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Concrete compressive characteristic strength analysis of pile caps with three piles

Análise da resistência característica à compressão do concreto em blocos sobre três estacas

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Abstract

In this paper a numerical analysis of three-pile caps is developed to study the influence of concrete compressive characteristic strength in pile caps resistance capacity. A three-pile cap model derived from Miguel's [1] work was adopted. From this model, variations on the compressive characteristic strength were made in order to observe modifications in its structural behavior. The numerical analysis was developed with finite element software ATENA 3D [2]. The results demonstrated that an increase in the compressive characteristic strength was not followed by a significant increment in pile cap's strength, since models' ruin were due to concrete splitting (opening cracks parallel to principal compressive stresses as a result of perpendicular tension stresses within the structure) and ties steel bars yielding. In the models analyzed high-tension stresses were developed along the struts and at the bottom of the pile cap's section, demonstrating that pile cap's ultimate resistance is not influenced by the compressive strength.

Keywords: pile caps, finite elements, reinforced concrete, foundation.

Resumo

Este trabalho tem por objetivo realizar uma análise numérica da influência da resistência característica à compressão do concreto (fck) em blocos sobre três estacas. Para tanto, foi utilizado um modelo-padrão de bloco sobre três estacas originalmente desenvolvido por Miguel [1]. A partir deste modelo foram realizadas variações na resistência à compressão do concreto de modo a se observar modificações no comportamento estrutural do elemento. A análise numérica é desenvolvida por meio de programa de computador baseado no MEF. Os resultados demonstraram que o aumento do fck não provocou um aumento significativo da resistência do bloco, visto que a ruína dos modelos ocorreu devido ao fendilhamento (desenvolvimento de tensões de tração perpendiculares às bielas comprimidas) e escoamento da armadura dos tirantes. Nos modelos analisados desenvolveram-se tensões de tração elevadas ao longo das bielas e na seção inferior do bloco, demonstrando que a resistência última dos blocos não é função da resistência à compressão.

Palavras-chave: blocos de concreto, elementos finitos, concreto armado, fundação.

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1. Introduction

Pile caps are an important structural element which transmits forces of the superstructure to the infrastructure. According to Fusco [3], pile cap's structural behavior must be sufficiently rigid to allow that its deformations do not affect the superstructure stresses neither the onsite foundation.

In the last decades, great advances in this area were achieved with the development of the struts and ties model to describe pile cap's structural behavior. Since 1980's, finite element software's evolution and structural monitoring allowed a deeper knowledge of pile caps inner stresses and, hence, allowed the development of less conservative and more realistic design models. Nowadays, it is possible to analyze pile caps' behavior in detail, observing stress flow in the struts and ties, crack pattern, plastic strains, among other relevant aspects.

Studies in this field have proved that the struts and ties model is the best representation of the structural behavior of pile caps. The struts and ties theory is the result of Blévot e Frémy's [4] pioneer work, which framework was the observation that most of the pile caps ruin were due to a brittle collapse as a result of concrete splitting.

Adebar et al. [5] and Miguel [1] researches proved pile caps collapse due to concrete splitting as a result of compressive stresses expansion (cracking increase with concrete collapse) followed by ties yielding.

Delalibera [6], through a statistical analysis of variance, determined four main variables that influence in the pile cap's struts stress flow and load bearing capacity, which are the column and piles cross-section dimensions, external vertical load eccentricity and pile caps height. In addition, through experimental and numerical models, [6] proved that the pile caps structural behavior is influenced by column and piles cross-section dimensions, struts angle, pile caps height and existence or not of splitting reinforcement.

In reference to pile caps design, most design codes recommend deep beams, bending or truss models. In spite of that, pile caps are volume structures that present discontinuity zones due to the non-dissipation of local disturbances and, therefore, Bernoulli's hypothesis is not applied. In this particular case, Saint Venant principle is applied. In pile caps, tensions are not uniform due to stresses concentration in the superior and inferior nodal zones which generates a discontinuity zone (D-region) in all the elements. According to Fusco [3], struts and ties model should be adopted to the elementary treatment of stresses distribution in the regularization zones of structures subjected to Saint Venant principle. Thus, bending theory and linear models do not correctly represent the behavior of pile caps.

Su and Chandler [7] noted the lack of an established design model. The authors affirmed that, in the last decades, struts and ties model has been one of the most popular and rational methods of structural analysis not submitted to bending. And the main design directives were given by national codes such as the Canadian [8], Australian [9] and Neo Zeland [10] Standards and the CEB-FIP. In the CEB-FIP:1973 [11] the procedures for the design of pile caps are described and in the *fib*-Model:2010 [12] design procedures are prescribed for structures or elements with discontinuities using struts and tie models.

Despite that, each of the design code has its own safety factors and different design methodologies. Brazilian ABNT NBR 6118:2007 [13] only mentions its preference to tridimensional struts and ties models in relation of linear and non-tridimensional ones. The references largely adopted in Brazil, according to Ramos [14] are the strut and tie model and the CEB-FIP code.

Clarke [15] observed that ties bars anchorage is positively influenced by struts confining action, which would dispense the use of hooks. Studies conducted by Rausch et al. [16], Miguel [1] and Delalibera [6] demonstrated that ties stresses are not constant, occurring a significant reduction in the inferior nodal zones. Moreover, at the border of the ties the strains are close to zero. Buttignol [17] has shown, through numerical analysis in pile caps with two and three piles, that the ties stresses are not constant through the bars and, at their borders, stresses were very low, dispensing the use of hooks.

In addition, adherence is not a determinant factor to pile cap's ultimate strength capacity, since ties bars slipping occur after pile cap's collapse. Clarke's [15] experimental results have shown that ties bars without hooks slipped only after the crushing of the struts.

Finally, splitting reinforcement can contribute to the increase of pile cap's ultimate strength capacity and to the control of cracking. Buttignol [17] analyzed numerical models with splitting reinforcement (steel bars disposed perpendicular to the struts in order to combat tensile stresses and to resist concrete splitting) as proposed by [6], demonstrating an increase in pile cap's resistance, which presented high strains in the struts cross-section as a result of tensile stresses action within this region.

1.1 Justification

Despite the advances on pile cap's research in the last decades, an analysis about the influence of the concrete compressive characteristic strength in the structural behavior of pile caps is still needed. There are few references about this matter which are concentrated in beams analysis.

According to Delalibera [6], an increase in pile cap's stiffness increases the element strength capacity. And the collapse of rigid pile caps occurs by concrete splitting followed by struts crushing. Therefore, it is preferable to increase pile cap's height (stiffness increment) than to increase concrete compressive strength.

Through numerical modeling of three-pile caps using the finite element software ATENA 3D, this paper demonstrates that the variation on the concrete compressive strength (f_{ck}) is not followed by a significant gain in pile cap's resistance. Moreover, high tensile stresses were observed within the struts and in the nodal zones.

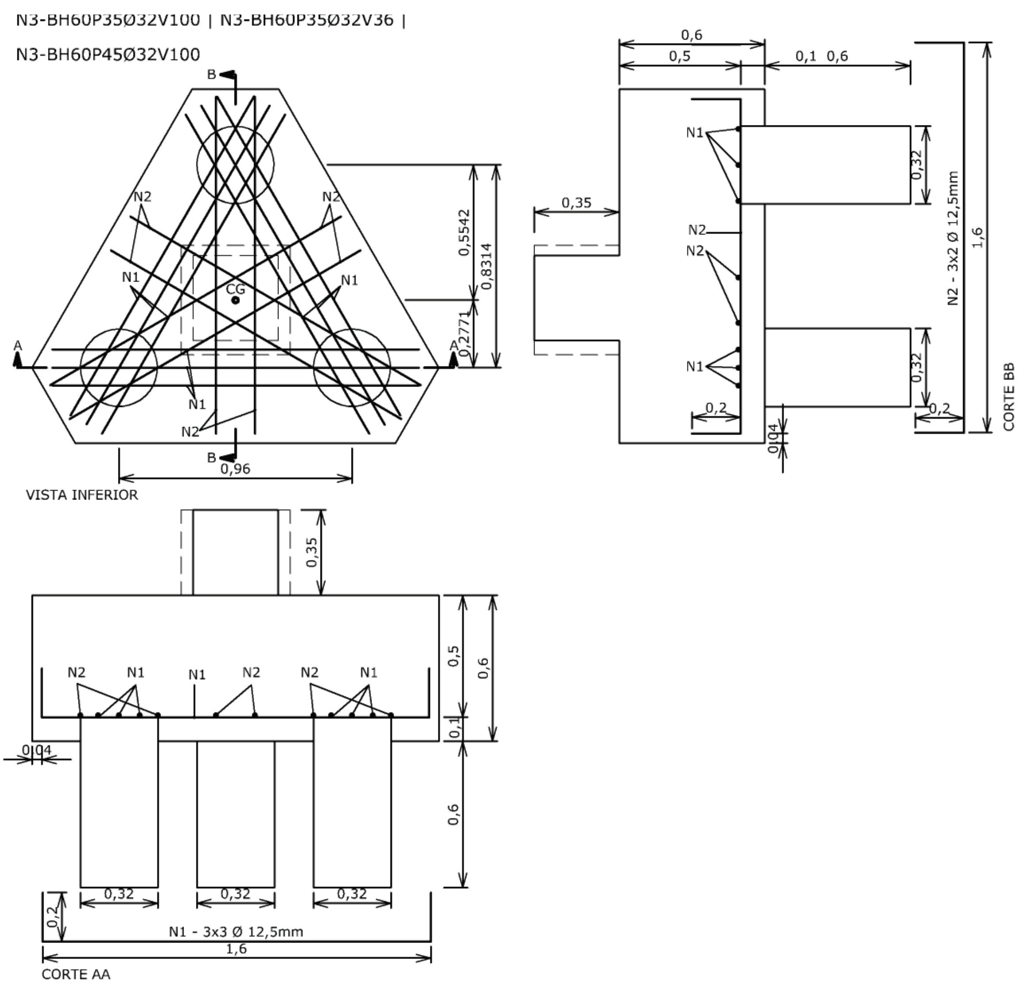
These results highlight the fact that in the case of a necessary increase in the ultimate strength capacity of pile caps, a simple increase in the concrete compressive strength will not result in a consistent benefit. To obtain an increase in the load bearing capacity, it will be necessary to adopt one of the hypotheses brought by [6] and cited before.

To sum up, this paper has the merit of bringing to light a fundamental property of pile cap's behavior, which must be taken in account by designers and constructors.

2. Analysis method

The analysis was developed with numerical modeling of three-pile caps with complementary reinforcement passing through

Figure 1 – Pile caps reinforcement details



the column's cross-section projection, as shown in Figure 1. The pile caps analyzed were originated from [1] research. All original geometric, materials characteristics and reinforcement configuration were maintained, varying only the concrete compressive strength. In total, three pile caps with three piles with different concrete compressive strength (f_{ck}) were modeled, as shown in Table 1.

2.1 Geometric model and reinforcement disposal

The pile caps have a prismatic shape with 60 cm height. The columns are rectangular with 35 cm height and 25 cm x 25 cm cross-section. The piles are cylinders with 32 cm diameter and 60 cm height.
The pile cap's reinforcement has principal ties (bars with hooks disposed above the piles and parallel to the sides of the pile cap) and complementary reinforcement passing through the cross-section projection of the column, as shown in the Figures 1 and 2. The column reinforcement have eight bars with 12,50 mm diameter and stirrups with 6,30 mm diameter, spaced with 10,0 cm, as shown in Figure 3.

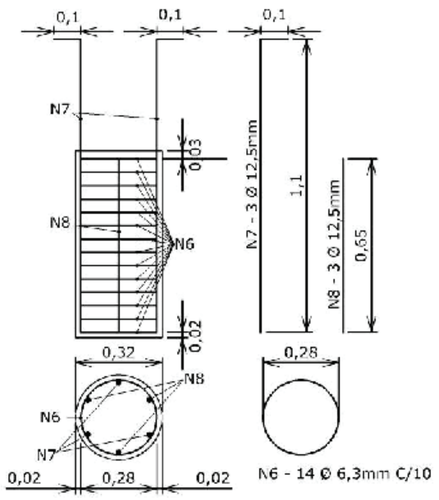
2.2 Numerical analysis

The software ATENA 3D [2], which has its main architecture based on the finite element theory and tridimensional non-linear analysis of reinforced concrete structures, was used to develop the numerical analysis. The ultimate load is calculated by an integer in time of force increments, applying the Arc-Length or Newton-Rhapson methods. To determine the structural behavior of the deformed structure either the Lagrange or the Euler formulation is used.

Table 1 – Numerical models analyzed

Model 1	Model 2	Model 3
Pile cap with f_{ck} = 30 MPa	Pile cap with f_{ck} = 35 MPa	Pile cap with f_{ck} = 40 MPa

Figure 2 – Piles reinforcement details

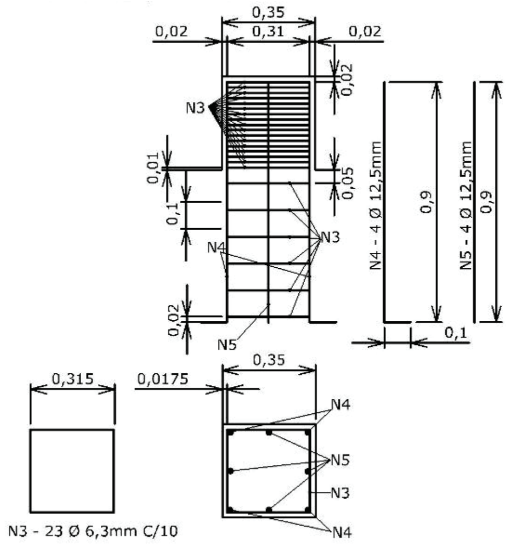


The numerical analysis is divided in three main parts, the pre-processing, the processing and the post-processing. In the pre-processing, the geometric shape of the structure is defined, with the reinforcement, the external load, the supports, the finite element mesh, the monitoring points and the analysis method (Newton-Rhapson or Arc-Length). In the processing, the numerical analysis is executed and the loading (increments of force) and reactions (stresses, strains and cracking) are monitored. In the post-processing, the results are analyzed with auxiliary graphic elements that show the structural behavior in different angles and situations.

Table 2 – Concrete properties

Properties	Pile caps	Piles and columns
Poisson's ratio (ν)	0,2	0,2
Especific fracture energy (G_f)	70,18 J/m^2	120,5 J/m^2
Modulus of elasticity (E_c)	34,03 GPa	43,69 GPa
Concrete compressive strength (f_{ck})	30 MPa (Model 1) 35 MPa (Model 2) 40 MPa (Model 3)	76,50 MPa
Ultimate concrete tensile strength(f_{tk})	2,58 MPa (Model 1) 2,86 MPa (Model 2) 3,13 MPa (Model 3)	4,82 MPa

Figure 3 – Column's reinforcement detail



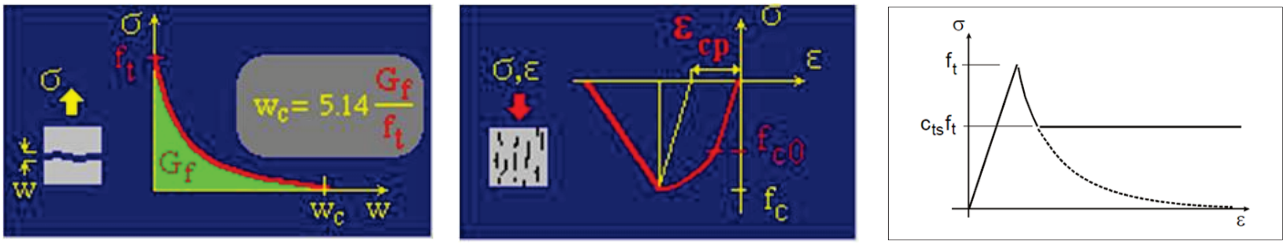
2.3 Materials specifications

A non-linear elastic-plastic behavior was assumed for the concrete, according to the characteristics shown in Table 2.
The concrete behavior, in the elastic regimen, follows Hook's Law which establishes linear relations in the stress-strain field. In the post-cracking stress regimen, structure's collapse plane is determined by Drucker-Prager (in compression) and Rankine criteria (in tension).
The Specific Fracture Energy (G_f), determined by the Equation 1, was originally proposed by Irwin [18] and corresponds to the tax relief of the potential energy stored in the system. Nowadays, this is an essential parameter of concrete structures numerical simulations, allowing the development of more sophisticated modeling forms. Its value corresponds to the internal area of the tension versus crack opening graphic shown in Figure 4a.

$$w_c = 5{,}14\,\frac{G_f}{f_t} \quad (1)$$

The software also considers the plastic strain effect in concrete, as shown in Figure 4b, and the tension stiffening effect, which is the concrete tension stress limit value that contributes to limit the crack expansion, increasing the structural stiffness. Its value is determined by the tension stiffening factor (c_{ts}), as shown in Figure 4c.
A perfect elastic-plastic behavior was assumed to the reinforcement, with the properties shown in Table 3. The steel yielding criterion was based on the von Mises definitions.
An elastic-isotropic material was assumed for the steel plates of the piles supports and for the columns superior cross-section, as specified in the Table 4.

Figure 4 – Concrete constitutive laws



A Stress *versus* opening cracks curve (ATENA)

B Plastic strain (ATENA)

C Tension stiffening curve (2)

2.4 Analysis method

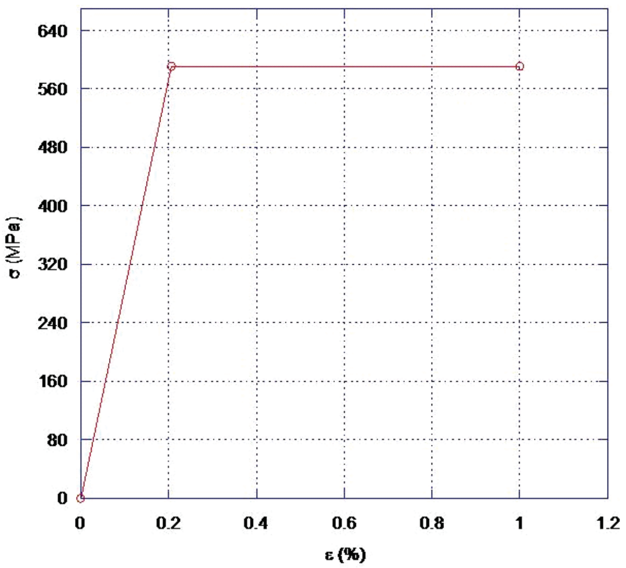
The Newton-Rhapson method was adopted in the numerical analysis, with a uniform load at the top of the column's cross-section and force increments of 50 N/cm². For the elements, a tetrahedral finite element mesh was adopted as shown in Figure 5, since the pile caps have a cylindrical mesh and ATENA software only generates hexahedral forms for prismatic elements.

The structure's load and displacement monitoring points were fixed respectively in the center of the column's superior cross-section and in the center of the pile cap's inferior section.

The piles vertical movement was restricted in all of their inferior cross-section as shown in Figure 5. In the contact-faces of the pile cap with the piles and with the column, a 3D element interface based on Mohr-Coulomb criterion was adopted, as shown in the table 5.

Table 3 – Reinforcement properties

Poisson's ratio (ν)	Modulus of elasticity (E_s)	Ultimate tensile strength (f_{yk})	Yielding (ε_{yd})	Ultimate strain (ε_{lim})
0,3	210 GPa	591 MPa	0,207%	1%



3. Results

3.1 Stiffness and bearing capacity

All three models displayed intense cracking in the pile cap's inferior

Table 4 – Steel plates properties

Poisson's ratio (ν)	Modulus of elasticity (E_s)	Yielding (f_{yk})
0,3	210 GPa	591 MPa

Figure 5 – Details of the finite element mesh, the supports and the applied load (ATENA)

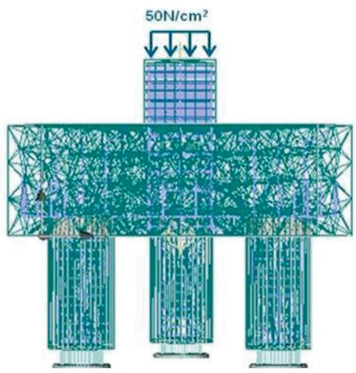


Table 5 – Contact elements properties

Contact element	Values
Normal stiffness parameter (K_{nn})	2,0 . 10^5 kN/m^3
Tangential stiffness parameter (K_{tt})	2,0 . 10^5 kN/m^3
Cohesion	0,0
Friction coefficient	0,0
Concrete ultimate tensile strength (f_{tk})	3,2 MPa

section and in the lateral faces within the struts, as illustrated by Figure 6. In addition, a fragile collapse was observed due to concrete crushing in the inferior nodal zones, concrete splitting and ties bars yielding.

The pile caps ultimate strength capacities were very close in all three models, as shown in Table 6, demonstrating that the increase in the concrete's compressive strength and tensile strength did not exert a significant influence in the pile cap's load bearing capacity. An increase of 33,33% in the concrete's compressive strength (from 30 MPa to 40 MPa) and of 21,20% in the concrete's tensile strength (from 2,58 MPa to 3,13 MPa) caused a meager increase of 13,32% in the pile cap's ultimate load, from 2.756 kN to 3.123 kN. Table 6 presents the correlations between the concrete compressive strength and ultimate load variations.

Figure 7 shows the load *versus* displacement curve where it is possible to notice the great similarity in the behavior of the three pile caps. It is important to stress that no variations in the pile caps stiffness were observed.

3.2 Cracking pattern

In all three models the first superficial cracks appeared in the center of the pile cap's inferior section, expanding through the center of the pile cap's span and towards the pile cap's superior faces. In the ultimate load, intense cracks occurred in the pile cap's inferior section and in its lateral sections, as can be seen in Figure 6.

Figure 6 – Crack pattern in the ultimate load (ATENA)

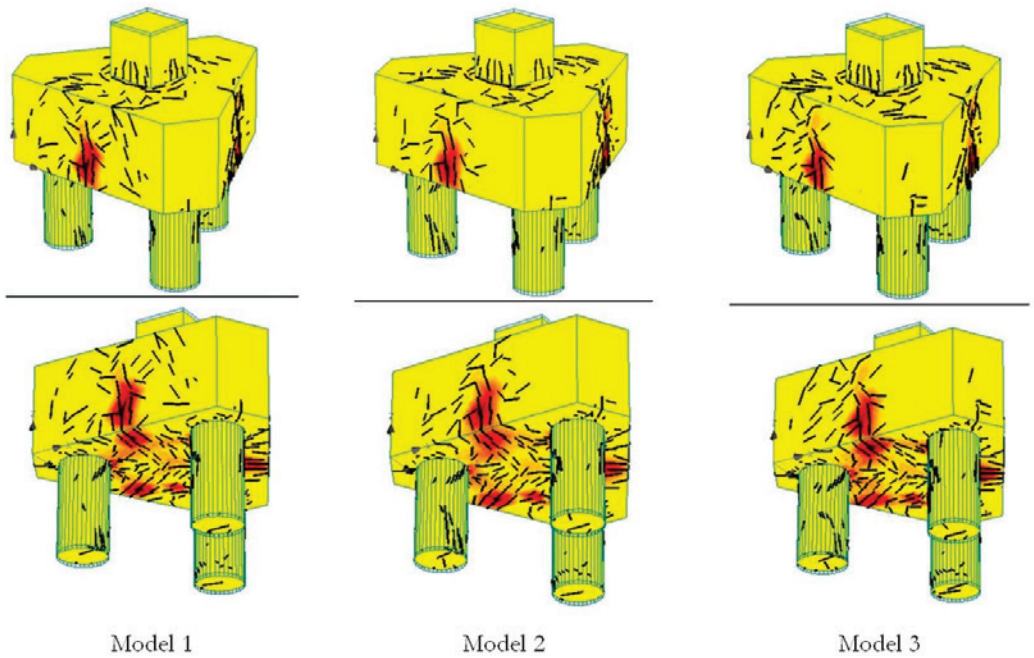
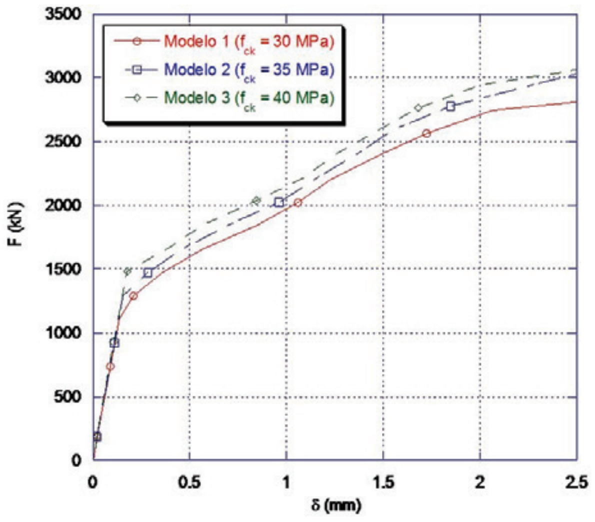


Table 6 – Ultimate load variation (Δf_{ck}) in relation to the concrete compressive strength

	f_{ck}	Δf_{ck}	f_{tk}	Δf_{tk} (%)	F_u (Ultimate load)	ΔF_u
Model 1	30 MPa	-	2,58	-	2.756 kN	-
Model 2	35 MPa	+16,66%	2,83	+9,69	2.940 kN	+6,68%
Model 3	40 MPa	+14,28%	3,13	+10,60	3.123 kN	+6,22%

Figure 7 – Pile caps load *versus* displacement graphic



The increase in the concrete compressive strength (f_{ck}) caused a reduction in the cracking intensity, as shown in Table 7, due to the increase in the concrete tensile strength. It is worth noting that the cracking process begins in the region where the structure reaches the ultimate tensile strength, thus beginning a microcracking process, which leads to stress reduction until the material reaches the critical opening crack (w_c) – Figure 4a –, when a complete separation of the crack sides takes place. Therefore, the higher the concrete's ultimate tensile strength, the higher the pile cap's cracking resistance.

Table 7 – Opening cracks values

Load values (kN)		F=920	F=1.840	F=2.750
Maximum opening crack on the pile cap's surface (mm)	Model 1	0,022	1,52	3,88
	Model 2	0,019	1,27	3,33
	Model 3	0,015	0,97	3,20

Having mentioned that, in Table 8 a correlation between the pile cap's crack intensity reduction according to the concrete tensile strength increase is presented. From model 1 to model 3 there was an increase of 21,32% (from 2,58 MPa to 3,12 MPa) in the concrete tensile stress and a reduction around 30% in the crack opening intensity.

3.3 Compressive struts stresses

In all three pile caps analyzed compressive struts were developed with equal divisions of the stress flow at the bottom of the column's cross-section in direction to the piles, as shown in Figure 8.
In addition, in all models struts compressive stresses were concentrated in the piles superior cross-section region closer to the column, corroborating Delalibera's [6] statement that in the inferior nodal zones the struts stresses are not uniformly distributed.
The concrete compressive strength led to a proportional increase in the struts compressive stresses, as demonstrated in Table 9. The concrete's compressive strength increase from 30 MPa to 40 MPa (+33,33) generated an increase of 38,09% in the struts stresses.
Notwithstanding, as shown in Figure 8, pile caps stress flow did not have a perceptible modification.
In all models, the inferior nodal zones stresses were higher than the concrete compressive strength (f_{ck}), indicating concrete collapse, as shown in Table 10.
There was also the development of tensile stresses in the nodal zones and along the struts which reached values higher than the concrete tensile strength, as can be seen in Table 11, demonstrating the concrete splitting.

3.4 Ties tension stresses

Tie bars yielded in the ultimate load in all three models. Moreover, in the inferior nodal zones an abrupt reduction in the ties tensile stresses occurred due to the positive action of the compressive struts in the steel bars.
From figures 9 to 11, it is possible to observe that, in the pile cap's span, reinforcement tensile stresses were practically constant with values around 590 MPa. Nonetheless, at the beginning of the nodal zones, tensile stresses were greatly reduced, reaching very low values, around 5 MPa in the borders of the bars and in the hooks. These results prove that tie hooks are not necessary, since tie bars anchorage is made almost totally in the inferior nodal zones which receive the positive influence of the compressive struts.
Ties tensile stresses in the inferior nodal zones and at the bars borders were not altered by the increase in the concrete compressive strength, as shown in the Figures 9, 10 and 11.

Table 8 – Opening cracks variation (Δω) in relation to the concrete tensile strength

		f_{tk}	Δf_{tk}	F=920	F=1.840	F=2.750
Opening cracks variation (%)	Model 1	2,58 MPa	-	-	-	-
	Model 2	2,83 MPa	+9,69%	-13,63%	-16,45%	-14,17%
	Model 3	3,13 MPa	+10,60%	-21,05%	-23,62%	-4,00%

Figure 8 – Principal compressive stress flow (ATENA)

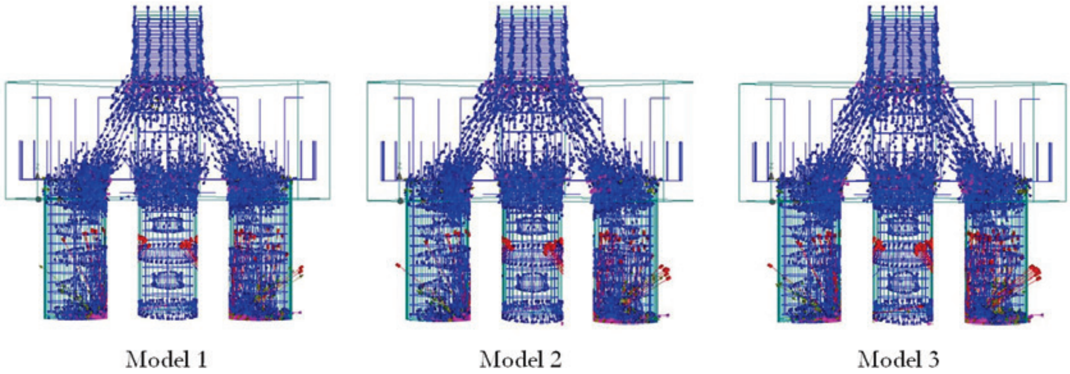


Table 9 – Struts compressive stress variation ($\Delta\sigma_c$) in relation to the f_{ck}

	Pile cap	f_{ck}	Δf_{ck}	Struts stress (σ_c)	$\Delta\sigma_c$
Struts compressive stress variation	Model 1	30 MPa	-	21 MPa	-
	Model 2	35 MPa	16,66%	25 Mpa	19,05%
	Model 3	40 MPa	14,28%	29 MPa	16,00%

4. Conclusion

The increase in the concrete compressive strength was not followed by a significant augment in the pile cap's load bearing capacity and structural behavior. In addition, pile cap's stiffness was not altered.

In all three models, a brittle collapse was observed due to concrete crushing in the inferior nodal zones, concrete splitting, and the yielding of the ties. Despite the small increment in the ultimate load caused by the concrete compressive strength increase, the influence of high tensile stresses through the struts and the nodal zones was determinant to the pile cap's collapse.

In all models analyzed, there was no perceptible variation in the crack pattern. Nevertheless, in percentage, there was a higher reduction in the pile caps opening cracks in relation to the increase of the concrete's tensile strength.

In all three models a concentration of compressive stresses in the piles cross-section closest to the column was observed.

Concrete compressive strength increase led to a proportional increase of the struts compressive stresses.

In all models, the ties reinforcement yielded and there was a significant decrease of the ties stresses in the inferior nodal zones due to the positive action of compressive struts stresses. At the ends of the bars and on the hooks stresses were practically null, thus corroborating that hooks anchorage in pile caps is unnecessary.

Table 10 – Compressive stresses in the nodal zones at the ultimate load

	Pile cap	Inferior nodal zones	Superior nodal zones
Compressive strength (MPa)	Model 1	> 30	24
	Model 2	> 35	28
	Model 3	> 40	30

5. Acknowledgments

To FEC-UNICAMP for the provision of all necessary tools for the development of this research which has resulted in a participation and presentation of a paper related to this subject in the 53º Brazilian Concrete Congress with a subsequent publication in the Congress Annals.

Table 11 – Pile caps tensile stresses at the ultimate load

Tensile stresses (MPa)	Model 1	> 2,58
	Model 2	> 2,86
	Model 3	> 3,13

Figure 9 – Model 1 ties tensile stresses in the anchorage zone (ATENA), where stresses values are highlighted in some ties points (values in MPa)

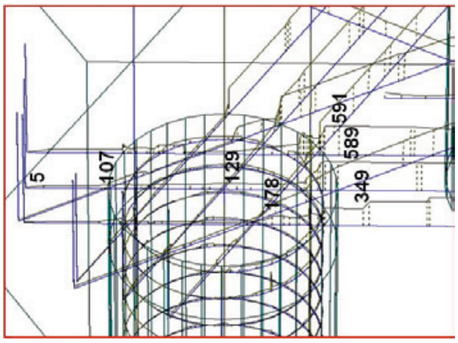


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Figure 10 – Model 2 ties tensile stresses in the anchorage zone (ATENA), where stresses values are highlighted in some ties points (values in MPa)

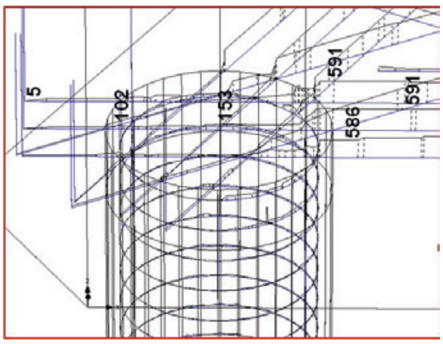
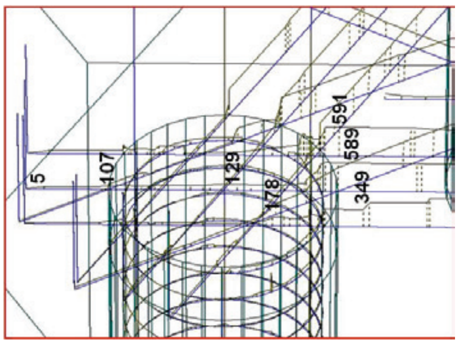


Figure 11 – Model 3 ties tensile stresses in the anchorage zone (ATENA), where stresses values are highlighted in some ties points (values in MPa)



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