Volume 6, Number 6 (December 2013) p. 875-902 • ISSN 1983-4195

IBRACON

REVISTA IBRACON DE ESTRUTURAS E MATERIAIS
IBRACON STRUCTURES AND MATERIALS JOURNAL

Comparative analysis between prediction models in codes and test data for shear strength

Análise comparativa entre modelos de predição de norma e dados de ensaios na determinação da resistência ao cisalhamento

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Abstract

Since the beginning of twentieth century, along with academic publications of Ritter and Mörsch, several studies have been done in order to understand shear strength in reinforced concrete elements. Approximately 1,200 laboratory tests results of reinforced concrete beams under shear stresses were used in a comparative analysis among values from prediction models of codes and laboratory tests results, enabling classification of the codes according to their applicability in several tests intervals. Although the Brazilian Code NBR 6118 (2007) showed good results in usual ranges of parameters, it presented unsatisfactory results on the following cases: low and medium shear transverse reinforcement rate.

Keywords: shear design, shear strength, standards comparison, standards applicability.

Resumo

Desde o início do século XX, com as publicações de Ritter e Mörsch, diversos modelos de cálculo foram desenvolvidos para tentar avaliar o valor da força cortante resistente em elementos em concreto armado. Com um banco de dados de cerca de 1.200 resultados de ensaios de laboratório de vigas de concreto armado, solicitadas por esforços de cisalhamento, efetuou-se a análise comparativa entre os valores de predição das principais normas e os resultados de ensaios, permitindo qualificar o modelo de predição das normas quanto sua aplicabilidade em diversos intervalos de ensaios. O modelo de predição da norma brasileira NBR 6118 (2007) [1] apresentou resultados satisfatórios nos intervalos usuais dos parâmetros, porém pouco satisfatórios para elementos com média e baixa taxa de estribos.

Palavras-chave: dimensionamento ao cisalhamento; resistência ao cisalhamento; comparação entre normas; aplicabilidade das normas.

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Received: 26 Feb 2013 • Accepted: 02 Oct 2013 • Available Online: 12 Dec 2013

1. Introduction

In several studies a discrepancy in noticed when laboratory results are compared to analytical values of reinforced concrete's shear strength. There are several phenomena that contribute to the behavior of reinforced concrete under tangential and axial loads which contribute to the previously mentioned discrepancy in estimating similar values to laboratory tests.

Leonhardt [2] presents a list of 21 factors that influence the shear strength of reinforced concrete elements, some with direct influence and others, indirect. Thus, creating a formulation considering the most significant factors becomes a complex activity, given the elevated number of factors that may influence the determination of shear strength.

Therefore, it is important to assess the formulations used within certain parameters ranges and especially in particular geometries.

2. Methodology

2.1 Determination on Codes prediction values

The results of Code's prediction models were obtained from electronic spreadsheets designed by the author, as well as the charts. The obtained data enabled the author to assess the influence of various Codes models when confronted to experimental tests.

This study used four Codes: ACI 318 (2008) [3]; CSA A23.3 - 04 (2004) [4]; EUROCODE 2 (2004) [5] and NBR 6118 (2007).

The prediction models of ACI, EUROCODE and NBR have different formulations to evaluate the resistance values for reinforced concrete elements due to strut under ultimate compression strength, described as shear-compression failure by Fusco [6]. The same models have different formulations for elements with and without stirrups under ultimate tensile strength, described as shear-tensile failure by Fusco [6]. CSA Code presents a single formulation for both shear-compression and shear-tensile failures.

Table 1 summarizes the formulations of the studied models, whereas the shear stress, τ, is represented by the ratio of shear force, V, and the section's effective area $b_w.d$.

In order to obtain the maximum shear strength according to EUROCODE and NBR's Model II, there were created some optimization scripts that vary the strut's angle. Limitation to maximum angle reduction was according to each Code, to strut's ultimate compression strength, and to bending reinforcement.

2.2 Database description

The database comprises a compilation of 1,235 laboratory tests results on reinforced concrete beams, being 547 reinforced concrete beans with stirrups and 688, without. This paper focus on reinforced concrete beams with stirrups. All tested beams presented longitudinal bars (for bending moment) and were perpendicularly loaded to them longitudinal axis. The loading can be a single concentrated load in mid span, equally spaced concentrated loads, or a knife-edge load along the entire beam.

The failure modes were separated in groups so each ultimate limit state could be represented.

The test data summary used in this paper is shown in Table 2, which contains results for elements with shear reinforcement. These results are related to shear force failure.

Table 1 – Prediction Models Equations in order to obtain ultimate shear strength

Codes	ULS Equations			Comments
	ULS shear-compression $\tau_{rd,c}$ (MPa)	ULS shear - tensile $\tau_{rd,t} = \tau_c + \tau_{sw}$ (MPa)		
		τ_c (MPa)	τ_{sw} (MPa)	
NBR 6118 Model I	0,27.(1-fc/250).fc	$\tau_c=\tau_{c0}=0{,}126.fc^{2/3}$	$0{,}9.\rho_w.f_y$	q=45°, fc ≤ 50 MPa
NBR 6118 Model II	0,54.(1-fc/250).fc.sen(2θ)	τ_{c0}.(τrd,c-τsd)/(τrd,c-τc0) <t_{c0}	$0{,}9.\rho_w.f_y$.cotgθ	30° ≤ q < 45°, fc ≤ 50 MPa
EUROCODE 2	0,45.n1.fc.sen(2θ)	0	$0{,}9.\rho_w.f_y$.cotgθ	n1 = 0,6, for fc ≤ 60 MPa n1 = 0,9-fc/200 > 0,5, for fc > 60 MPa 1 ≤ cot q ≤ 2,5
CSA	0,25.fc	$\beta.fc^{1/2}$	$0{,}9.\rho_w.f_y$.cotgθ	β e θ are obtained by simple equations as a function of axial strain fc ≤ 64 MPa
ACI 318	0,83.fc1/2	$0{,}17.fc^{1/2}$	$\rho_w.f_y$	–

Where: fc=concrete compression strength (MPa); ρ_w=transverse reinforcement ratio; fy=steel tensile strength (MPa) ϕ=angle of inclination of compression concrete strut (°); $\tau_{rd,c}$=shear strength of concrete strut (MPa); $\tau_{rd,t}$=shear - tensile strength (MPa); τ_c: shear strength provided by concrete (MPa); τ_{sw}: shear resistance provided by steel stirrups (MPa).

2.3 Analysis criteria

After selecting some database results, it was possible to assess and compare those results to the Codes' prediction models.
This paper names as "accepted" all the parameters used within an established range by a given Code. On the other hand, "denied" defines a parameter which is found outside the range under analysis. Thus, using parameters outside the valid range would be, for example, to calculate a predictive value using a code resistance value of concrete above the maximum allowed.

This paper considered that the section strength obtained throughout the test is the ultimate strength.
Illustrated in Table 3, the ratio between the laboratory's ultimate value, V_{exp}, and ultimate analytical value, V_u is shown as V_{exp}/V_u. In general terms, V_u comprises the influence of concrete's shear strength (V_c) and stirrup's strength (V_s). Thus, $V_u = V_c + V_s$.
This paper names the shear strength as V_d, concrete's partial safety coefficient as Φ_c, and Φ_s for stirrups' partial safety coefficient. The ultimate safety coefficient will be stated as Φ. For elements with stirrups, ultimate safety factor, Φ depends on the weighing between V_c and V_s. For example, elements with high stirrups rate, Φ

Table 2 – Database for RC beams with stirrups (part 1)

Author	Number of tests	d (cm)	f_c (MPa)	ρ_l %	ρ_{sw} %	a/d	$\rho_{sw.fyk}$ (MPa)
Anderson, N. S. Ramirez, J. A.(7)	13	34,5 - 42,53	28,69 - 42,76	2,31 - 2,65	0,39 - 0,53	2,15 - 2,65	2,14 - 2,83
Angelakos, D.; Bentz, E. C.; Collins M. P. (8)	6	92,50	21 - 80	0,5 - 1,01	0,08	2,92	0,40
Bahl (1968)[a]	4	30 - 120	25,1 - 26,8	1,26	0,15	3,00	0,66
Bresler ; Scordelis (9)	10	39 - 46,61	23,17 - 38,75	1,8 - 3,66	0,1 - 0,38	2,35 - 6,98	0,33 - 1,26
Bresler ; Scordelis (1966)[a]	22	45,67 - 46,25	23,17 - 26,75	1,67 - 2,34	0,1 - 0,21	3,95 - 4,01	0,35 - 0,7
Cho, S. H. (10)	24	21,50	52 - 73	3,77	0,2 - 1,8	1,5 - 2,5	0,78 - 6,98
Cladera, A.; Marí, A. R. (11)	11	35,1 - 35,3	49,9 - 87	2,28 - 2,99	0,11 - 0,24	3,06 - 3,08	0,58 - 1,29
Clark, A. P. (1951)[b]	51	31,37 - 39,4	13,79 - 87	1,63 - 3,42	0,24 - 1,22	1,16 - 3,08	1,13 - 4,04
Collins e Kuchma (12)	4	45,9 - 92	71 - 75	1,03 - 1,36	0,13 - 0,16	2,5 - 2,72	0,65 - 0,8
Debaiky, S. Y.; Elniema, E. I. (13)	9 2	26,00	17,23 - 31,4	1,93 - 3,02	0,2 - 0,42	1,92 - 3,46	0,63 - 1,33
Elstner, Moody, Viest, Hognestad (1955)[b]		30,50	23 - 24	4,76	0,95 - 1,47	2,00	2,98 - 4,81
Elzanaty, Nilson e Slate (14)	3	26,60	20,7 - 62,8	2,5 - 3,3	0,17	4,00	0,65
Fernandes, G. B. (15)	5	28,00	61,1 - 78,5	4,1 - 6,18	0,25 - 0,38	3,57 - 5,36	2,14 - 3,21
Fukuhara, Kokusho (16)	19	34 - 36	20 - 32	0,61 - 3,21	0,12 - 1,13	1,76 - 2,35	0,63 - 5,01
Guralnick (1960)[c]	9	30,6 - 31	17 - 38	1,41 - 4,38	0,24	2,95 - 2,99	1,26
Haddadin, Hong, Mattock (17)	22	38,10	13 - 44,9	3,79 - 7,58	0,19 - 1,26	2,5 - 6	0,68 - 4,77
Hsiung, W.; Frantz, G. C.(18)	4	41,91	43,00	1,82	0,21 - 0,22	3,00	0,62
Karayiannis e Chalioris (1999)[a]	8	26,00	26,00	1,47 - 1,96	0,04 - 0,25	2,77 - 3,46	0,11 - 0,64
Kokusho, Kobayashe, Mitsugi, Kumagai (1987)[a]	9	34,00	20 - 38	3,16	0,15 - 1	1,76	2,09 - 14,31
Kong, P. Y. L.; Rangan, B. V. (1998)[d]	43	19,8 - 54,2	63,6 - 89,4	1,66 - 4,47	0,1 - 0,26	1,75 - 3,3	0,6 - 1,49
Krefeld, W. J.; Thurston, C. W. (19)	20	45,57	15,73 - 48,49	2,22	0,06 - 0,16	4,02	0,21 - 0,64
Lee, Kim; Mansour (20)	4	24,4 - 26,4	42,00	2,67 - 3,6	0,22 - 0,32	2 - 4	0,79 - 1,15

[a] data obtained in Zararis (37); [b] data obtained in Reineck (38); [c] data obtained in Collins (39); [d] data obtained in Bentz (40)
[e] data obtained in Cladera (41); [f] data obtained in Bette (42)

Table 2 – Database for RC beams with stirrups (part 2)

Author	Number of tests	d (cm)	f_c (MPa)	ρ_l %	ρ_{sw} %	a/d	$\rho_{sw.fyk}$ (MPa)
Leonhardt; Walther (1962)[b]	4	27,00	28,2 - 30,4	2,47	0,41 - 0,59	2,78	1,52 - 1,63
Leonhardt; Walther (1961)[b]	1	82,50	23,84	9,44	2,83	3,03	11,71
Leonhardt; Walther (1963)[b]	1	27,00	28,16	2,02	2,02	0,58	9,39
Levi, F.; Marro, P. (1988)[d]	7	94,00	25 - 60	3,5 - 5,3	0,84 - 1,25	4,20	4,03 - 6
Lyngberg, B. S. (1976)[e]	2	54,00	25,7 - 26,6	3,88	0,53	2,78	3,43 - 3,57
Matsuzaki, Nakano, Watanabe (1990)[f]	8	33,60	23 - 37	2,88	0,19 - 1,18	1,79	1,29 - 8,59
Mattock, A. H.; Wang, Z. (21)	8	31,5 - 34	20 - 34,1	2,07 - 3,16	0,24 - 0,47	1,76 - 3	0,84 - 4,13
McGormley; Creary e Ramirez (1996)[a]	12	41,90	35,3 - 56,7	3,03	0,34	3,27	1,45
Moody, K. G.; Viest, I. M.; Elstner, R. C.; Hognestad, E.(1954)[d]	2	53,34	22,42 - 25,38	4,25	0,52 - 0,95	1,52	1,7 - 2,88
Moretto, O. (22)	5	46,4 - 49,5	23 - 33	3,99	0,27	1,64 - 1,75	0,85 - 1,02
Mphonde; Frantz (23)	12	29,80	22,1 - 83	3,36	0,12 - 0,38	3,60	0,35 - 1,03
Nishiura, Makitani, Shindou (1993)[f]	6	33,60	20 - 33	2,88	0,4 - 0,89	2,38	3,32 - 7,39
Ozcebe, G; Ersoy, U.; Tankut, T. (24)	13	31 - 32,5	58 - 82	1,93 - 4,43	0,14 - 0,28	3 - 5	0,35 - 0,71
Peng (25)	8	27,40	29,3 - 33,7	2,70	0,05 - 0,37	3,10	0,3 - 1,68
Piyamahant Songkramc	4	35,90	41,5 - 46,15	1,06	0,04 - 0,08	3,00	0,12 - 0,28
Placas, A.; Regan, P. E. (26)	44	25,4 - 26,4	12 - 57	0,98 - 4,16	0,14 - 0,84	3,36 - 7,2	0,38 - 2,25
Rajagopalan, K. S.; Ferguson, P. M. (27)	3	26,42 - 26,59	27,04 - 33,93	1,71 - 1,74	0,21 - 0,23	4,16 - 4,23	0,71 - 0,72
Rangan, B. V. (28)	4	56,30	30,2 - 36,5	8,35 - 9,81	1,53 - 3,19	2,49	7,42 - 15,47
Rodriguez, Bianchini, Viest, Kesler (1959) (29)	12	30,9 - 32,6	19 - 25	2,6 - 2,74	0,37 - 1,11	1,99 - 2,29	1,28 - 3,51
Roller, J. J.; Russell, H. G. (1990) (30)	10	55,88 - 76,2	72,42 -125,32	1,73 - 7,29	0,08 - 1,76	2,5 - 3	0,34 - 8,05
Sarsam, K. F.; Al-Musawi, J. M. S. (31)	14	23,2 - 23,5	39 - 80,1	2,23 - 3,51	0,09 - 0,19	2,5 - 4	0,76 - 1,53

[a] data obtained in Zararis (37); [b] data obtained in Reineck (38); [c] data obtained in Collins (39); [d] data obtained in Bentz (40)
[e] data obtained in Cladera (41); [f] data obtained in Bette (42)

approaches the value Φs. In the most general case, Φ is obtained as shown in Table 3.

NBR 6118 (2007) states that there are three main factors that reduces the element reliability when under shear loading: the variability of the strength of the materials, the difference between the trial tests and the effective structure, and deviations on construction site. Moreover, considerations on the kind of failure (brittle and ductile) and risks tolerance should be taken.

There a several considerations in order to guarantee the structures' reliability: construction execution detail, construction tolerances, material strength, maximum and minimum number of reinforcement bars, etc. These considerations differ from the reliability considerations on the Codes prediction models, which presents similar results to those obtained in laboratory tests. This paper only analyses the codes predictions models.

As stated in Table 3, the safety coefficients comprise two values. The first one, Φ_{mat}, is responsible for ensuring safety due to the variability and the possibility of a low material strength. The second, Φ_{mod}, is responsible for the security by the inaccuracy of the model representation.

Table 2 – Database for RC beams with stirrups (part 3)

Author	Number of tests	d (cm)	f_c (MPa)	ρ_l %	ρ_{sw} %	a/d	$\rho_{sw.fyk}$ (MPa)
Simplício (32)	5	27 - 35,4	69,3 - 73,5	2,33 - 2,96	0,1 - 0,22	3,3 - 3,8	0,75 - 1,54
Swamy e Andriopoulos (1970)[c]	10	9,5 - 13,2	25,9 - 29,4	1,97 - 3,95	0,06 - 0,6	3 - 5	0,17 - 1,33
Takagi, Okudeh, Nitta (1989)f	19	35,20	32 - 36	3,09	0,19 - 1,21	2,27	1,48 - 12,9
Tompos, E. J.; Frosch, R. J. (33)	4	42,55 - 85,09	35,8 - 42,7	1,00	0,08 - 0,15	3,00	0,41 - 0,72
Xie, Y. et. al. (34)	9	19,81 - 20,32	42,4 - 108,7	3,2 - 4,54	0,49 - 0,78	1 - 4	1,59 - 2,53
Yoon, Y. S.; Cook W. D.; Mitchell D. (35)	9	65,50	36 - 87	2,80	0,08 - 0,24	3,05	0,35 - 1,02
Zararis e Papadakis (1999) (36)	9	23,50	20,8 - 23,9	0,68 - 1,37	0,06 - 0,27	3,60	0,16 - 0,73
TOTAL	**547**	**9,5 - 120**	**12 - 125,32**	**0,5 - 9,81**	**0,04 - 3,19**	**0,58 - 7,2**	**0,11 - 15,47**

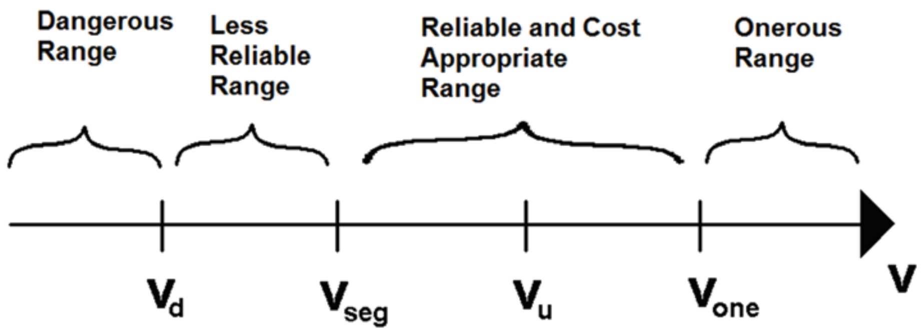
[a] data obtained in Zararis (37); [b] data obtained in Reineck (38); [c] data obtained in Collins (39); [d] data obtained in Bentz (40)
[e] data obtained in Cladera (41); [f] data obtained in Bette (42)

After, similar safety criteria among the Codes were defined based on the previously presented values, V_{exp}, V_u and V_d.
In order to assist the reliability analysis of the codes prediction models in determining the shear strength, a quality analysis based on the V_{exp}/V_u ratio within four ranges (as shown in Figure 1 and Table 4) was undertaken [43]. V_{seg} is obtained from $V_u * \Phi_{mod}$, being Φ_{mod} already described before. Indirectly related to monetary costs, V_{one}, defines another boundary. When a laboratory test gives a highly reliable value (i.e. above $1.1V_u$), it is considered an onerous situation in terms of wasting material.
The criteria analysis is based on determining four ranges to compare V_{exp}.
The first range indicates that if $V_{exp} < V_d$, the code prediction model is considered dangerous and induces to unreliable values and is likely to fail. The second range indicates that if $V_{exp} \leq V_d \leq V_{seg}$, the model presents a low reliability. The third range, $V_{seg} \leq V_{exp} \leq V_{one}$ defines an optimum range, because the closer to unity the ratio V_{exp}/V_d is, the greater the strength of materials will be (Φ_{mat} related). On the other hand, the greater the ratio V_{exp} / V_{seg} is, the more reliable the model is (Φ_{mod} related).
The fourth range defines if $V_{exp} > V_{one}$, there is a cost issue due to material waste. The security level is extremely higher than regular levels. This result induces to an elevated material consumption. The summary of the analysis intervals is expressed in Table 4.
The boundary values of the third range are to be defined. For beams with stirrups, it was necessary to determine Φ_{mod}'s average, which is considered as approximately equal to $\sqrt{\phi_{med}}$, being Φ_{med} the average of reducing strength coefficients of all Codes prediction models. From all the Codes [CSA (2004), Eurocode (2004), ACI (2008) and NBR 6118 (2007)], the partial safety coefficient

Table 3 – Equations

$Vd = \dfrac{V_c}{\gamma_c} + \dfrac{V_s}{\gamma_s}$	Determination of design shear resistance, using the division by γ
$Vd = \phi_c.V_c + \phi_s.V_s$	Determination of design shear resistance, using multiplication by Φ
$\phi = \dfrac{Vd}{V_u} = \dfrac{\phi_c.V_c + \phi_s.V_s}{V_c + V_s}$	Determination of ultimate safety factor, Φ
$\phi = \phi_{mat}.\phi_{mod}$	Coefficients of ultimate safety factor, Φ, being Φ mat: partial safety coefficient provided by the variability and the possibility of a low material strength and Φ mod: safety coefficient provided by the inaccuracy of the model representation
$\phi_{mat} \cong \phi_{mod} \cong \sqrt{\phi}$	Assumption used to define Φ mat and Φ mod

Figure 1 – Comparison criteria intervals for Test shear force, V_{exp}. V_d – Design Value, V_{seg} – Reliable limit value, V_u – Ultimate analytical value, and V_{one} – Onerous limit value



$$V_{seg} = \sqrt{\phi_{med}}.V_u$$

$$V_{one} = 1{,}1.V_u$$

was equal to 0.78, and the value of 0.90 to Φ_{mod} was adopted. As the analysis criteria should be the same for all codes, the partial safety factor Φ_{med} is to be determined as an average value of all Codes prediction models.
Thus, the four groups of Table 4 were defined for "reinforced concrete with stirrups cases" and are shown at Table 5.

3. Results

From the analysis criteria defined in previous section, charts were made indicating data percentage, V_{exp} / V_u, according to each given range, and show in Table 4 and Table 5.
For each equation from Codes' prediction models (V_d, V_{seg}, V_u e V_{one}) a comparison was made with a laboratory test result V_{exp}. Based on those results and on Table 4 analysis criteria, the percentage of data belonging to each range (first to fourth) was stated. Analysis charts were shown containing the four ranges in its respective codes.

3.1 Analysis of reinforced concrete beams with stirrups

In order to organize the findings will be presented results obtained in the analysis of the values of the predictions models of codes and experimental values for reinforced concrete elements with stirrup.

3.1.1 RC beam with stirrups, "valid" parameters and shear-tensile failure

All the parameters are considered within the Codes requirements.
Mechanical Ratio of Stirrups, psw.fyk, lower than 1 MPa
According to Figure 2, it is possible to consider:

- NBR 6118 (2007) Model I is less recommended, because it presented 2% of the cases in range 1 (considered less reliable).
- NBR 6118 (2007) Model II is less recommended as well, because 8% of the results are in range 1.
- EUROCODE (2004) prediction model presented a fair result, although 89% of the results are in range 4 (considered costly in monetary terms).
- ACI (2008) prediction models showed good results, being the recommended on among the previous three analyses, presenting 15% of results in range 3, and irrelevant values for ranges 1 and 4.
- CSA (2004) model had fair results, due to great results in range 3.

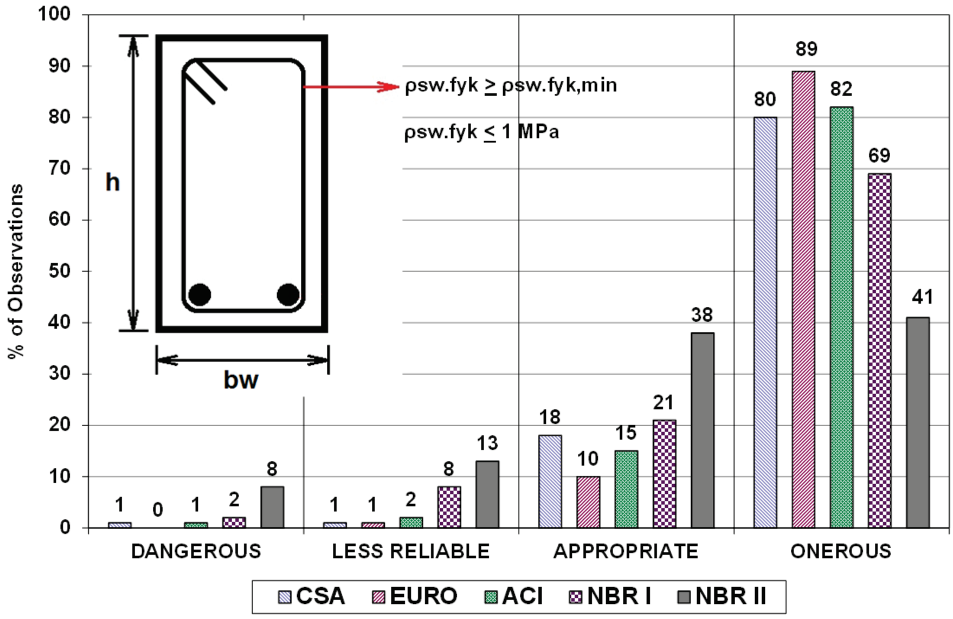
Table 4 – Intervals of comparison criteria

Comparison criteria	Description
$V_{Exp} < \phi_{med}.V_u$	Dangerous
$\phi_{med}.V_u \le V_{Exp} < \sqrt{\phi_{med}}.V_u$	Less reliable
$\sqrt{\phi_{med}}.V_u \le V_{Exp} \le 1{,}1.V_u$	Reliable and costly appropriate
$V_{Exp} > 1{,}1.V_u$	Onerous

Table 5 – Definition of the third interval of Table 4 for elements with stirrups

Elements	Comparison criteria	Description
With stirrup	$0{,}9.V_u \le V_{Exp} \le 1{,}1.V_u$	Reliable and costly appropriate

Figure 2 – Application of comparison criteria for $\rho_{sw}.f_{yk}$ ≤1 MPa, defined in this paper, for elements with valid parameters and stirrups



Mechanical Ratio of Stirrups, psw.fyk, greater than 1 MPa and lesser than 2 MPa

■ According to Figure 3, EUROCODE (2004) model presented unsatisfactory results due to its unreliability, presenting 12% of

Figure 3 – Application of comparison criteria for $\rho_{sw}.f_{yk}$ between 1 and 2 MPa, defined in this paper, for elements with valid parameters and stirrups

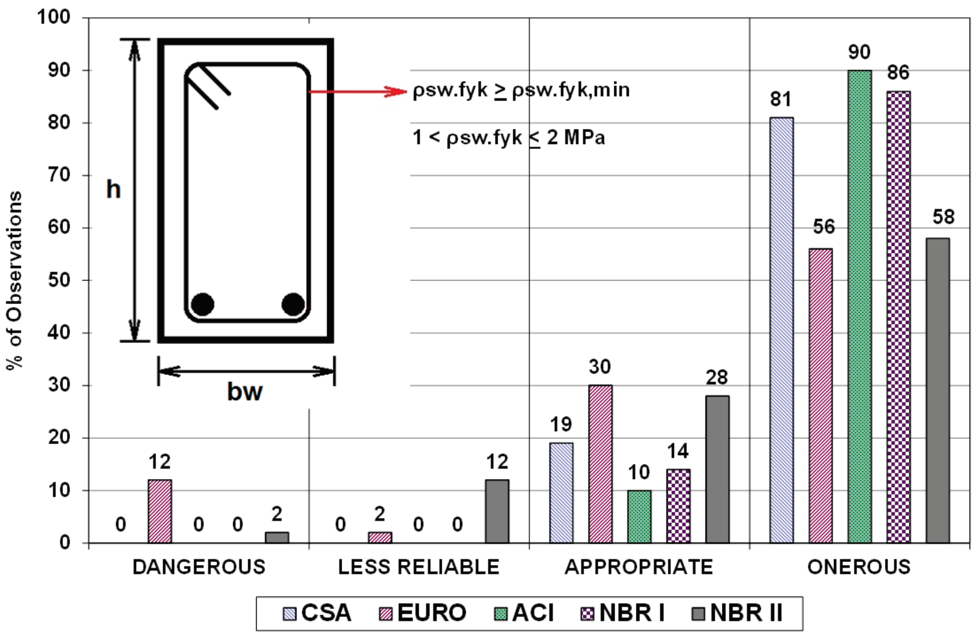
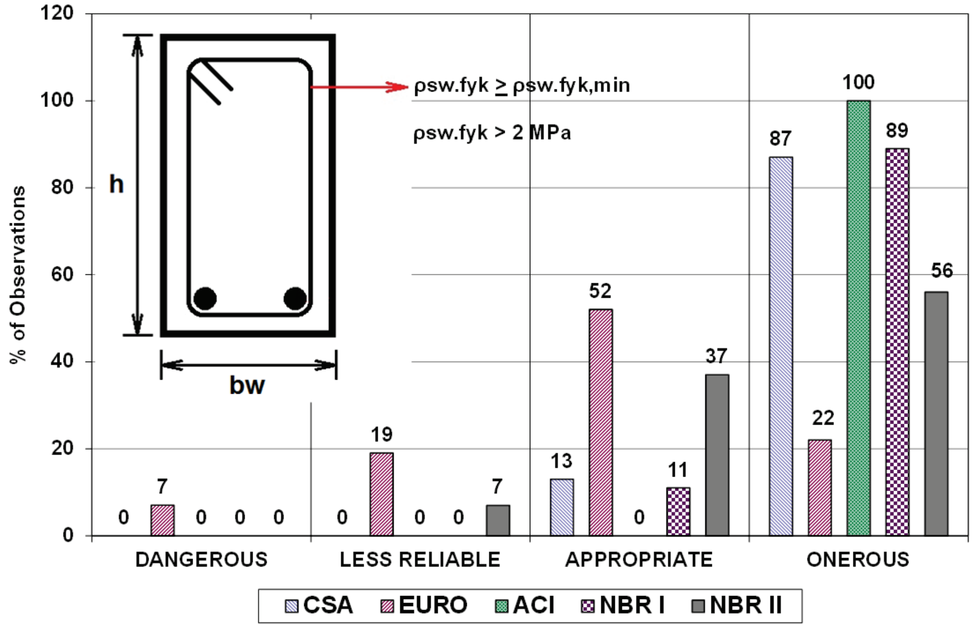


Figure 4 – Application of comparison criteria for $\rho_{sw}.f_{yk}$ greater than 2 MPa, defined in this work, for elements with valid parameters and stirrups



the case scenarios within range 1.

- As NBR 6118 (2007) Model II presented 2% within range 1, it was considered less recommended.
- NBR 6118 (2007) Model I presented 86% of results within range 4. Although onerous monetarily, its use is recommended with caution.

Figure 5 – Distribution of models results, according to the comparison criteria for elements with stirrups and mechanical reinforcement ratio less than the minimum (invalid range)

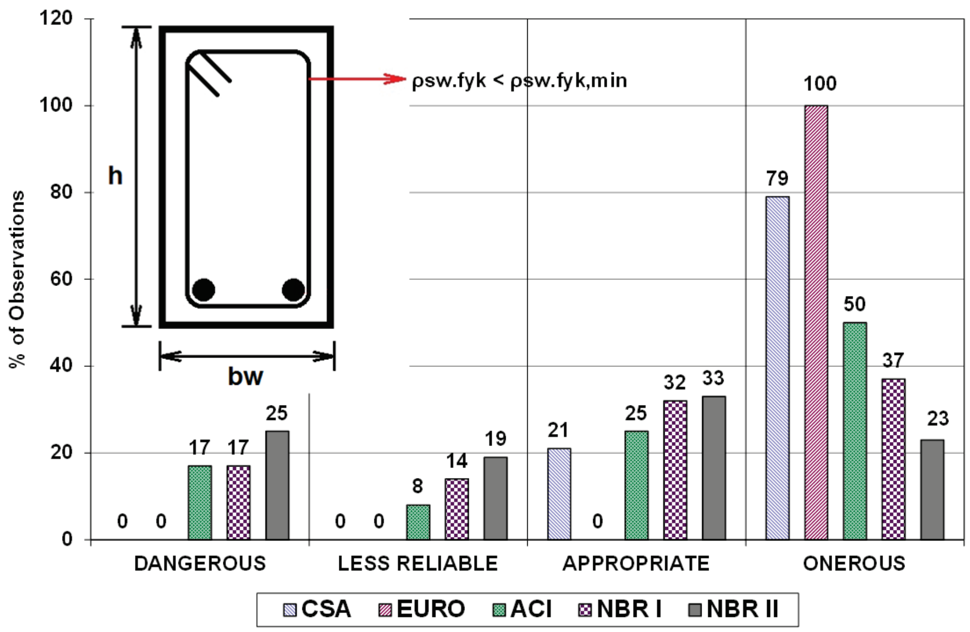


Table 6 – Database used for beams with stirrup elements under axial stress

Author	Number of tests	b_w (cm)	d (cm)	f_c (MPa)	ρ_l %	ρ_{sw} %	a/d	σ_n (MPa)*	$\rho_{sw.fyk}$ (MPa)
Mattock and Wang (1984)	11	15,0	31,5	24,5 - 26	2,61	0,23 - 0,46	2,87	-12 a 0	0,83 - 1,66
Haddadin, Hong and Mattock (1971)	21	17,8	38,0	15,5 - 30,2	3,81	0,19 - 1,22	2,5 - 4,25	-5 a 3	0,68 - 4,78
TOTAL	**32**	**15 - 17,8**	**31,5 - 38,1**	**15,5 - 30,2**	**2,61 - 3,81**	**0,19 - 1,22**	**2,5 - 4,25**	**-12 a 3**	**0,68 - 4,78**

* Compression < 0 e Tensile > 0

- ACI (2008) model is recommended for this case scenario, although there are 90% of the results in range 4 and 10% in range 3.
- The calculation model I of NBR 6118 (2007) presented 86 % of the predictions considered onerous , although safe. Therefore , its use is recommended with caution.
- CSA (2004) models presented no results in ranges 1 and 2, 19% in range 3 and 81% in range 4, being considered the most recommended for this case scenario.

Mechanical Ratio of Stirrups, psw.fyk, greater than 2 MPa

- According to Figure 4, EUROCODE Prediction model showed unsatisfactory results in terms of reliability, due to a 7% rate in range 1.
- ACI (2008) model present a full rate inside range 4. Although onerous, it was considered a fair result.
- NBR 6118 (2007) presented 89% inside range 4, being also considered a fair result. The code model of CSA (2004) showed no results within ranges 1 and 2, 19% in range 3 and 89% in range 4.
- NBR 6118 (2007) presented satisfactory results with 37% in range 4 and 56% in range 3, being recommended its use.

3.1.2 RC beam with stirrups, "denied" parameters and shear-tensile failure

Mechanical Ratio of Stirrups, psw.fyk, lesser than Codes minimum requirement, psw.fyk,min

According to Figure 5, NBR 6118 (2007) Model I is less recommended, because there is 25% of the results within range 1. Model II is also less recommended, presenting 17% of the results within the same range.

ACI (2008) models presented 17% of results in range 1, being considered less recommended as well.

EUROCODE (2004) models presented 100% of results inside range 4, being all reliable, although costly.

Figure 6 – Reliability analysis for cases under compression, for elements with stirrups

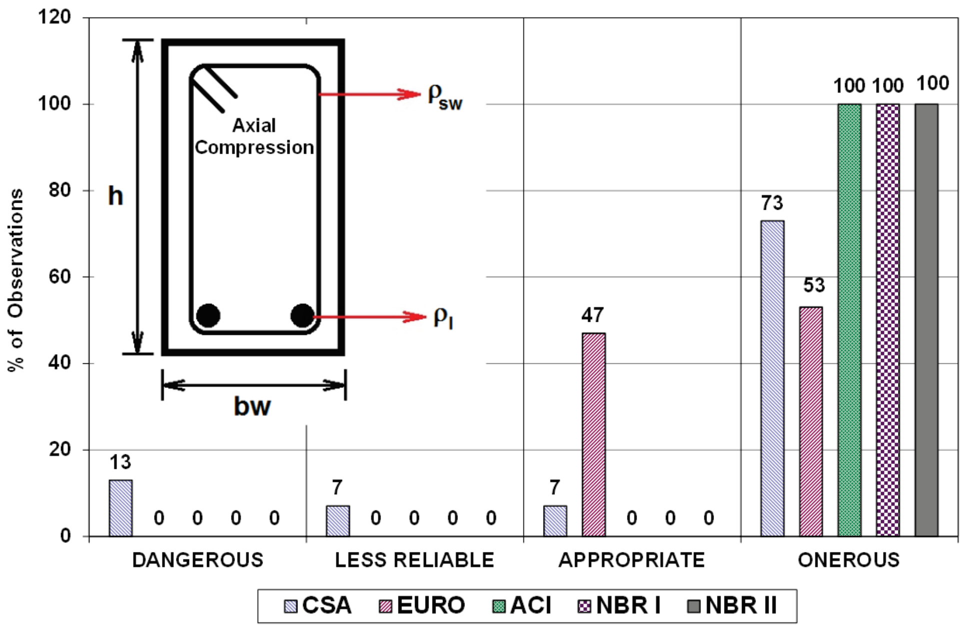
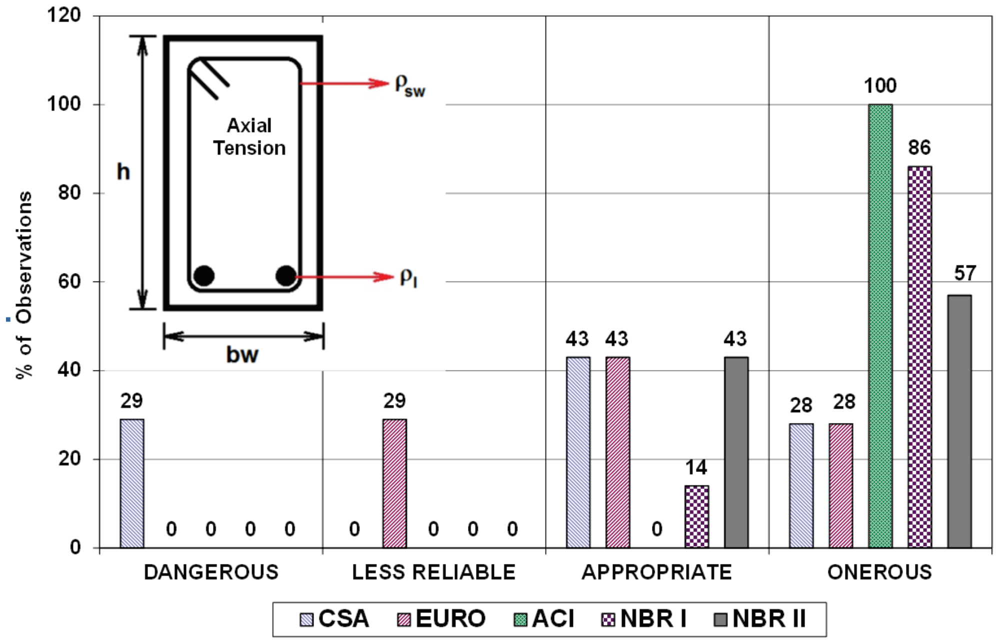


Figure 7 – Reliability analysis for cases under tension, for elements with stirrups



Considered recommended to use, CSA (2004) models presented 79% of results inside range 4, 21% in range 3 and no results in range 1 or 2.

3.1.3 RC Beams with stirrups under axial loading and shear-tensile failure

The summary of the database is presented in Table 6.

RC under axial compression

Figure 6 illustrates the reliability of reinforced concrete elements with stirrups under axial compression. CSA (2004) was considered less recommended because it presented 13% of the results in range 1.

NBR 6118 (2007) Models I and II, as well as ACI (2008) model presented 100% of their results within range 4, being considered reliable, although onerous. Thus, these models have been qualified for use as recommended with caution due to higher costs.

EUROCODE (2004) models showed 53% of results in range 4, while 47% was considered appropriate (range 3). Thus, it was considered recommended for use in these conditions.

RC Beams under axial tensile loading

Figure 7 represents this scenario.

While CSA (2004) models presents 29% of results in range 1, EUROCODE (2004) presented the same amount in range 2. Since element under axial tensile loading is an issue when designing RC

Table 7– Qualification of prediction's models, for safety use, for elements with stirrups

	Elements with stirrups					
	Valid ranges			Invalid ranges	Axial stress	
	$\rho_{sw}.f_{yk} \leq 1$	$1 < \rho_{sw}.f_{yk} \leq 2$	$\rho_{sw}.f_{yk} > 2$	$\rho_{sw}.f_{yk} \leq \rho_{sw}.f_{yk,min}$	Compression	Tensile
CSA	R	R	CC	R	LS	LS
EURO	CC	LS	LS	CC	R	LS
ACI	R	CC	CC	LS	CC	CC
NBR I	LS	CC	CC	LS	CC	R
NBR II	LS	LS	R	LS	CC	R

Obs: The mechanical ratio of stirrups, $\rho_{sw}.f_{yk}$, are expressed in MPa

Table 8 – Abbreviations used to qualify prediction models in code regarding its use

Qualification regarding the use	Abbreviations	Criteria*
Recommended	R	–
Less recommended for safety reason	LS	% Dangerous > 1%
Caution/Cost	CC	% Onerous ≥ 85%
Caution/Safety	CS	% Less Reliable ≥ 10%

* Criterion for guidance

structures, ACI's results were also considered less recommended. ACI (2008) models showed 100% of results within range 4.
NBR 6118 (2007) Models I and II presented the best results for this scenario with, respectively, 86% and 57% within range 4. There was no percentage within ranges 1 and 2 and respectively 14% and 43% of results in range 3. Both models are recommended for use in such conditions.

4. Conclusions

Table 7 presents a summary of all analyzes performed in this paper, based on analysis criteria, explained in previous sections. In this table there are conclusions on reliability of RC concrete beams with stirrups.
In order to clarify, acronyms are indicated and properly described in Table 8.
The nomenclature "less recommended for safety reasons", "NR", was used on each Code that presented more than 1% of results within range 1.
Defined as Caution / Cost, "CC" is named for all the cases that presented at least more than 85% within range 4. In this case, caution is recommended because although the structure can be reliable, elevated costs can be considered for that design.
Caution/Safety, "CS", indicates all the codes with more than 10% of results in range 2, because some results can influence a decrease in certain safety coefficients factors.
Since the nomenclatures are presented and described, final conclusions can be made in the following paragraphs.
In order to obtain the maximum value for ultimate shear strength, all codes equations were taken into account so this could be done. Example given, when using NBR 6118 (2007) Model II, in order to obtain the ultimate shear strength, the minimum strut angle took into account considerations on concrete's strut crushing and yielding phenomena on bending reinforcement.
The term "safe" used on "Caution/Safety" is not related to Codes safety. When this paper recommends not to use a certain Code due to lack of safety, it is related to the prediction models reliability.
The analysis criteria presented in this paper, all the laboratory results from the database and the prediction models results, helped to organize in a simple and clear way different manners of designing a RC beams under shear loading.
Table 9 indicates which models are recommended and less recommended for designing purposes. The recommended Codes are those which presented safe prediction results and, possibly, less costly when considering material consumption. Also, there can be found in Table 9 the conclusions for all the analysis on a RC beam with stirrups within the four ranges.
Table 9 shows that CSA (2004) models are recommended in 2 out of 5 scenarios; NBR 6118 (2007) are the most recommended in 2 cases, and finally, ACI (2008) and EUROCODE (2004) are recommended for one case each.
NBR 6118 (2007) models were less recommended for mechanical ratio of stirrups lesser than 1MPa. Model II was the least recommended for ratios lesser than 2MPa. For both models, the use of minimum transverse reinforcement must be taken into account.
CSA (2004) models, based on the Modified Compression Field Theory, showed for elements without axial loading the best average of cases defined as recommended for use. For members under compression load-

Table 9 – Prediction models recommended and less recommended for analysis of elements with stirrups, being: CSA - CSA (2004), EURO - Eurocode (2004), ACI - ACI (2008), NBR I - I model the NBR 6118 (2007) NBR and II - II model of NBR 6118 (2007)

Qualification of the prediction model	Elements with stirrups				
	Valid ranges			Axial stress	
	$\rho_{sw}.f_{yk} \leq 1$	$1 < \rho_{sw}.f_{yk} \leq 2$	$\rho_{sw}.f_{yk} > 2$	Compression	Tensile
Most recommended	CSA/ACI	CSA	NBR II	EURO	NBR I e NBR II
Less recommended	NBR I/NBR II	EURO/NBR II	EURO	CSA	CSA/EURO

Obs: The mechanical ratio of stirrups, $\rho_{sw}.f_{yk}$, are expressed in MPa ing EUROCODE (2004) models showed good results, while the most recommended models under tensile loadings were NBR 6118 (2007). It is noticeable that when the strut's angle on Model II is equal to 39º, the results are similar to those calculated by Model I. Therefore, as long as there is a limitation on 39º on strut angle, both models can give the same results.

5. Acknowledgments

Acknowledgements to Professor Maria Elena Santos Taqueda.

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