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On the precision and accuracy of the acoustic birefringence technique for stress evaluation

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ABSTRACT

This paper presents a numerical procedure for estimation of the precision and accuracy of the acoustic birefringence technique for evaluation of residual and applied stresses in civil structures and components. This procedure accounts in an automatic and systematic way for the uncertainties in the input data and their propagation throughout the calculations. The acoustic birefringence is defined from the speeds of two mutually orthogonal volumetric waves of normal incidence, but when the use of a pulse-echo measurement system is feasible, the birefringence can be defined directly from the time-of-flight of the waves, since they travel the same physical space. The times-of-flight of the waves are estimated by coupling the mathematical techniques of cross correlation and data interpolation, whereas the material's acoustoelastic constant is determined via a weighted linear regression. As an example, the estimation of the precision and accuracy in the evaluation of the stresses in a beam under bending is discussed.

Keywords: Ultrasound; Acoustic Birefringence; Stress Measurements; Precision and Accuracy.

INTRODUCTION

This paper proposes a numerical procedure for estimation of the experimental precision and accuracy of the acoustic birefringence technique as used at the Instituto de Engenharia Nuclear (IEN) of the Comissão Nacional de Energia Nuclear (CNEN) for evaluation of residual and applied stresses in nuclear civil structures and components. The acoustic birefringence is defined from the speeds of two mutually orthogonal volumetric waves of normal incidence, but when the use of an ultrasonic pulse-echo measurement system is feasible, the birefringence can be defined directly from the time-of-flight of the waves since they travel the same physical space. In the pulse–echo mode, the time-of-flight of the ultrasonic wave can thus be regarded as the primary variable for stress measurement by the acoustic birefringence technique. For the fundamentals of ultrasonic wave propagation in elastic, homogeneous, isotropic, and anisotropic solids, see [1].

This paper starts with the basic concepts and equations of the acoustic birefringence technique for stress measurement and the fundamentals of experimental error analysis focusing on the statistics concepts of interest. Then follows a description of the techniques available at IEN's ultrasonic laboratory for generating the ultrasonic waves and estimating their times-of-flight, and for calculating the acoustic birefringence at a material point. Next the numerical procedure proposed to estimate the experimental precision and accuracy of the birefringence technique for stress evaluation is detailed. The precision is characterized by the relative error given by the ratio between the standard deviation of the individual measures and their average value, and the accuracy, by the ratio between the average value of the individual measures and a reference value. To exemplify the procedure developed, the estimation of the experimental precision and accuracy in the evaluation of the stresses acting in a beam under simple bending is discussed.

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2. MATERIALS AND METHODS

The acoustical birefringence B is the normalized difference of the speeds V_{31} and V_{32} of two shear waves polarized orthogonally along the material symmetry axes x_1 and x_2 and propagating through the thickness (x_3 axis) of a component with flat and parallel surfaces [1–3]. B is evaluated according to Eq. (1)

$$B = -2\left(t_{31} - t_{32}\right) / \left(t_{31} + t_{32}\right), \tag{1}$$

where the times-of-flight t_{31} and t_{32} are used to replace the speeds since, in principle, waves start exactly at the same point and travel the same distance. B is the sum of the anisotropy from the material texture B_0 and from the internal and applied stresses. For a homogeneous material, the last contribution depends linearly by means of the acoustoelastic constant m from the difference of the principal stresses T_1 and T_2 aligned with material symmetry axes x_1 and x_2,

$$B = B_0 + m\left(T_1 - T_2\right). \tag{2}$$

B_0 and m can be characterized through a tensile test by linearly fitting the values of the birefringence at increasing loading. Eq. (1) and Eq. (2) show that the birefringence technique can provide a thickness-averaged and relative stress magnitude only.

2.1. Fundamentals of experimental error analysis

In this work, the basic statistical concepts of *arithmetic mean* (average value) and *standard deviation* of individual repeated measurements, *standard deviation of the mean*, and *Pearson correlation coefficient* of two random sets (a scaled version of *covariance*) are employed for interpretation of the experimental data. The characterization of the material acoustoelastic constant m and of the initial birefringence B_0 (Eq. 2) is done by linearly fitting the values of birefringence at increasing loading. Error propagation throughout the calculations is based on differential calculus. Finally, the principal stress difference ($\Delta T = T_1 - T_2$) is estimated by solving Eq. (2) in reverse order assuming B, B_0 and m as independent variables (input data). These statistical concepts and their use here are briefly reviewed in the following subsections [4, 5].

2.1.1. Statistical errors: basic concepts

The result of the measurement of a quantity should be given by the best estimate of its expected value and the uncertainty associated with this estimate. The best estimate for the expected value of a quantity from a sample of N direct measurements of a variable x $\{x_1, x_2, ..., x_N\}$, is the mean or average value of these measurements, and the uncertainty associated with this estimate is the standard deviation of this mean or average value.

Thus, the result of the measurement of a quantity is defined by,

$$\bar{x} \pm \sigma_{\bar{x}} \tag{3}$$

where

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i \,, \quad \text{(= mean or average value)} \tag{4}$$

$$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}} \quad \text{(= standard deviation of the mean)} \tag{5}$$

$$\sigma_x = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N}} \quad \text{(= standard deviation of individual measurements)} \tag{6}$$

If the number of direct measurements N is not sufficiently large for its standard deviation to be an adequate estimate of the standard deviation of the totality of the possible measurements (which is usually the case at the experiments carried out at IEN's ultrasonic laboratory), the standard deviation of individual measurements σ_x is replaced by s_x, termed the *experimental standard deviation* of individual repeated measurements,

$$s_x = \sqrt{\frac{N}{N-1}}\,\sigma_x = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{(N-1)}} \tag{7}$$

so that the *standard deviation of the mean* becomes,

$$\sigma_{\bar{x}} = \frac{s_x}{\sqrt{N}} = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N(N-1)}} \tag{8}$$

The covariance of two sets of random variables x and y extracted from two samples of N direct measures, $\{ x_1, x_2, \dots, x_N \}$ and $\{ y_1, y_2, \dots, y_N \}$ is given by

$$\sigma_{xy} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})(y_i - \bar{y}) = \overline{xy} - \bar{x}\,\bar{y} = \sigma_{yx} \tag{9}$$

and Pearson correlation coefficient (r) of the sets x and y by

$$r = \frac{\sigma_{xy}}{\sigma_x\, \sigma_y} \qquad (-1 \le r \le 1). \tag{10}$$

The two sets of random variables x and y are said to be strongly correlated when the value of r is close to unity, and to be uncorrelated when r is null.

2.1.2. Data fitting: weighted linear regression

Assume that there is a strong correlation among the measurements associated with a pair of variables (x, y) and that a relationship of cause and effect exist between them described by a functional relationship $y = f(x)$. The determination of this relationship is known as curve fitting (or regression). The simplest functional relationship is when the Pearson correlation coefficient (r) among the pairs (x_i, y_i) of N measurements of the variables x and y is close to unity. In this case, the variation of y as a function of x can be expressed as a linear relationship,

$$y = f(x) = a\,x + b \tag{11}$$

where the angular (a) and linear (b) coefficients of the straight line can be estimated by the method least squares by minimizing the functional expression,

$$S(a,b) = \sum_{i=1}^{N} \left[\frac{y_i - (a\,x_i - b)}{\sigma_i} \right]^2 \tag{12}$$

where

$$\sigma_i^2 = \sigma_{y_i}^2 + a^2\, \sigma_{x_i}^2 \tag{13}$$

σ_{x_i} and σ_{y_i} are the uncertainties in the measurements of x_i and y_i, leading to

$$a = r\,\frac{\sigma_y}{\sigma_x} \quad \text{and} \quad b = \bar{y} - a\,\bar{x}. \tag{14}$$

The uncertainties in the values of the angular (a) and linear (b) coefficients are given by

$$\sigma_a = \frac{1}{\sigma_x \sqrt{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}}} \quad \text{and} \quad \sigma_b = \sigma_a \sqrt{\sum_{i=1}^{N} \frac{x_i^2}{N}} \tag{15}$$

The identification of the material acoustoelastic constant m and initial birefringence B_0 (and their uncertainties) is done by applying the weighted linear regression described above to the linear function given by Eq. (2)

relating the birefringence B with the principal stress difference ΔT ($\Delta T = T_1 - T_2$). The input data for this procedure are the values of the B_i determined at N loading levels ΔT_i and their associated uncertainties σ_{B_i} and $\sigma_{\Delta T_i}$ (see subsection 2.3.1). Thus,

$$B = f(\Delta T) = m\,\Delta T + B_0\,, \tag{16}$$

where

$$m = r\,\frac{\sigma_B}{\sigma_{\Delta T}}; \quad B_0 = \bar{B} - m\,\Delta\bar{T} = \bar{B} - m\left(\bar{T}_1 - \bar{T}_2\right), \tag{17}$$

and

$$\sigma_m = \frac{1}{\sigma_{\Delta T}\sqrt{\sum_{i=1}^{N}\frac{1}{\sigma_i^2}}}; \qquad \sigma_{B_0} = \sigma_m\sqrt{\sum_{i=1}^{N}\frac{\Delta T_i^2}{N}}. \tag{18}$$

with

$$\sigma_i^2 = \sigma_{B_i}^2 + m^2\,\sigma_{\Delta T_i}^2 \tag{19}$$

2.1.3. Error propagation

Error propagation is determined using the formulas provided by differential calculus based on the Taylor expansion of a multivariate function. Thus, if a variable ϕ depends of M other variables $X = (x_1, x_2, \cdots, x_M)$, according to some function *f(X),* and if the N measurements of each one of the independent variables are distributed around the mean value $\bar{X} = (\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_M)$, such that around this neighborhood *f(X)* can be approximated by the first terms of a Taylor expansion, i.e.,

$$\Phi = f(X) = f(\bar{X}) + \sum_{k=1}^{M}\left(\frac{\partial f}{\partial x_k}\right)_{\bar{X}}(x_k - \bar{x}_k), \tag{20}$$

The estimate for the expected value of ϕ is given by the value of *f(X)* at the mean value point $\bar{X}$,

$$\bar{\Phi} = f(\bar{X})\,, \tag{21}$$

and the uncertainty associated with each indirect measure of ϕ by

$$\sigma_\Phi = \sqrt{\sum_{k,l}^{M}\left(\frac{\partial f}{\partial x_k}\right)_{\bar{X}} V_{kl}\left(\frac{\partial f}{\partial x_l}\right)_{\bar{X}}} \tag{22}$$

where V_{kl} is the covariance of the sets $x_k = (x_{k_1}, x_{k_2}, ..., x_{k_N})$ and $x_l = (x_{l_1}, x_{l_2}, ..., x_{l_N})$,

$$V_{kl} = \sigma_{x_k x_l} = \frac{1}{N}\sum_{i=1}^{N}(x_{ki} - \bar{x}_k)(x_{li} - \bar{x}_l) = \sigma_{x_l x_k} \tag{23}$$

and the uncertainty in the mean value of ϕ is

$$\sigma_{\bar{\Phi}} = \frac{\sigma_\Phi}{\sqrt{N}}. \tag{24}$$

The multivariate functions considered in this work are given by Eq. (1), for the birefringence B, and Eq. (2), for the difference of the principal stress $\Delta T = (T_1 - T_2)$. The former is a *two-variable* function of the times-of-flight t_{31} and t_{32}. Applying the above procedure to account for the propagation of errors in the calculations, the expected value of the birefringence is

$$\overline{B} = f(\overline{t}_{31}, \overline{t}_{32}) = -2\left(\frac{\overline{t}_{31} - \overline{t}_{32}}{\overline{t}_{31} + \overline{t}_{32}}\right) \tag{25}$$

with uncertainty

$$\sigma_{\overline{B}} = \sqrt{\left(\frac{4\,\overline{t}_{32}}{(\overline{t}_{31} + \overline{t}_{32})^2}\right)^2 \sigma^2_{\overline{t}_{31}} + \left(\frac{4\,\overline{t}_{31}}{(\overline{t}_{31} + \overline{t}_{32})^2}\right)^2 \sigma^2_{\overline{t}_{32}} - \frac{32\ \overline{t}_{31}\ \overline{t}_{32}}{N\,(\overline{t}_{31} + \overline{t}_{32})^4}\ \sigma_{t_{31}\,t_{32}}}\,. \tag{26}$$

Eq. 2 for the principal stress difference should be treated as a subtraction and division of numbers with *independent* uncertainties, i.e. ($\overline{B} \pm \sigma_{\overline{B}};\ B_0 \pm \sigma_{B_0};\ m \pm \sigma_m$). Applying the general procedure to account for the propagation of errors, the expected value of the principal stress difference is given by

$$\Delta\overline{T} = f(\overline{B}, B_0, m) = \left(\frac{\overline{B} - B_0}{m}\right) \tag{27}$$

with uncertainty

$$\sigma_{\Delta\overline{T}} = \left(\frac{\overline{B} - B_0}{m}\right)\sqrt{\frac{\sigma^2_{\overline{B}} + \sigma^2_{B_0}}{\left(\overline{B} - B_0\right)^2} + \left(\frac{\sigma_m}{m}\right)^2}\,. \tag{28}$$

2.1.4. Precision and accuracy

The *experimental precision* is characterized by *relative error* given by the ratio between the standard deviation of the mean (Eq. 8) and the absolute value of the mean (Eq. 4), whereas the *experimental accuracy* is the relative error obtained by dividing the absolute value of the difference between the principal stress difference (Eq. 27) and the reference value x_{REF}, by the,

$$PRECISION = \frac{\sigma_{\overline{x}}}{|\,\overline{x}\,|} \tag{29}$$

and

$$ACCURACY = \frac{\left|\Delta\overline{T} - X_{REF}\right|}{|\,X_{REF}\,|} \tag{30}$$

The number of significant figures used to report the results are determined considering the number of significant figures of the input data and the standard deviation of the mean of the variable under consideration (times-of-flight, birefringence, acoustoelastic constant and stresses) constrained by the precision of the input data.

2.2. The experimental procedure used at IEN/CNEN

At IEN's ultrasonic laboratory two different techniques are employed for propagating the ultrasonic shear waves and acquiring the electronic signals at a determined point: the *continuous* and the *pair-to-pair* techniques. For shortness only the former will be considered hereafter. The waves' time-of-flight is determined from their electronic signals using the mathematical techniques of cross correlation and data interpolation [6].

In the *continuous technique*, an initial direction aligned with one of the symmetry axes is chosen (say direction 1) and kept fixed while a sequence of N (usually five to ten) ultrasonic shear waves is generated and polarized in this direction, their signals captured and their times-of-flight $(t_{31})_i$ (i =1, N) determined using the mathematical techniques of cross correlation and data interpolation. Only after the series of signals acquisition is completed, the transducer is rotated to the direction orthogonal to the first one (direction 2) and a similar series of measurements are made leading to the set $(t_{32})_i$ (i =1, N). These results are then used to determinate the waves' average time-of-flight in each orthogonal direction ($\bar{t}_{31}$,$\bar{t}_{32}$) and their corresponding uncertainties ($\sigma_{\bar{t}_{31}}$,$\sigma_{\bar{t}_{32}}$) that are then finally used to calculate the birefringence value $\bar{B}$ and its uncertainty $\sigma_{\bar{B}}$.

2.3. The numerical procedure proposed

It should be initially recalled that the birefringence technique can only provide a thickness-averaged and relative stress magnitude. The thickness-averaged constraint is due to the application of *volumetric* shear waves, and the *relative* stress constraint relates to the fact that only the principal stress *difference* and not their *nominal* values can be generally estimated. An additional limitation comes from the fact that Eq. (2) refers to stress states in which the principal stresses are aligned with the material symmetry axes. In some applications, however, as in a beam under bending to be presented, it is possible to obtain the stress distribution along a beam's cross-section normal to the deformation plane by using shear waves propagating on the cross-section's plane and orthogonally polarized along the material symmetry axes on the deformation plane. For this example, the nominal value of the principal stress can also be estimated for the most external fibers on the deformation plane, since one the principal stress acting there has a null value. For stress states in which the principal stresses are not aligned with the material symmetry axes, a modified version of Eq. (2) must be employed [1,7].

The numerical procedure is now summarized in two steps: *material characterization* (estimation of m and B_0 parameters) and *stress estimation* (estimation of the principal stress difference $\Delta T = T_1 - T_2$).

2.3.1. Material characterization

Material characterization is done by a uniaxial loading test (T_1 or T_2 is null) at different stress levels below the material yielding stress. The direction of applied loading should coincide with one of the material symmetry axes 1 or 2, but the specific choice may affect the parameters' results and deserves further study [8]:

a) For each load level $\bar{T}_k \pm \sigma_{\bar{T}_k}$ [k = 1, P (P = nº of stress levels at the loading test)], estimate the waves' *average* time-of-flight in each orthogonal direction ($\bar{t}_{31}$,$\bar{t}_{32}$) and their corresponding uncertainties ($\sigma_{\bar{t}_{31}}$,$\sigma_{\bar{t}_{32}}$) using Eq. (4) and Eq. (8), and then estimate the birefringence mean value $\bar{B}$ and its uncertainty $\sigma_{\bar{B}}$ using Eq. (4) and Eq. (8) once again;
b) For the set of values $\bar{B}_k$ and T_k and associated uncertainties $\sigma_{\bar{B}_k}$ and $\sigma_{\bar{T}_k}$, estimate the parameters m and B_0 and their uncertainties $\sigma_{\bar{m}}$ and σ_{B_0} applying Eq. (17) to Eq. (19).

The number of significant figures to be retained in the results is based on the number of significant digits and relative error (precision) of the input data and on the standard deviation of the mean of the computed variables [6].

2.3.2. Estimation of the principal stress difference

With the material parameters characterized, Eq. (2) can be applied in reverse order to estimate the principal stress difference in selected points of a structure under loading:

a) First select the technique (*continuous* in this work) for propagating the ultrasonic shear waves and acquire and treat the data (the waves' time-of-flight) accordingly to obtain the expected value of the birefringence $\bar{B}$ and its uncertainty $\sigma_{\bar{B}}$ (Eq. 25 and Eq. 26);
b) Apply Eq. 2 in reverse order to estimate the expected value of the principal stress difference $\Delta\bar{T} = (T_1 - T_2)$ and its uncertainty; $\sigma_{\Delta T}$ (Eq. 27 and Eq. 28);
c) Determine the experimental precision and accuracy (if a reference solution is available) of the result using Eq. 29 and Eq. 30.

The number of significant figures to be retained in the results is based, as in the previous case, on the number of significant digits and relative error (precision) of the input data and on the standard deviation of the of the mean of the computed variables.

3. RESULTS AND DISCUSSION

To illustrate the numerical procedure proposed for estimation of the precision and accuracy of the acoustic birefringence ultrasonic technique as used at IEN Lab, the behavior of a beam under bending is examined. The beam specimen was manufactured from 20 MnMoNi 55 steel and had 107 mm height, 95 mm thickness and 895 mm span length (see Figure 1).

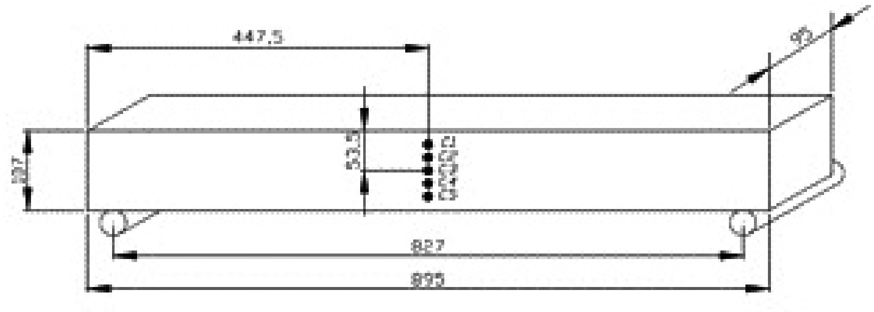

Figure 1: Beam dimensions (mm) [7].

The beam was simply supported in two points 827 mm apart, and then loaded up to 42,000 Kgf (96% of the material yield limit) at its central region at half of the beam length. The ultrasonic signals were acquired first for the beam in the unloaded condition, and then loaded.

Only points located along the beam height and located at half of its length were selected for the measurements (Figure 1). The *continuous* technique was used to acquire the ultrasonic signals. Shear waves were propagated along the beam thickness and polarized along the material symmetry axes x_1 (longitudinal direction: length) and x_2 (transversal direction: height). In each point, 5 pairs of signals were acquired to determine the wave average time of flight, making a total of 10 signals acquisition per point. The first 5 signals were acquired with the shear wave polarized along the beam's longitudinal direction and the remaining 5 ones with the shear wave polarized along its transversal direction (the material symmetry directions). A data acquisition system using a dual element transducer of 0,5 MHz was mounted to generate, receive, and treat the shear wave echoes to determine the waves' time-of-flight.

3.1. Determination of the acoustoelastic constant and of the initial birefringence

To obtain the acoustoelastic constant, which is the angular coefficient of the straight line that approximates the relationship between the wave velocity and the applied load, a sample of the beam material with a length of 60 mm and a cross section of 40 × 40 mm^2 was subjected to a loading (compression) program consisting of 6 load increments of 5,000 Kgf each at its central point. Shear waves were propagated along the beam thickness and polarized along the material symmetry axes x_1 and x_2, and their times of flight were acquired at each load level. DUTRA [7] considered two alternative load testing in which the direction of applied compression load is alternatively aligned with the material symmetry axes x_1 and x_2, finding two different values for the acoustoelastic constant. Here, the acoustoelastic constant for the load aligned with the material direction 2 (transversal direction) was used to evaluate the principal stress difference.

Applying the procedure indicated in Section 2.2 for the *continuous* technique, and considering the time-of-flight of the longitudinal and transversal waves indicated in Tables 1 and 2, the following expected values (and uncertainties) where obtained for the acoustoelastic constant and the initial birefringence:

Table 1: Waves' time-of-flight (without loading).

CONTINUOUS TECHNIQUE	0 Kgf	
MEASUREMENT	TIME-OF-FLIGHT (LONGITUDINAL) (s)	TIME-OF-FLIGHT (TRANSVERSAL) (s)
1	0.000024504	0.000024660
2	0.000024504	0.000024660
3	0.000024504	0.000024660
4	0.000024504	0.000024660
5	0.000024504	0.000024660
Mean	0.000024504	0.000024660
Uncertainty[a]	2.50e-10	2.50e-10

[a] Including the resolution of the equipment and the effects of data interpolation.

Table 2: Waves' time-of-flight (under loading).

CONTINUOUS TECHNIQUE	5,000 Kgf		10,000 Kgf		15,000 Kgf	
MEASUREMENT	TIME-OF-FLIGHT (LONGI-TUDINAL) (s)	TIME-OF-FLIGHT (TRANS-VERSAL) (s)	TIME-OF-FLIGHT (LONGI-TUDINAL) (s)	TIME-OF-FLIGHT (TRANS-VERSAL) (s)	TIME-OF-FLIGHT (LONGITU-DINAL) (s)	TIME-OF-FLIGHT (TRANS-VERSAL) (s)
1	0.000024504	0.000024660	0.000024516	0.000024612	0.000024516	0.000024624
2	0.000024516	0.000024660	0.000024516	0.000024624	0.000024528	0.000024636
3	0.000024516	0.000024660	0.000024516	0.000024636	0.000024528	0.000024624
4	0.000024516	0.000024660	0.000024516	0.000024636	0.000024528	0.000024636
5	0.000024516	0.000024660	0.000024516	0.000024636	0.000024528	0.000024624
Mean	0.000024514	0.000024660	0.000024516	0.000024629	0.000024526	0.000024629
Uncertainty[a]	2.4e-09	2.5e-10	2.5e-10	4.8e-09	2.4e-09	2.9e-09
	20,000 Kgf		**25,000 Kgf**		**30,000 Kgf**	
1	0.000024528	0.000024624	0.000024552	0.000024612	0.000024552	0.000024636
2	0.000024540	0.000024612	0.000024552	0.000024612	0.000024552	0.000024612
3	0.000024540	0.000024612	0.000024552	0.000024612	0.000024552	0.000024600
4	0.000024528	0.000024612	0.000024552	0.000024612	0.000024552	0.000024600
5	0.000024528	0.000024612	0.000024552	0.000024612	0.000024552	0.000024600
Mean	0.000024533	0.000024614	0.000024552	0.000024612	0.000024552	0.000024610
Uncertainty[a]	2.9e-09	2.4e-09	2.5e-10	2.5e-10	2.5e-10	7.0e-09

[a] Including the resolution of the equipment and the effects of data interpolation.

Table 3: Material acoustoelastic constant and initial birefringence.

Acoustoelastic constant:	$m = -0.000230 \pm 0.000002$ (Kgf/mm^2)$^{-1}$
Initial birefringence (average value):	$B_0 = 0.0006535 \pm 0.000024$

3.2. Principal stress difference estimation by the birefringence technique

Applying the *continuous* technique for the stress estimation (Section 2.2), the following times-of-flight and birefringence were determined at points C1 and C5 at the center cross- section of the beam (Figure 1).

The values of the initial birefringence B_0 at points C1 and C5 (Table 4) indicate that the material of the beam is *acoustically heterogeneous*. To account for this material characteristic, as a first approximation, the local values of B_0 were used instead of the average value indicated in Table 3. With this consideration, the expected principal stress difference (with uncertainties) at points C1 and C5 were then estimated from Eq. (27) and Eq. (28) as indicated in Table 5.

3.3. Principal stresses determination by the strength of materials theory

For the case of simply supported, rectangular beam subjected to a uniformly distributed load acting on a small area of the beam midway between the supports, the magnitudes of the principal stresses T_1 (direction longitudinal, x_1) and T_2 (direction transversal, x_2) are given by the elementary theory of the strength of materials [9] as:

a) For points at the central section $x_1 = l/2$ such that $0 \le x_2 \le h/2$ (tensile region)

$$T_1\left(\frac{l}{2}, x_2\right) = \frac{12}{bh^3}\frac{q\,d}{4}\left(l - \frac{d}{2}\right)x_2; \quad T_2\left(\frac{l}{2}, x_2\right) = 0 \tag{31}$$

b) For points at the central section $x_1 = l/2$ such that $-h/2 \le x_2 \le 0$ (compression region)

$$T_1\left(\frac{l}{2}, x_2\right) = \frac{12}{bh^3}\frac{q\,d}{4}\left(l - \frac{d}{2}\right)x_2; \quad T_2\left(\frac{l}{2}, x_2\right) = 0 \tag{32}$$

Table 4: Waves' time-of-flight and birefringence at selected points of the beam.

LOAD	0 Kgf		42,000 Kgf	
POINT (MEASUREMENT)	TIME-OF-FLIGHT (LONGITUDINAL) (S)	TIME-OF-FLIGHT (TRANSVERSAL) (S)	TIME-OF-FLIGHT (LONGITUDINAL) (S)	TIME-OF-FLIGHT (TRANSVERSAL) (S)
C1 (1st measurement)	0.000058770	0.000058650	0.000058560	0.000058680
C1 (2nd measurement)	0.000058810	0.000058650	0.000058560	0.000058650
C1 (3rd measurement)	0.000058780	0.000058650	0.000058560	0.000058650
C1 (4th measurement)	0.000058780	0.000058650	0.000058560	0.000058680
C1 (5th measurement)	0.000058780	0.000058650	0.000058560	0.000058680
Mean (time-of-flight)	0.000058784	0.000058650	0.000058560	0.000058668
Uncertainty[a]	6.8e-09	2.5e-10	2.5e-10	7.4e-09
Mean (birefringence)	–0.0023		0.0018	
Uncertainty[a]	0.0001		0.0001	
C5 (1st measurement)	0.000058920	0.000059000	0.000059070	0.000058920
C5 (2nd measurement)	0.000058960	0.000059000	0.000059040	0.000058920
C5 (3rd measurement)	0.000058960	0.000059000	0.000059040	0.000058920
C5 (4th measurement)	0.000058960	0.000059000	0.000059040	0.000058950
C5 (5th measurement)	0.000058960	0.000059000	0.000059070	0.000058920
Mean (time-of-flight)	0.000058952	0.000059000	0.000059052	0.000058926
Uncertainty[a]	8.0e-09	2.5e-10	7.4e-09	6.0e-09
Mean (birefringence)	0.0008		–0.0021	
Uncertainty[a]	0.0001		0.0002	

[a] Including the resolution of the equipment and the effects of data interpolation.

Table 5: Principal stress difference at points C1 e C5 (birefringence technique).

POINT	m (mm²/Kgf)	B_0	$\bar{B}$	$\Delta T(= T_1 - T_2)$ (Kgf/mm²)	RELATIVE ERROR (%)
C1	–0.000230 ± 0.000002	–0.0023 ± 0.0001	0.0018 ± 0.0001	–17.8 ± 0.6	3.37
C5	–0.000230 ± 0.000002	0.0008 ± 0.0001	–0.0021 ± 0.0002	12.6 ± 1.0	7.94

Table 6: Geometric and loading data.

GEOMETRIC DATA			UNIFORMLY DISTRIBUTED LOADING		
LENGTH (l) (mm)	THICKNESS (b) (mm)	HEIGHT (h) (mm)	APPLIED LOAD (Kgf)	PISTON DIAMETER (d) (mm)	LOAD/LENGTH (q) (Kgf/mm)
827	95	107	42,000	127	331

Table 7: Principal stress difference at points C1 e C5 (strength of materials).

POINT	COORDINATES (mm)		STRESS T1 (Kgf/mm²)	STRESS T2 (Kgf/mm²)	DELTA (T1 – T2) (Kgf/mm²)
	X_1	X_2			
C1	447.5	–35.0	–14.95	0.0	–14.95
C5	447.5	35.0	14.95	0.0	14.95

where, q is the distributed load acting on length d, l is the distance between the supports, h is the height of the beam, b is the thickness of the beam (along the x_3 axis). The geometric and loading data are compiled in Table 6, while the nominal stresses determined at points C1 and C5 are shown in Table 7.

3.4. Evaluation of the precision and accuracy of the experimental procedure

Considering Eq. (29) and Eq. (30), and the results in Tables 5 and 7, the precision and accuracy in the estimation of principal stress difference at points C1 and C5 are summarized in Table 8.

Table 8: Precision and accuracy in the estimation of the principal stress difference.

DELTA (T1 – T2) (Kgf/mm^2)				
POINT	BIREFRINGENCE TECHNIQUE	PRECISION (%)	STRENGTH OF MATERIALS	ACCURACY (%)
C1	–17.8 ± 0.6	3.4	–14.95	19.1
C5	12.6 ± 1.0	7.9	14.95	15.7

4. CONCLUSIONS

In this work, a numerical procedure has been proposed for *estimation of the precision and accuracy* of the acoustic birefringence technique as used in the Instituto de Engenharia Nuclear (IEN) for evaluation of residual and applied stresses in nuclear civil structures and components. This procedure accounts in an *automatic and systematic* way for the uncertainties in the input data and their propagation throughout the calculations. For the case showed here, the acoustic birefringence technique provided precise and accurate results. When assessing these results, however, it should be kept in mind that the material of the beam showed an acoustically *heterogeneous* behaviour and because of that additional approximation had to be introduced in the analysis.

Future work shall be directed to the analysis of structures in which the principal stresses are not aligned with the material axes of symmetry and to a more appropriate treatment of acoustically heterogeneous materials. This will require the modification of the equation relating the principal stress difference and the birefringence as discussed by THOMPSON *et al*. [7].

5. BIBLIOGRAPHY

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[6] BITTENCOURT, M.S.Q., *"Desenvolvimento de um sistema de medida do tempo decorrido da onda ultrasônica e análise do estado de tensões em materiais metálicos pela técnica da birrefringência acústica"*. Tese (Doutorado em Ciências), Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2000.

[7] THOMPSON, R.B., LEE, S.S., SMITH, J.F., "Angular dependence of ultrasonic wave propagation in a stressed, orthorhombic continuum: theory and application of the measurement of stress and texture", *The Journal of the Acoustical Society of America*, v. 80, n. 3, pp. 921–931, 1986. http://dx.doi.org/10.1121/1.393915.

[8] DUTRA, M.A.M., *"Avaliação acustoelástica do aço 20 MnMoNi 55, material estrutural do vaso de pressão dos reatores nucleares de Angra II e Angra III"*. Dissertação (Mestre em Ciências), Instituto de Engenharia Nuclear, Rio de Janeiro, RJ, 2009.

[9] TIMOSHENKO, S.P., Strength of materials. 2nd ed., New York: D. Van Nostrand Company, Inc., 1949.

Erratum

In the article "On the precision and accuracy of the acoustic birefringence technique for stress evaluation", with the DOI code number: https://doi.org/10.1590/1517-7076-RMAT-2022-0146, published at Matéria (Rio de Janeiro) 27(3):e20220146:

On page 2 where it was written:
These statistical concepts and their use here are briefly reviewed in the following subsections [4].

2.1.1. Statistical errors: basic concepts

Consider a sample of N direct measurements of a variable x {x1, x2, ..., xN }, the standard estimate for the expected result of the measurement is […]

If the number of measurements N is not sufficiently large (which is usually the case at the experiments carried out at IEN's ultrasonic laboratory), […]

It should read:
These statistical concepts and their use here are briefly reviewed in the following subsections [4, 5].

2.1.1. Statistical errors: basic concepts

The result of the measurement of a quantity should be given by the best estimate of its expected value and the uncertainty associated with this estimate. The best estimate for the expected value of a quantity from a sample of N direct measurements of a variable x {x1, x2, ..., xN }, is the mean or average value of these measurements, and the uncertainty associated with this estimate is the standard deviation of this mean or average value.

Thus, the result of the measurement of a quantity is defined by, […]

If the number of direct measurements N is not sufficiently large for its standard deviation to be an adequate estimate of the standard deviation of the totality of the possible measurements (which is usually the case at the experiments carried out at IEN's ultrasonic laboratory), […]

On page 3, equation 8 where it was written:

$$\sigma_{\bar{x}} = \frac{s_x}{\sqrt{N}} = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \overleftrightarrow{\overline{x}})^2}{N(N-1)}}$$

It should read:

$$\sigma_{\bar{x}} = \frac{s_x}{\sqrt{N}} = \sqrt{\sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N(N-1)}}$$

On page 4 where it was written:
(see subsection 5.1). Thus, […]

It should read:
(see subsection 2.3.1). Thus, […]

On page 4, equations 20 where it was written:

$$\phi = f(X) = f(\bar{X}) + \sum_{k=1}^{M} \left(\frac{\partial f}{\partial x_k} \right)_{\bar{X}} (x_k - \bar{x}_k)$$

It should read:

$$\Phi = f(X) = f(\bar{X}) + \sum_{k=1}^{M} \left(\frac{\partial f}{\partial x_k} \right)_{\bar{X}} (x_k - \bar{x}_k)$$

On page 4, equations 21 where it was written:

$$\bar{\phi} = f(\bar{X})$$

It should read:

$$\bar{\Phi} = f(\bar{X})$$

On page 4, equations 22 where it was written:

$$\sigma_{\phi} = \sqrt{\sum_{k,l}^{M} \left(\frac{\partial f}{\partial x_k} \right)_{\bar{X}} V_{kl} \left(\frac{\partial f}{\partial x_l} \right)_{\bar{X}}}$$

It should read:

$$\sigma_{\Phi} = \sqrt{\sum_{k,l}^{M} \left(\frac{\partial f}{\partial x_k} \right)_{\bar{X}} V_{kl} \left(\frac{\partial f}{\partial x_l} \right)_{\bar{X}}}$$

On page 4, equations 24 where it was written:

$$\sigma_{\bar{\phi}} = \frac{\sigma_{\phi}}{\sqrt{N}}$$

It should read:

$$\sigma_{\bar{\Phi}} = \frac{\sigma_{\Phi}}{\sqrt{N}}$$

On page 5 where it was written:

$$\bar{B} = f(\bar{t}_{31}, \bar{t}_{32}) = 2\left(\frac{\bar{t}_{31} - \bar{t}_{32}}{\bar{t}_{31} + \bar{t}_{32}} \right)$$

It should read:

$$\bar{B} = f(\bar{t}_{31}, \bar{t}_{32}) = -2\left(\frac{\bar{t}_{31} - \bar{t}_{32}}{\bar{t}_{31} + \bar{t}_{32}} \right)$$

On page 5 where it was written:
The *experimental precision* is characterized by *relative error* given by the ratio between the standard deviation of the mean (Eq. 8) and the absolute value of the mean (Eq. 4), whereas the *experimental accuracy* is defined by the ratio between the standard deviation of the mean (Eq. 8) and the […]

It should read:
The *experimental precision* is characterized by *relative error* given by the ratio between the standard deviation of the mean (Eq. 8) and the absolute value of the mean (Eq. 4), whereas the *experimental accuracy* is the relative error obtained by dividing the absolute value of the difference between the principal stress difference (Eq. 27) and the reference value Xref by the […]

On page 5 where it was written:
In the *continuous technique*, an initial direction aligned with one of the symmetry axes is chosen (say direction 1) and kept fixed while a sequence of *N* (usually five to ten) ultrasonic shear waves are generated and polarized […]

I should read:
In the *continuous technique*, an initial direction aligned with one of the symmetry axes is chosen (say direction 1) and kept fixed while a sequence of *N* (usually five to ten) ultrasonic shear waves is generated and polarized […]

On page 5, equation 29 where it was written:

$$PRECISION = \frac{\sigma_x}{|\bar{x}|}$$

It should read:

$$PRECISION = \frac{\sigma_{\bar{x}}}{|\bar{x}|}$$

On page 5, equation 30 where it was written:

$$ACCURACY = \frac{\Delta\bar{T}}{|x_{REF}|}$$

It should read:

$$ACCURACY = \frac{|\Delta\bar{T} - X_{REF}|}{|X_{REF}|}$$

On page 5 where it was written:
The waves' time-of-flight is determined from their electronic signals using the mathematical techniques of cross correlation and data interpolation [5].

It should read:
The waves' time-of-flight is determined from their electronic signals using the mathematical techniques of cross correlation and data interpolation [6].

On page 6 where it was written:
For stress states in which the principal stresses are not aligned with the material symmetry axes, a modified version of Eq. (2) must be employed [1,6].

It should read:
For stress states in which the principal stresses are not aligned with the material symmetry axes, a modified version of Eq. (2) must be employed [1,7].

On page 6 where it was written:

2.4. MATERIAL CHARACTERIZATION

It should read:

2.3.1. Material characterization

On page 6 where it was written:
The direction of applied loading should coincide with one of the material symmetry axes 1 or 2, but the specific choice may affect the parameters' results and deserves further study [7]:

It should read:
The direction of applied loading should coincide with one of the material symmetry axes 1 or 2, but the specific choice may affect the parameters' results and deserves further study [8]:

On page 6 where it was written:
The number of significant figures to be retained in the results is based on the number of significant digits and relative error (precision) of the input data and on the standard deviation of the mean of the computed variables [5].

2.5. Estimation of the principal stress difference

With the material parameters characterized, Eq. (3) can be applied in reverse order to estimate the principal stress difference in selected points of a structure under loading:

a) First select the technique (*continuous* in this work) for propagating the ultrasonic shear waves and acquire and treat the data (the waves' time-of-flight) accordingly (Section 4) to obtain the expected value of the birefringence $\overline{B}$ and its uncertainty $\sigma_{\overline{B}}$ (Eq. 25 and Eq. 26);
b) Apply Eq. 2 in reverse order to estimate the expected value of the principal stress difference $\Delta\overline{T} = (T_1 - T_2)$ and its uncertainty; $\sigma_{\Delta T}$ (Eq. 27 and Eq. 28);
c) Determine the experimental precision and accuracy (if a reference solution is available) of the result according to the Section 3.4.

It should read:
The number of significant figures to be retained in the results is based on the number of significant digits and relative error (precision) of the input data and on the standard deviation of the mean of the computed variables [6].

2.3.2. Estimation of the principal stress difference

With the material parameters characterized, Eq. (2) can be applied in reverse order to estimate the principal stress difference in selected points of a structure under loading:

d) First select the technique (*continuous* in this work) for propagating the ultrasonic shear waves and acquire and treat the data (the waves' time-of-flight) accordingly to obtain the expected value of the birefringence $\overline{B}$ and its uncertainty $\sigma_{\overline{B}}$ (Eq. 25 and Eq. 26);
e) Apply Eq. 2 in reverse order to estimate the expected value of the principal stress difference $\Delta\overline{T} = (T_1 - T_2)$ and its uncertainty; $\sigma_{\Delta T}$ (Eq. 27 and Eq. 28);
f) Determine the experimental precision and accuracy (if a reference solution is available) of the result using Eq. 29 and Eq. 30.

On page 7 where it was written:
Applying the procedure indicated in Section 5.1.1 for the continuous technique, [...]

It should read:
Applying the procedure indicated in Section 2.2 for the continuous technique, [...]

On page 8, Table 2 header, where it was written:
5000 Kgf
10000 Kgf
15000 Kgf
20000 Kgf
25000 Kgf
30000 Kgf

It should read:
5,000 Kgf
10,000 Kgf
15,000 Kgf
20,000 Kgf
25,000 Kgf
30,000 Kgf

On page 8, section 3.2., where it was written:
Applying the continuous technique for the stress estimation (Section 5.2), …
… were used instead of the average value indicated in Table 5.
... and Eq. (28) as

It should read:
Applying the continuous technique for the stress estimation (Section 2.2), …
… were used instead of the average value indicated in Table 3.
... and Eq. (28) as indicated in Table 5.

On page 8, section 3.3., where it was written:
… are given by the elementary theory of the strength of materials [8] as: ...

It should read:
… are given by the elementary theory of the strength of materials [9] as: ...

On page 9, Table 4 header, where it was written:
42000 Kgf

It should read:
42,000 Kgf

On page 9, Table 4, line “Mean (birefringence)”, where it was written:
-0,0023
0,0018
0,0008
-0,0021

It should read:
-0.0023
0.0018
0.0008
-0.0021

On page 9, Table 4, line “Uncertainty[a]”, where it was written:
0,0001
0,0001
0,0001
0,0002

It should read:
0.0001
0.0001
0.0001
0.0002

On page 9, Table 5, where it was written:

Table 5: Principal stress difference at points C1 e C5 (birefringence technique).

POINT	m (mm²/Kgf)	B_0	$\bar{B}$	ΔT (=$T_1 - T_2$) (Kgf/mm²)	RELATIVE ERROR (%)
C1	–0.000230 ± 0.000002	–0.0023 ± 0.0001	–0.0018 ± 0.0001	–17,8 ± 0.6	3,37
C5	–0.000230 ± 0.000002	0.0008 ± 0.0001	0.0021 ± 0.0002	12,6 ± 1.0	7,94

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C5	–0.000230 ± 0.000002	0.0008 ± 0.0001	–0.0021 ± 0.0002	12.6 ± 1.0	7.94

On page 9, Table 6, column "APPLIED LOAD (Kgf)" where it was written:
42000

It should read:
42,000

On page 10, section 4, where it was written:
This will require the modification of the equation relating the principal stress difference and the birefringence as discussed by THOMPSON et al. [6].

It should read:
This will require the modification of the equation relating the principal stress difference and the birefringence as discussed by THOMPSON et al. [7].

On page 10, section "Bibliography" where it was written:
…

[5] BITTENCOURT, M.S.Q., "*Desenvolvimento de um sistema de medida do tempo decorrido da onda ultrasônica e análise do estado de tensões em materiais metálicos pela técnica da birrefringência acústica*". Tese (Doutorado em Ciências), Universidade Federal do Rio de Janeiro, Rio de Janeiro, 2000.

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[8] TIMOSHENKO, S.P., Strengthen of materials. 2nd ed., New York: D. Van Nostrand Company, Inc., 1949.

It should read:

…

[5] TAYLOR, J. R., Error analysis: the study of uncertainties in physical measurements. 2nd ed., University Science Books, 1997.

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