. Researchers were present in each team and followed the interactions of each team without any interventions and used logged interactions of each team as the source of data and considered each team as a unit of analysis.

For each team, their VMT environment (http://vmt.mathforum.org/VMTLobby/) contain two dynamic spaces: the whiteboard and the chat (see Figure 1). The environment creates a log of each team's discursive interactions (chat postings and whiteboard inscriptions). Using a VMT-Replayer, researchers can exhaustively review the enfolding interactions. The prospective teachers engaged the following task:

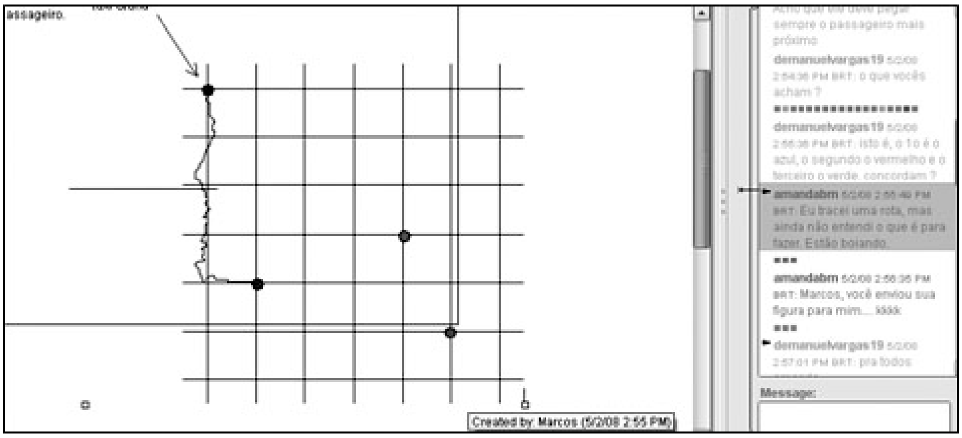
The Taxicab Problem

A taxi driver is given a specific territory of a town, shown below. All trips originate at the taxi stand. One very slow night, the driver is dispatched only three times; each time, she picks up passengers at one of the intersections indicated on the map. To pass the time, she considers all the possible routes she could have taken to each pick-up point and wonders if she could have chosen a shorter route. What is the shortest route from a taxi stand to each of three different destination points? How do you know it is the shortest? Is there more than one shortest route to each point? If not, why not? If so, how many? Justify your answer.

Accompanying this statement, the participants have a map, actually, a 6 x 6 rectangular grid on which the left, uppermost intersection point represents the taxi stand. The three passengers are positioned at different intersections as blue, red, and green dots, respectively, while their respective distances from the taxi stand are one unit east and four units south, four units east and three units south, and five units east and five units south (see Figure 1).

In the next section, we first present discursive practices interlocutors employ as they interact collaboratively to understand and resolve mathematical tasks. Later we discuss how these interactions influence the growth of their mathematical ideas.

Figure 1. The whiteboard, with Taxicab Problem grid, and chat spaces of VMT



Results

The future teachers exhibited diverse interlocution patterns. From a session that contained 178 turns of chat, in turns 51 to 59 below, participants' refine their ideas about the problem and about how to determine the minimum number of shortest paths from the taxi stand to destination points[2].

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[2] We have translated the data from Portuguese to English.

51	Elder	The nº of paths to blue is the combination of (5,1), that is, (b,b,b,b,d) where b=down and d=right
52	amandabm	Then for the blue point there are 5, red 7, and green 10
53	demanuelvargas19	that's it Amanda
54	Marcos	that's it yes
55	Marcos	But problem is to know how many shortest paths exists
57	Elder	For the red point there are (4,3), (d,d,d,d,b,b,b)
58	demanuelvargas19	that is it Elder
59	Elder	all of these cases have the shortest path, they are equal

This chat excerpt evidences interlocution patterns that are informative, interpretive, and evaluative. Informatively, *Elder* introduces the notion of combination (turn 51). In turn 52, *amandabm* interprets an unstated aspect of *Elder*'s statement, concerning the total number of units or street blocks from the taxi stand to each given destination point, and frames her contribution as a question. Afterward, both demanuelvargas19 and Marcos evaluate *amandabm*'s contribution, and the latter reminds *amandabm* about their goal. Before turn 51, in the fifth previous chat turn, *amandabm* confirms her understanding of the metric in this problem but expresses a doubt about how the team was constructing paths, as she says: "Wow…I think that our interpretation was completely wrong after this explanation" (turn 46). It appears that for this participant how the group constructs paths does not contribute insights.

In the following interactive chat sequence, *amandabm* tries to understand a new contribution from *Elder*. Again, their interlocution evidences informative, interpretative, and evaluative interlocution.

155	Elder	I think that the nº of paths is actually 2C(b+d,d)
156	Wallace	for example, to arrive at blue, we must use 5 steps, being 4 down and 1 to the right
157	amandabm	Wow, but that would give 840 paths for the red? Is it really that or did I make a mistake with the formula?
158	Elder	You made a mistake
159	Wallace	well, to go there will be 35 paths
160	Wallace	also to return

While *amandabm* tries to follow *Wallace*'s reasoning, *Elder* judges *amandabm*'s statement as incorrect. Wallace also negotiates the meaning of *Edler*'s statement in turn 155, and senses that since there are 35 shortest paths to reach the red destination point, and says "also to return," meaning 35 paths to return from there to the taxi stand.

Shortly thereafter, *amandabm* offers how to calculate a combination, and is informed by Wallace how to write correctly her formula.

171	amandabm	*I also typed wrong: C= p! / (n – p!)
172	Wallace	Amanda, C(n,p)=n!/(n-p!)]
173	amandabm	Wow... Thanks... I really made a mistake...

In this sequence of turns, *Wallace* informs *amandabm* how to write correctly the idea she seems to express in turn 171. Here is a case of informative interlocution, and *amandabm* responds appreciatively.

Discussion

Our case study indicates that the conversational interactions among participants can advance their individual and collective actions. Through their discursive interactions, the participants impose meaning and structure, make decisions about what to do and how to do it, and interpret the reasonableness of their actions and solutions. They take intellectual risks as well as consider each other's thinking.

The four properties of interlocution appear in various moments in the participants' discursive interactions. The same interlocution can evidence different interlocution properties and, consequently, different aspects of mathematical thinking emerge. Furthermore, different interlocution interactions yield different outcomes and influence the development of mathematical ideas and reasoning in diverse ways. For instance, though the informative interaction of Wallace with *amandabm* corrected the latter's formula (turn 172), the evaluative discursive interactions that *Wallace* and *Elder* engage with *amandabm* failed to help that participant to distinguish between permutations and combinations.

Regarding prospective teachers' PCK (for instance, epistemological + didactic + mediation) on this particular virtual environment, the space and task encourages and facilitates collaboration, and stimulates particular behaviors; continuously interacting and making observations, written and inscriptions observations lead to questions and specific to general cases.

Importantly, the data suggest that among our four interlocution properties (1) interpretive and negotiatory interlocution have the potential for advancing the mathematical understanding of individual learners working in a small group and (2) the personal or individual understanding of a learner is intermeshed with the understanding of his or her interlocutors. In sum, interpretive and negotiatory interlocutions support the development of the participants' mathematical ideas and reasoning. This observation raises instructional questions for us about how best to help learners in an online, collaborative environment engage these properties of interlocution as a deliberate feature of their conversational interactions. We suspect that the theoretical and pedagogic application of deliberative discourse (MICHAELS, O'CONNOR *et al.*, 2008) may yield interesting insights for future research on the development of mathematical ideas and reasoning among learners in online, collaborative environments, solving challenging, open-ended problems.

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