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Analysis of the efficiency of strengthening design models for reinforced concrete columns

Análise da eficiência de modelos para dimensionamento do reforço em pilares de concreto armado

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Abstract

This paper develops a comparative analysis of the main design models used for predicting the strengthening of reinforced concrete columns subjected to uniaxial compression. The study evaluated four strengthening design models with concrete jackets and eleven strengthening design models with wrapping in Carbon Fiber-Reinforced Polymer (CFRP). All models consider the effect of confinement provided by the transverse steel reinforcement and the CFRP sheet wrapping on the gain in resistance of the column. For the validation, a database was formulated containing 135 experimental results of columns tested by several researches, which was used to analyze all design models and identify which was best for expressing the behavior of the strengthened column. At the end of the study, one confinement design model with transverse reinforcement and eleven design models with confinement provided by CFRP sheet wrapping and transverse steel reinforcement which showed the best resistance predictions were selected.

Keywords: strengthening, confinement, carbon fibers, concrete jacketing, design models.

Resumo

Este trabalho desenvolve uma avaliação comparativa dos principais modelos empíricos de dimensionamento utilizados no reforço de pilares de concreto armado submetidos a carregamento axial centrado. Foram avaliados quatro modelos de confinamento por armaduras transversais, utilizados no dimensionamento do reforço por aumento de seção transversal de concreto, e onze modelos para o dimensionamento do reforço por encamisamento por polímero reforçado com fibras de carbono. Todos eles consideram o efeito do confinamento, proporcionado pela armadura transversal e pelo reforço com fibras, no ganho de resistência do pilar. Para a validação, foi montado um banco de dados contendo 135 pilares ensaiados em diversas pesquisas, ao qual foram aplicados os modelos em análise de modo a identificar aqueles que melhor expressam o comportamento do pilar reforçado. Ao final do trabalho, foi selecionado um modelo de confinamento por armadura transversal e onze combinações entre modelos de confinamento por fibras de carbono e armadura transversal que conduziram às melhores previsões de resistência dos pilares do banco de dados.

Palavras-chave: encamisamento, confinamento, fibras de carbono, aumento da seção transversal de concreto, modelos de dimensionamento.

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1. Introduction

Reinforced concrete structures are designed and built to withstand the stresses imposed throughout their life cycles. Occasionally, however, in the case of constructive defects or accidents, the constructions require retrofitting to improve their structural strength, increase their load capacity, expand their life cycles, or change the function of the building. This show the importance of developing adequate strengthening design models for concrete structures to guarantee their technical and economic viability.

Among the strengthening techniques for concrete structures, the use of concrete jackets and Carbon Fiber-Reinforced Polymer (CFRP) sheets is highlighted. In recent years, these techniques have been heavily used in columns, significantly increasing their load capacity.

Strengthening with a concrete jacket involves wrapping the column in a concrete layer. According to Takeuti [1], transverse and longitudinal steel reinforcements can be added to the concrete jacket, improving the column resistance for loads. In the case of strengthening by wrapping with CFRP, the column is surrounded by a composite material formed by carbon fibers filled with epoxy resin. Both techniques are effective, and it is necessary to carry out adequate studies of cost, availability of trained labor, and impact on the layout of the building to evaluate the most adequate solution.

The literature contains several design methods to evaluate the gain resistance of strengthened columns. However, since these models are mostly empirical, there is significant variability in the coefficients suggested by different authors. Moreover, the variation in the results of design models is due to the different considerations adopted by each author. Therefore, different amounts of reinforcement and CFRP sheet wrapping are obtained.

Thus, the main objective of this paper is to compare the common design models available in the literature. For this purpose, they were applied to a database with 135 columns tested in the laboratory and the models that achieved the highest efficiency were selected to represent the results of tests by means of statistical inference analysis. To evaluate strengthening with a concrete jacket, the design models proposed by Cusson and Paultre [2], Saatcioglu and Razvi [3], Frangou *et al*. [4], and the fib Model Code 2010 [5] were used. With regard to strengthening with CFRP sheet wrapping, 11 empirical models available in the literature were evaluated.

2. Strengthening models for reinforced concrete columns

The capacity of strengthened concrete columns subjected to uniaxial compression is calculated from the sum of resistances of concrete and steel reinforcement in the longitudinal direction. However, several researches showed the importance of the confinement by a concrete jacket for the capacity of the strengthened concrete columns.

According to Carrazedo [6], when the columns are loaded by longitudinal loads, they show lateral expansion because of the Poisson coefficient. However, when the columns are laterally restrained, triaxial compression stresses are induced, generating a gain in longitudinal resistance in the element. The confinement can be induced by CFRP sheet wrapping or transverse steel reinforcement.

This paper considers that in the columns wrapped by CFRP, the confinement is provided by both the transverse reinforcement and the CFRP sheet wrapping. For columns strengthened with concrete jackets, the confinement is provided by both transverse steel reinforcement in the concrete core and transverse steel reinforcement in the concrete jacket.

2.1 Confinement due to transverse steel reinforcement

The basic principle of strengthening with a concrete jacket is that the resistance of the strengthened column is due to the concrete and the longitudinal steel present in the core and in the concrete jacket. Takeuti [1] adds that the transverse steel reinforcement in

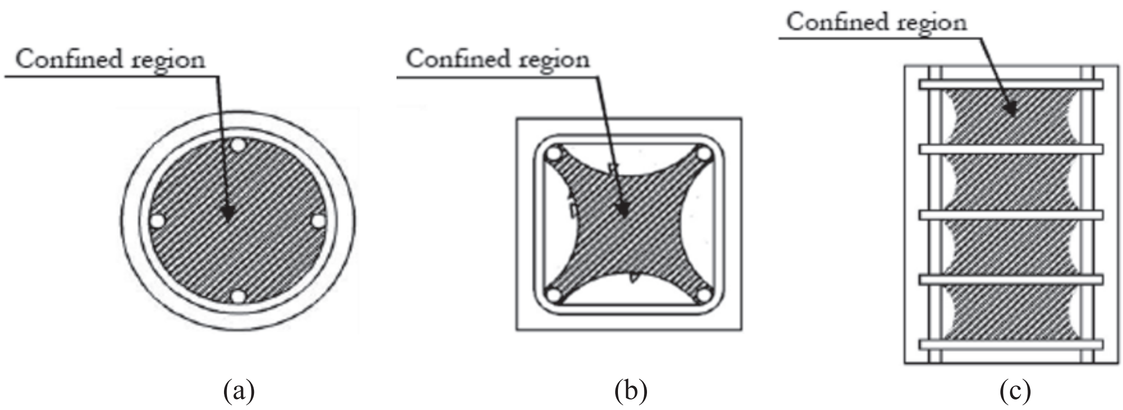

Figure 1
Column region that is confined by transverse reinforcement.. (a) Cross-section of circular confined columns; (b) Cross-section of rectangular confined columns; (c) Longitudinal section of confined columns (confinement between stirrups)
Source: Adapted from Cusson and Paultre [2]

the core and in the concrete jacket ensures the confinement of the column, which consequently increases the strength of column.
The four design models analyzed considered that there is an internal area of the concrete core defined by the transverse reinforcement which is effectively confined, as shown in the hatched area in Figure 1. In circular columns, the confined area of the concrete core is the same as the edge of the transverse steel reinforcement. In columns with square or rectangular sections, there is an arching action of the confining stress due to transverse reinforcement, generating stress peaks in the corners where the transverse reinforcement meets the longitudinal reinforcements. The part of the section outside the confined area is considered as concrete cover and does not contribute to the strength of the column [2–4].
Takeuti [1] and Carrazedo [6] point out that cracking and spalling of concrete cover can happen with the application of axial loading on the column. Therefore, these authors recommend disregarding the concrete cover external to the transverse reinforcement.

2.1.1 Cusson and Paultre model

Based on experimental tests, Cusson and Paultre [2] defined a relationship between the strength gain of confined concrete and the effective confinement index (fle/fc) defined from the nominal lateral pressure of transverse reinforcement, which is given by Equation (1):

$$f_{cc} = f_c\left[1 + 2.1\left(\frac{f_{le}}{f_c}\right)^{0.7}\right] \qquad (1)$$

where:
f_{cc} is the compressive strength of confined concrete,
f_c is the compressive strength of the original concrete,
f_{le} is the nominal lateral pressure.
The authors adopt a confinement effectiveness coefficient, K_e, for a rectangular cross-section, which was evaluated by Mander, Priestley, and Park [7] as follows:

$$f_{le} = f_l K_e$$
with:
$$K_e = \frac{\left(1 - \sum\frac{(w_i)^2}{6c_xc_y}\right)\left(1 - \frac{s'}{2c_x}\right)\left(1 - \frac{s'}{2c_y}\right)}{(1-\rho_l)} \qquad (2)$$

where:
f_l is the lateral pressure of transverse reinforcement,
w_i is the clear spacing between adjacent longitudinal steel bars,
c_x and c_y are the dimensions of the column core perpendicular to the directions x and y, respectively, measured between centers of the transverse reinforcements,
s' is the clear spacing of stirrups,
ρ_l is the longitudinal reinforcement ratio in the core section.
Mander, Priestley, and Park [7] also establish the coefficient of confinement effectiveness K_e for circular columns reinforced with conventional stirrups and with spirals according to Equations (3) and (4), respectively.

$$K_e = \frac{\left(1 - \frac{s'}{2d_i}\right)^2}{(1-\rho_l)} \qquad (3)$$

$$K_e = \frac{\left(1 - \frac{s'}{2d_i}\right)}{(1-\rho_l)} \qquad (4)$$

where:
d_i is the diameter of circular stirrups or spiral between bar centers,
The lateral confining stress on the concrete for rectangular columns is obtained from Equation (5):

$$f_l = \frac{f_{y,t}}{s}\left(\frac{A_{s,tx} + A_{s,ty}}{c_x + c_y}\right) \qquad (5)$$

where:
$f_{y,t}$ is the yield strength of transverse reinforcement,
$A_{s,tx}$ and $A_{s,ty}$ are the total areas of the transverse reinforcement parallel to the y-axis and x-axis, respectively, corresponding to twice the cross-sectional area of the stirrups,
s is the center-to-center spacing between stirrups.
Cusson and Paultre [2] do not evaluate the lateral confining stress on the concrete for circular columns.

2.1.2 Saatcioglu and Razvi's model

The second model analyzed was proposed by Saatcioglu and Razvi [3] and was based on the same confinement principle as was used by Cusson and Paultre [2]. The difference lies in the empirical correlation between the variables.
The gain in concrete strength is evaluated as a function of the nominal lateral pressure by Equation (6).

$$f_{cc} = f_c + 6.7 f_{le}^{0.83} \qquad (6)$$

The nominal lateral pressure is given by:

$$f_{le} = k_2 f_l$$
with:
$$k_2 = 0.26\sqrt{\left(\frac{b_c}{s}\right)\left(\frac{b_c}{w_i}\right)\left(\frac{1}{f_l}\right)} \leq 1.0 \text{ for rectangular sections,} \qquad (7)$$
$$k_2 = 1.0 \text{ for circular sections.}$$

and the lateral pressure of transverse reinforcement is given by Equation (8).

$$f_l = \frac{\sum 2A_{s,t} f_{y,t}\sin\theta}{sb_c} \qquad (8)$$

where:
b_c is the distance between the centers of the longitudinal bars,
$A_{s,t}$ is the area of the transverse reinforcement,
θ is the angle between the transverse reinforcement and b_c and is equal to 90° for rectangular columns.

2.1.3 Model of Frangou *et al.*

Frangou *et al.* [4] proposed a model to evaluate the strength of confined concrete based on Eurocode 8 (CEN [8]) recommendations. This model differs from the others in that it considers the gain resistance of concrete as a function of its mechanical confinement rate ω_w, as shown in Equations (9) and (10).

$$f_{cc} = f_c(1.125 + 1.25\alpha'\omega_w) \text{ if } \alpha'\omega_w \geq 0.1$$
$$f_{cc} = f_c(1 + 2.5\alpha'\omega_w) \text{ if } \alpha'\omega_w \leq 0.1 \qquad (9)$$

$$\omega_w = \frac{A_{s,t}\pi d}{\frac{\pi d^2}{4}s}\frac{f_{y,t}}{f_c} = \frac{4A_{s,t}f_{y,t}}{dsf_c} \qquad (10)$$

where:
α' is a reduction factor, calculated from Equation (11),
d is the diameter of the concrete section confined by the stirrups.
To evaluate the effective confinement on the column, Eurocode 8 (CEN [8]) uses a reduction factor α' given by:

$$\alpha' = \alpha_n \alpha_s$$
with:
$$\alpha_n = \frac{c_x c_y - 2\left(\frac{{c_x}^2 + {c_y}^2}{6}\right)}{c_x c_y - A_{s,l}} \text{ for rectangular cross-sections,}$$
$$\alpha_n = \frac{1}{3} \text{ for square cross-sections,}$$
$$\alpha_n = 1.0 \text{ for circular cross-sections,} \quad (11)$$
$$\alpha_s = \frac{\left(c_x - \frac{s}{2}\right)\left(c_y - \frac{s}{2}\right)}{c_x c_y} \text{ for rectangular cross-sections,}$$
$$\alpha_s = \left(\frac{1-s}{2d_i}\right)^2 \text{ for circular cross-sections with conventional stirrups,}$$
$$\alpha_s = \left(\frac{1-s}{2d_i}\right) \text{ for circular cross-sections with spirals.}$$

where:
$A_{s,l}$ is the total area of longitudinal reinforcement of the column.

2.1.4 *fib* Model Code

The fib Model Code 2010 [5] determines the gain resistance of the transverse reinforced confined column from Equation (12):

$$f_{cc} = f_c\left[1 + 3.5\left(\frac{f_{le}}{f_c}\right)^{\frac{3}{4}}\right] \quad (12)$$

The nominal lateral confinement pressure for circular and rectangular cross-sections is given by Equations (13) and (14), respectively.

$$f_{le} = \omega_c f_c\left(1 - \frac{s}{d_i}\right) \text{ for a section confined by spirals,}$$
$$f_{le} = \omega_c f_c\left(1 - \frac{s}{d_i}\right)^2 \text{ for a section confined by conventional stirrups} \quad (13)$$
with:
$$\omega_c = \frac{2A_{s,t} f_{y,t}}{s d_i f_c}$$

$$f_{le} = \omega_c f_c\left(1 - \frac{s}{c_x}\right)\left(1 - \frac{s}{c_y}\right)\left(1 - \frac{\sum b_c^2}{6 c_x c_y}\right)$$
with: (14)
$$\omega_c = min\left(\frac{A_{s,tx} f_{y,t}}{s c_x f_c}, \frac{A_{s,ty} f_{y,t}}{s c_y f_c}\right)$$

2.2 *CFRP confinement models*

The design models for confinement with CFRP sheet wrapping are based on the same confinement principles as are used for confinement with steel reinforcement. The load capacity of the column is

Table 1
Expressions for evaluating the compressive strength of confined concrete with FRP

Reference	Confinement type	f_{cc}
Samaan *et al.* [10]	GFRP	$f_c + 6.0 f_{l,f}^{0.7}$
Miyauchi *et al.* [11]	CFRP	$f_c\left[1 + 3.5\frac{f_{l,f}}{f_c}\right]$
Kono *et al.* [12]	CFRP	$f_c\left(1 + 0.0572 f_{l,f}\right)$
Toutanji [13]	CFRP GFRP	$f_c\left[1 + 3.5\left(\frac{f_{l,f}}{f_c}\right)^{0.85}\right]$
Saafi *et al.* [14]	CFRP GFRP	$f_c\left[1 + 2.2\left(\frac{f_{l,f}}{f_c}\right)^{0.84}\right]$
Spoelstra and Monti [15]	CFRP GFRP	$f_c\left[0.2 + 3\left(\frac{f_{l,f}}{f_c}\right)^{0.5}\right]$
Fardis and Khalili [16]	GFRP	$f_c\left(1 + 2.05\frac{f_{l,f}}{f_c}\right)$
Karbhari and Eckel [17]	CFRP GFRP AFRP	$f_c\left[1 + 2.1\left(\frac{f_{l,f}}{f_c}\right)^{0.87}\right]$
Mirmiran and Shahawy [18]	GFRP	$f_c + 4.269 f_{l,f}^{0.587}$
Shehata, Carneiro, and Shehata [19]	CFRP	$f_c\left(1 + \beta\frac{f_{l,f}}{f_c}\right)*$

* β is equal to 2.0 for a circular section, 0.85 for a square section, and 0.7 for a rectangular section

guaranteed by the strength of the confined concrete and the longitudinal steel of the core.

There are several researches in the literature about the confinement of concrete by CFRP sheet wrapping. Table 1 lists some of empirical models that were analyzed in this paper, which depend of the strength of confined concrete. On the other hand, the strength of confined concrete as a function of the strength of the existing concrete and the lateral pressure from the CFRP sheet wrapping and can be calculated from Equation (15).

$$f_{l,f}=\frac{2nt_f f_f k_a}{D} \text{ for circular columns}$$
$$f_{l,f}=\frac{f_{l,f(b)}+f_{l,f(h)}}{2}=\frac{nt_f f_f k_a(b+h)}{b+h} \text{ for rectangular columns} \quad (15)$$

where:
$f_{l,f}$ is the lateral pressure from the CFRP sheet wrapping,
n is the number of CFRP sheets,
t_f is the thickness of the CFRP sheet,
f_f is the tensile strength of the CFRP sheet,
k_a is the confinement effectiveness coefficient. For circular columns, it is considered to be full confinement, that is, k_a = 1.0,
D is the diameter of the circular columns,
b and h are the width and height of the cross-section of the rectangular columns, respectively.

Note from Equation (15) that lateral pressure in the column depends on the tensile strength of the CFRP sheet, which is directly influenced by several properties of the FRP (Fiber Reinforced Polymers), such as the modulus of elasticity and deformation of fibers, thickness, and number of FRP layers. Several researches have carried out tests to propose expressions that already include these basic parameters for the main types of commercialized FRP systems, that is, CFRP, GFRP (Glass Fiber Reinforced Polymers), and AFRP (Aramid Fiber Reinforced Polymers). Thus, the expressions obtained from tests with other types of fibers can also be efficiently applied in the calculation of the confinement with CFRP sheets and are included in Table 1.

The expressions in Table 1 were obtained from tests with columns strengthened only with FRP sheets, without transverse or longitudinal reinforcements. Thus, they depend only on the lateral pressure due to the FRP jacket. For columns strengthened with CFRP sheets and transverse reinforcement, the equations shown in Table 1 can be associated with the confinement models with transverse reinforcement described in Subsection 2.1, as shown in Figure 2 and discussed in the next section.

2.3 Models for evaluation of confinement with transverse reinforcement and FRP sheet wrapping

The confinement of the concrete core of columns with transverse reinforcement is well-known. However, there are still doubts about the interaction between FRP sheet wrapping and transverse reinforcement used to confine the concrete core of columns. Carrazedo [6] considers that the interaction of FRP sheets and transverse reinforcement in the confinement of concrete can be evaluated by adding the strength gain obtained for each strengthening system individually. That is, initially the transverse reinforcement confines the concrete of the column and offers a resistance gain of $f_{cc,e}$. Subsequently, the FRP sheet wrapping provides a resistance gain of $f_{cc,f}$ to the unconfined concrete core. The total resistance of the confined concrete is given by Equation (16):

$$f_{cc}=f_c+f_{cc,f}+f_{cc,e} \quad (16)$$

Another proposal that considered this interaction was presented by Machado [9] and was based on the recommendations of ACI 440. According to this author, the strength of the confined concrete of the reinforced column can be evaluated from an empirical equation that considers the lateral pressure generated by the strengthening system and the strength of the original concrete, as shown in Equation (17).

$$f_{cc}=f_c\left[2.25\sqrt{1+\frac{7.9f_l}{f_c}}-\frac{2f_l}{f_c}-1.25\right] \quad (17)$$

This formulation is based on the hypothesis that the total lateral pressures on the column are due to the sum of the lateral

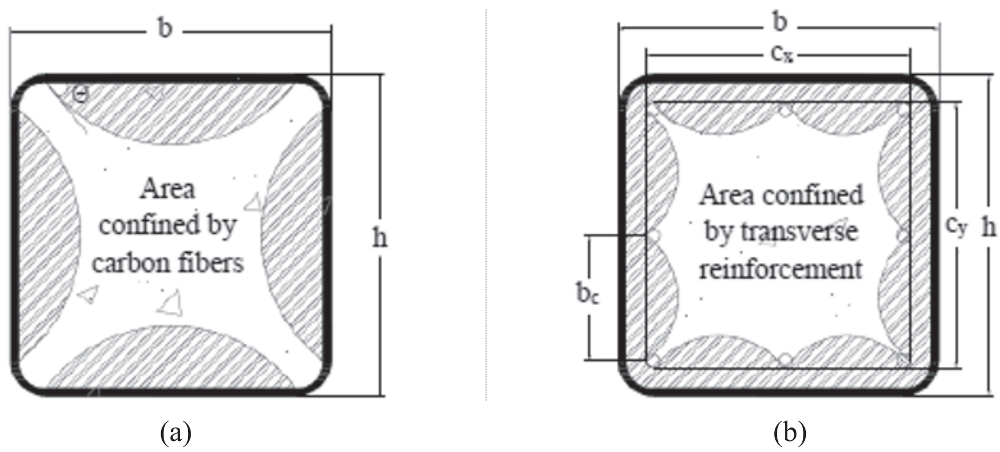

Figure 2
Effectively confined area of column. (a) Confinement with CFRP; (b) Confinement with transverse reinforcement.
Source: Modified from Machado [9]

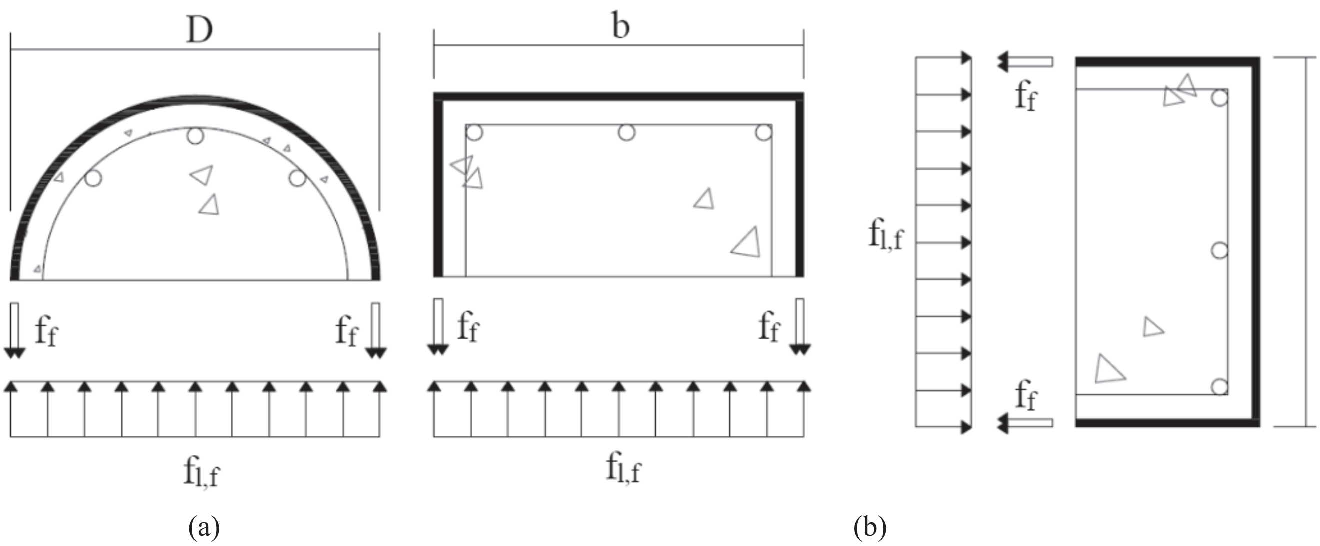

Figure 3
Lateral pressure due to CFRP sheet wrapping. (a) Circular columns; (b) Rectangular columns
Source: Modified from Machado [9]

pressures of the different strengthening systems, that is, FRP sheet wrapping and transverse reinforcement, as shown in Figure 3. Then, the total lateral pressure f_l is calculated from Equation (18), in a different procedure from Equation (16), which evaluates the strength of confined concrete independently from each strengthening system.

$$f_l = f_{l,e} + f_{l,f} \quad (18)$$

where:
$f_{l,e}$ is the lateral pressure from the transverse reinforcement,
$f_{l,f}$ is the lateral pressure from the CFRP sheet wrapping.
The lateral pressures from the CFRP sheet wrapping and the transverse reinforcement are evaluated by Equations (15) and (19), respectively. In circular sections, the pressure distribution is uniform, while the lateral pressure is proportional to the cross-sectional dimensions of the rectangular column.

$$f_{l,e} = \frac{2A_{s,t} f_{y,t} k_b}{s d_i} \text{ for circular columns}$$

$$f_{l,e} = \frac{A_{s,t} f_{y,t} k_b (b+h)}{sbh} \text{ for rectangular columns} \quad (19)$$

where:
k_b is the coefficient of confinement effectiveness,
Full confinement of circular columns section is considered, thus k_b = 1.0. For rectangular columns, Machado [9] uses Equation (20) to evaluate the confinement effectiveness coefficients k_a and k_b required in Equations (15) and (19).

$$k_a = k_b = 1 - \frac{(b-2r')^2 + (h-2r')^2}{3bh(1-\rho_t)}$$

$$r' = \frac{h}{b} \leq 1.5 \quad (20)$$

$$\rho_t = \frac{A_{s,t}}{A_g}$$

where:
ρ_t is the ratio between transverse reinforcement and the column section area,
A_g is the cross-sectional area of the column.

3. Analysis of design models for strengthening of reinforced concrete columns

The analysis of the design models described was done using a database of 135 columns that have been tested and are available in the literature. The design models shown in Section 2 were applied to this database and the results were compared to the resistance gain observed in the experimental tests. The analysis was subdivided into two parts, that is, strengthening with a concrete jacket and strengthening with CFRP sheet wrapping. Later, strengthening with both transverse reinforcement and CFRP sheet wrapping was also analyzed.

3.1 Strengthening with concrete jacket

A set of four columns tested by Takeuti [1] was used to evaluate the efficiency of the design models for predicting the resistance of reinforced columns strengthened with concrete jackets. All columns had an original square cross-section of 15 × 15 cm and were placed in concrete jackets of either 3 or 4 cm thick. Longitudinal and transverse reinforcements were added to the core and concrete jacket, as shown in Figure 4. The database for this strengthening system is small since there are few studies in the literature on reinforcement by wrapping with a concrete jacket.
The four design models for evaluating the confinement with transverse reinforcement were applied to this database. The strength of the column with the concrete jacket was determined by adding the strength of longitudinal reinforcements to the strength of concrete

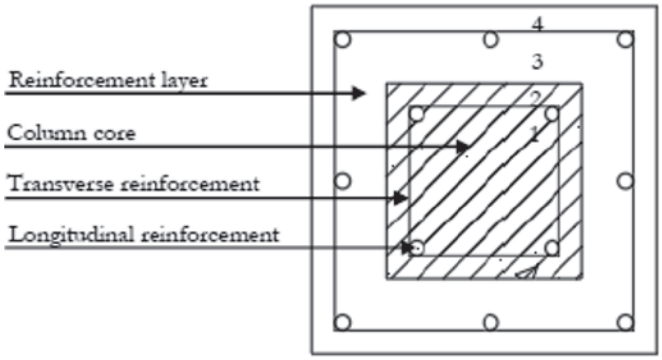

Figure 4
Cross-sectional area of column strengthened with concrete jacket

shown in Regions 1, 2, and 3 in Figure 4. Region 4, which is external to the transverse reinforcement, was disregarded. Only the cross-section of the original column was considered to be confined by the transverse reinforcement. Region 1 was confined due to transverse reinforcement placed on the concrete core and on the concrete jacket, while Region 2 was confined only by the transverse reinforcement placed on the concrete jacket.

Table 2 shows the comparison between the strength predicted by each model and the strength obtained experimentally for each column. It is observed that, on average, all models predicted values within the acceptable range for safety , that is, values for the ratio of maximum theoretical strength to maximum experimental strength ($F_{u,theor} / F_{u,exp}$) smaller than one. In addition, all models showed similar effectiveness, although different expressions were used, with an average difference of 10% from values obtained in the tests.

3.2 Strengthening with CFRP sheet wrapping

Several tests of strengthening of reinforced concrete columns with CFRP have been presented in the literature. Three situations shown in Figure 5 were analyzed. Initially, the confinement models with CFRP sheet wrapping (Table 1) were applied to columns that were wrapped with FRP and without transverse reinforcement. Then, the confinement models with transverse reinforcement shown in Subsection 2.1 were applied to circular columns with transverse reinforcement and without the presence of the CFRP sheet. The results

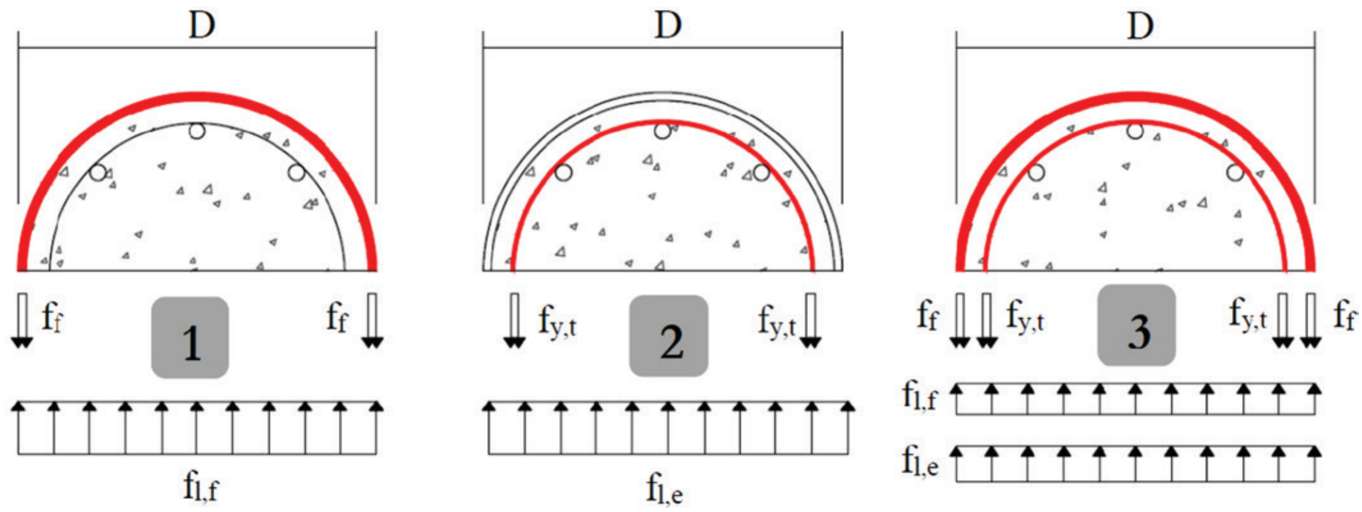

Figure 5
Confinement generated due to the FRP and the transverse reinforcement. Step 1: Confinement with FRP; Step 2: Confinement with transverse reinforcement; Step 3: Confinement with FRP and transverse reinforcement

Table 2
Comparison between the design models for strengthening with concrete jacket $\Delta = F_{u,theor}/F_{u,exp}$

Model	$F_{u,exp}$ (kN)	Cusson and Paultre [2]		Saatcioglu and Razvi [3]		Frangou *et al.* [4]		fib Model Code 2010 [5]	
		$F_{u,theor}$ (kN)	Δ	$F_{u,theor}$ (kN)	Δ	$F_{u,theor}$ (kN)	Δ	$F_{u,theor}$ (kN)	Δ
S1C1S	1540	1356	0.88	1394	0.91	1324	0.86	1340	0.87
S1C2S	1749	1276	0.73	1295	0.74	1320	0.75	1259	0.72
S2C1S	1850	1841	0.99	1876	1.01	1813	0.98	1823	0.99
S1C2S	1840	1749	0.95	1765	0.96	1780	0.97	1727	0.94
Mean	–	–	0.89	–	0.90	–	0.89	–	0.88
CV*	–	–	0.13	–	0.13	–	0.12	–	0.13

* CV = coefficient of variation

Table 3
Database of reinforced circular columns strengthened with FRP

Reference	Column	Dimensions			FRP						Experimental conditions		
		D (mm)	H (mm)	λ	Fiber type	t_f (mm)	ξ_f (‰)	E_f (MPa)	f_f (MPa)	n	f_c (MPa)	f_l (MPa)	$f_{cc,exp}$ (MPa)
Carrazedo [6]	C1	190	570	12	CFRP	0.130	11.92	218950	2610	1	26.16	3.57	38.81
	C2	190	570	12	CFRP	0.130	10.89	218950	2384	2	26.16	6.53	53.08
Shehata, Carneiro, and Shehata [19]	C1-25a	150	300	8	CFRP	0.165	15.00	235000	3525	1	25.60	7.76	43.90
	C2-30a	150	300	8	CFRP	0.165	15.00	235000	3525	1	29.80	7.76	57.00
	C1-25b	150	300	8	CFRP	0.165	15.00	235000	3525	2	25.60	15.51	59.60
	C2-30b	150	300	8	CFRP	0.165	15.00	235000	3525	2	29.80	15.51	72.10
Samaan *et al.* [10]	DA11	153	305	8	GFRP	0.240	–	–	579.2	6	30.86	10.94	53.66
	DA13	153	305	8	GFRP	0.240	–	–	579.2	6	30.86	10.94	56.50
	DB11	153	305	8	GFRP	0.240	–	–	579.2	6	29.64	10.94	67.12
	DB12	153	305	8	GFRP	0.240	–	–	579.2	6	29.64	10.94	55.29
	DB13	153	305	8	GFRP	0.240	–	–	579.2	6	29.64	10.94	60.23
	DC11	153	305	8	GFRP	0.240	–	–	579.2	6	31.97	10.94	59.06
	DC12	153	305	8	GFRP	0.240	–	–	579.2	6	31.97	10.94	60.79
	DA21	153	305	8	GFRP	0.220	–	–	579.2	10	30.86	16.71	72.92
	DA22	153	305	8	GFRP	0.220	–	–	579.2	10	30.86	16.71	65.67
	DA23	153	305	8	GFRP	0.220	–	–	579.2	10	30.86	16.71	77.99
	DB21	153	305	8	GFRP	0.220	–	–	579.2	10	29.64	16.71	74.56
	DB22	153	305	8	GFRP	0.220	–	–	579.2	10	29.64	16.71	93.02
	DB23	153	305	8	GFRP	0.220	–	–	579.2	10	29.64	16.71	71.77
	DC21	153	305	8	GFRP	0.220	–	–	579.2	10	31.97	16.71	77.35
	DC22	153	305	8	GFRP	0.220	–	–	579.2	10	31.97	16.71	77.08
	DA31	153	305	8	GFRP	0.212	–	–	579.2	14	30.86	22.56	85.72
	DA33	153	305	8	GFRP	0.212	–	–	579.2	14	30.86	22.56	86.76
	DB31	153	305	8	GFRP	0.212	–	–	579.2	14	29.64	22.56	86.22
	DB32	153	305	8	GFRP	0.212	–	–	579.2	14	29.64	22.56	114.66
	DB33	153	305	8	GFRP	0.212	–	–	579.2	14	29.64	22.56	87.44
	DC31	153	305	8	GFRP	0.212	–	–	579.2	14	31.97	22.56	86.11
	DC32	153	305	8	GFRP	0.212	–	–	579.2	14	31.97	22.56	83.99
Eid, Roy, and Paultre [20]	N1	152	300	8	CFRP	0.381	13.40	78000	1045	1	32.10	5.24	39.71
	N2	152	300	8	CFRP	0.381	13.40	78000	1045	2	32.10	10.48	57.58
	N3	152	300	8	CFRP	0.381	13.40	78000	1045	3	33.60	15.72	74.24
	M1	152	300	8	CFRP	0.381	13.40	78000	1045	1	48.00	5.24	59.80
	M2	152	300	8	CFRP	0.381	13.40	78000	1045	2	48.00	10.48	80.04
	M3	152	300	8	CFRP	0.381	13.40	78000	1045	3	48.00	15.72	99.84
Wang *et al.* [21]	C1H0L1M	305	915	12	CFRP	0.167	17.79	244000	4340	1	24.50	4.75	35.00
	C1H0L2M	305	915	12	CFRP	0.167	17.79	244000	4340	2	24.50	9.51	55.30
	C2H0L1M	204	612	12	CFRP	0.167	17.79	244000	4340	1	24.50	7.11	46.10
	C2H0L2M	204	612	12	CFRP	0.167	17.79	244000	4340	2	24.50	14.21	65.20
Lee *et al.* [22]	S0F1	150	300	8	CFRP	0.110	18.04	250000	4510	1	36.20	6.61	41.70
	S0F2	150	300	8	CFRP	0.110	18.04	250000	4510	2	36.20	13.23	57.80
	S0F3	150	300	8	CFRP	0.110	18.04	250000	4510	3	36.20	19.84	69.10
	S0F4	150	300	8	CFRP	0.110	18.04	250000	4510	4	36.20	26.46	85.40
	S0F5	150	300	8	CFRP	0.110	18.04	250000	4510	5	36.20	33.07	104.30

Table 4
Comparison between theoretical and experimental results for columns strengthened with FRP

Reference	$f_{cc,theor}/f_{cc,exp}$									
	Fardis and Khalili [16]	Karbhari and Eckel [17]	Mirmiran and Shahawy [18]	Miyauchi et al. [11]	Samaan et al. [10]	Saafi et al. [14]	Toutanji [13]	Spoelstra and Monti [15]	Kono et al. [12]	Shehata, Carneiro, and Shehata [19]
Mean	0.90	0.96	0.78	1.20	1.04	0.99	1.29	1.01	0.85	0.89
CV	0.12	0.12	0.17	0.12	0.12	0.12	0.11	0.11	0.15	0.12

Table 5
Results of student's t–test for columns strengthened with FRP. t-critical = 2.01808

Reference	T
Fardis and Khalili [16]	5.57413
Karbahari and Eckel [17]	2.83641
Mirmiran and Shahawy [18]	8.84600
Miyauchi *et al.* [11]	–7.66059
Samaan *et al.* [10]	–0.83592
Saafi *et al.* [14]	1.08314
Toutanji [13]	–11.17020
Spoelstra and Monti [15]	–0.10101
Kono *et al.* [12]	6.71226
Shehata, Carneiro and Shehata [19]	6.08063

were analyzed to verify the effectiveness of each model separately to predict the resistance of strengthened concrete columns. Finally, the interaction between the two strengthening systems was investigated in columns with transverse reinforcement and wrapped with FRP. For this purpose, the association between the calculation models presented in Sections 2.1 and 2.2 was investigated.

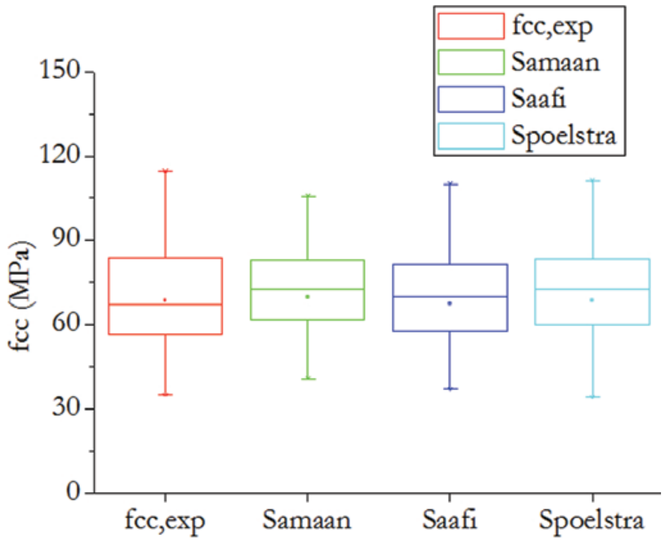

Figure 6
Variability of confinement models with FRP when compared to experimental values

3.2.1 Columns strengthened only with CFRP

The first analysis considered a database with 43 columns strengthened only with FRP (Table 3). The database contained twenty-two columns tested by Samaan et al. [10], two tested by Carrazedo [6], four tested by Shehata, Carneiro, and Shehata [19], six tested by Eid, Roy, and Paultre [20], four tested by Wang et al. [21], and five tested by Lee *et al.* [22]. They were all short columns, with a circular cross-section, a diameter varying between 150 and 305 mm, and concrete compressive strength varying from 24.5 to 48 MPa. Twenty-one columns were strengthened with glass fibers and twenty-two with CFRP. Table 3 compares the strength of confined concrete obtained from confinement models with FRP ($f_{cc,theor}$) with the values obtained from the experiments ($f_{cc,exp}$). These columns did not have longitudinal reinforcement, so their strength was due only to the strength of confined concrete.

A general view shows that the models of Fardis and Khalili [16], Karbhari and Eckel [17], Samaan *et al.* [10], Saafi *et al.* [14], and Spoelstra and Monti [15] achieved the best predictions, with an error of less than 10% compared to the experimental values. The coefficient of variation for all models also remained acceptable at around 12%.

Due to the bigger database, a bilateral paired Student's t-test was performed. The t-test is used to determine whether two sets of data are significantly different from each other [25]. The population variance was unknown and a significance level (α) of 10% was used for analysis. Table 5 shows the results obtained for the test variable (t) and the critical value of this variable (t critical). From this

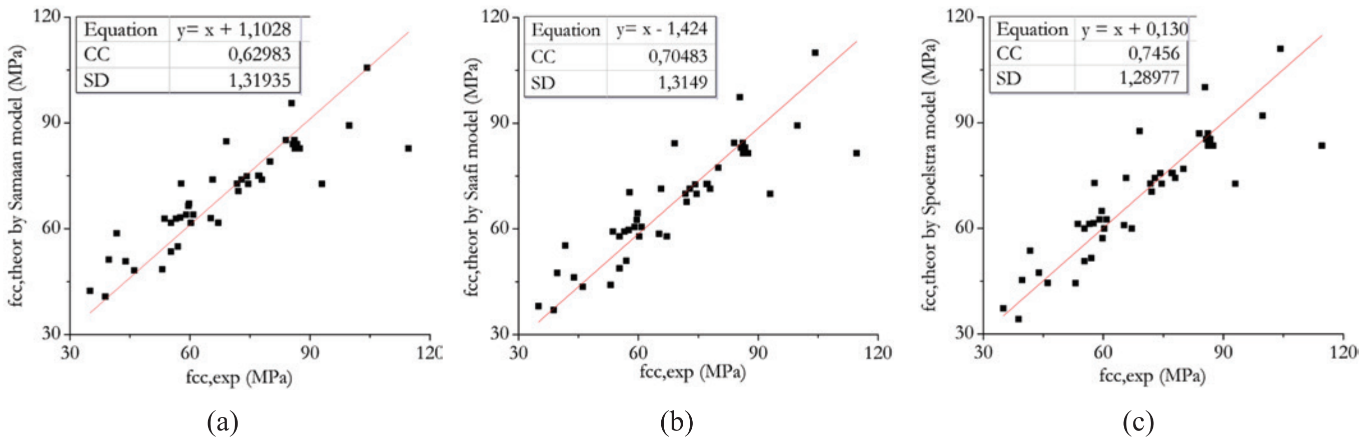

Figure 7
Comparison of theoretical and experimental results of compressive strength of confined concrete (f_{cc}) due to confinement with FRP. (a) Samaan's model; (b) Saafi's model; (c) Spoelstra's model. CC: Correlation coefficient; SD: Standard deviation

Table 6
Database of reinforced columns confined with transverse reinforcement

Reference	Model	Dimensions			Longitudinal reinforcement			Transverse reinforcement					Experimental conditions	
		D (mm)	H (mm)	λ	ϕl (cm)	n°	$f_{y,l}$ (MPa)	Type	ϕt (cm)	s (mm)	$f_{y,t}$ (MPa)	c (cm)	f_c (MPa)	$f_{cc,exp}$ (MPa)
Carrazedo [6]	C0S50	190	570	12	0.8	6	554.8	Spirals	0.50	50	756	1.5	26.16	39.44
	C0S25	190	570	12	0.8	6	554.8	Spirals	0.50	25	756	1.5	28.86	60.52
Eid, Roy, and Paultre [20]	C4NP0C	303	1200	16	1.6	6	423	Spirals	1.13	100	456	2.5	31.70	2930
	C4N1P0C	303	1200	16	1.6	6	423	Spirals	1.13	100	456	2.5	36.00	3235
	C2NP0C	303	1200	16	1.6	6	423	Spirals	1.13	65	456	2.5	31.70	3000
	C2N1P0C	303	1200	16	1.6	6	423	Spirals	1.13	65	456	2.5	36.00	3490
Wang *et al.* [21]	a	500	1500	12	1.6	12	295	Spirals	1.20	52	310	2.5	28.00	38.0
	b	500	1500	12	1.6	12	295	Spirals	1.20	52	340	2.5	31.00	48.0
	c	500	1500	12	1.6	12	295	Spirals	1.20	52	340	2.5	33.00	47.0
	1	500	1500	12	1.6	12	295	Spirals	1.20	41	340	2.5	28.00	51.0
Mander, Priestley, and Park [7]	2	500	1500	12	1.6	12	295	Spirals	1.20	69	340	2.5	28.00	46.0
	3	500	1500	12	1.6	12	295	Spirals	1.20	103	340	2.5	28.00	40.0
	4	500	1500	12	1.6	12	295	Spirals	1.00	119	320	2.5	28.00	36.0
	5	500	1500	12	1.6	12	295	Spirals	1.00	36	320	2.5	28.00	47.0
	6	500	1500	12	1.6	12	295	Spirals	1.60	93	307	2.5	28.00	46.0
	7	500	1500	12	2.8	8	296	Spirals	1.20	52	340	2.5	31.00	52.0
	8	500	1500	12	2.4	11	260	Spirals	1.20	52	340	2.5	27.00	49.0
	9	500	1500	12	2.0	16	286	Spirals	1.20	52	340	2.5	31.00	52.0
	10	500	1500	12	1.6	24	295	Spirals	1.20	52	340	2.5	27.00	50.0
	11	500	1500	12	1.6	36	295	Spirals	1.20	52	340	2.5	27.00	54.0
	12	500	1500	12	1.6	24	360	Spirals	1.20	52	340	2.5	31.00	52.0
	C2H2L0M	204	612	12	1.0	6	312	Stirrups	0.60	60	397	1.5	24.50	30.1
	C2H2L0M	204	612	12	1.0	6	312	Stirrups	0.60	60	397	1.5	24.50	30.1
	C2H2L0M	204	612	12	1.0	6	312	Stirrups	0.60	60	397	1.5	24.50	30.1
	C2H2L0M	204	612	12	1.0	6	312	Stirrups	0.60	60	397	1.5	24.50	30.1

analysis, it is concluded that only the models of Samaan *et al.* [10], Saafi *et al.* [14], and Spoelstra and Monti [15] are not significantly different from the experimental results. Therefore, it is possible to accept the hypothesis that only these models can predict a strength of confined concrete equal to the values observed in tests at a significance level of 10% (Figure 6). When comparing the results predicted by these models with the experimental results for this data set of columns tested, a correlation coefficient that ranges from 0.63 to 0.75 is obtained, as shown in Figure 7, and the best correlation is presented by the model of Spoelstra and Monti [15] (Figure 7 (c)).

3.2.2 Columns strengthened only with transverse reinforcement

The database used for the second analysis contains 25 circular columns reinforced with transverse and longitudinal reinforcements

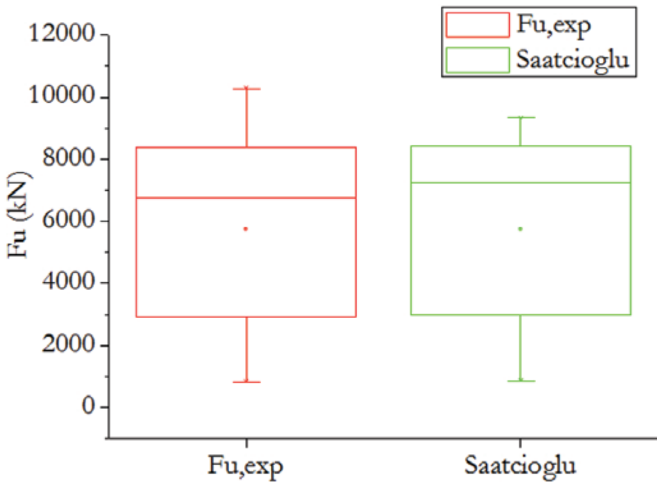

Figure 8
Variability of Saatcioglu and Razvi's model [3] when compared to experimental values

Table 7
Comparison between theoretical and experimental results for columns strengthened with transverse reinforcement

Reference	$F_{u,theor}/F_{u,exp}$			
	Cusson and Paultre [2]	Saatcioglu and Razvi [3]	Frangou *et al.* [4]	*fib* Model Code 2010 [5]
Mean	0.90	1.02	0.88	0.97
CV	0.06	0.08	0.06	0.08

Table 8
Results of student's t-test for columns strengthened with transverse reinforcement. t-critical = 1.71088

Reference	t
Cusson and Paultre [2]	5.40745
Saatcioglu and Razvi [3]	–0.02044
Frangou *et al.* [4]	5.91667
fib Model Code 2010 [5]	1.99189

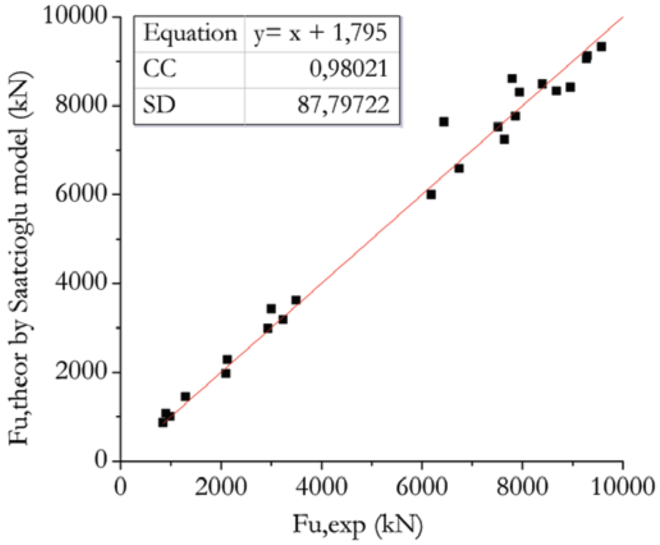

Figure 9
Comparison of theoretical and experimental results of ultimate load of the column (F_u) for Saatcioglu and Razvi's model [3]. CC: Correlation coefficient; SD: Standard deviation

(Table 6). Two columns were tested by Carrazedo [6], four by Eid, Roy, and Paultre [20], four by Wang *et al.* [21], and fifteen by Mander, Pristley, and Park [7]. All columns were short, with diameters varying between 190 and 500 mm and concrete compressive strength ranging from 24.5 to 36 MPa. The results of this analysis are presented in Table 7. The columns' resistance in this analysis was evaluated by adding the confined concrete resistance and the resistance due to the longitudinal reinforcement. The contribution of the concrete cover of the column was disregarded.
The comparison shows that all four design models predicted the experimental columns' strength with a difference of less than 12% and a coefficient of variation of around 8%. Saatcioglu and Razvi's [3] overestimated the resistance, since the authors considered the coefficient of confinement effectiveness to be equal to 1.0 for circular columns. However, this model showed a difference from the experimental results of only 2%.
From the bilateral paired Student's t-test with a level of significance (α) of 10% (Table 8), it is concluded that only the prediction by Saatcioglu and Razvi's model [3] is not significantly different from the experimental results. Thus, only this model can predict the strength of reinforced concrete column equal to the value observed in tests of columns confined with transverse reinforcement at a significance level of 10% (Figure 8). When comparing the results predicted by Saatcioglu and Razvi's model [3] with the experimental results for this data set of columns, a correlation coefficient of 0.98 is obtained (Figure 9).

3.2.3 Columns strengthened with CFRP and transverse reinforcement

The database for this analysis contains 63 columns, that is, six columns tested by Huang *et al.* [23], four tested by Carrazedo [6], thirteen tested by Eid, Roy and Paultre [20], fourteen tested by Lee *et al.* [22], nineteen tested by Yin *et al.* [24], and eight tested by Wang *et al.* [21]. All columns were short, with diameters varying from 150 to 305 mm and compressive concrete strength varying from 24.5 to 50.8 MPa. Six columns were strengthened with GFRP sheet wrapping and 57 with CFRP sheet wrapping. Moreover, 12 columns were made with conventional stirrups and 51 were made with circular spiral reinforcements (Table 9). For this analysis, the theoretical resistance of strengthened columns was calculated using Equation (16). Four design models with confinement by transverse reinforcement, shown in Subsection 2.1, were combined with ten models of confinement by FRP, shown in Subsection 2.2, generating a total of 40 combinations. Moreover, Machado's proposal [9] of considering simultaneous confinement by CRFP and stirrups was analyzed. The results are shown in Table 10. In this analysis, the columns' strength was

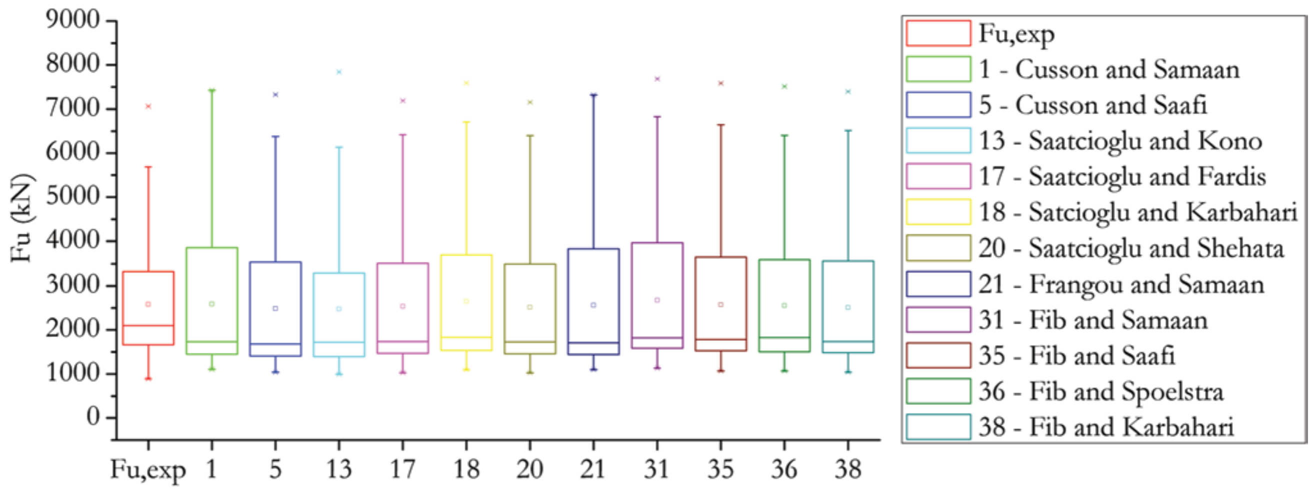

Figure 10
Variability of confinement models with FRP and transverse reinforcement when compared to experimental values.

evaluated by adding the resistance due to confined concrete to the resistance of the longitudinal reinforcement.

The general view shows that the results of all 41 analyses were close to the values obtained from the tests. However, all combinations showed a coefficient of variation greater than 20%.

From the results of the bilateral paired Student's t-test with a level of significance (α) of 10% shown in Table 11, it can be concluded that the predicted and experimentally determined resistance were not significantly different in eleven combinations (Table 12). Thus, only these models can predict the strength of a reinforced concrete column equal to the value observed in tests of columns confined with transverse reinforcement and CFRP sheet wrapping at a significance level of 10% (Figure 10). When comparing the results predicted by these eleven models with the experimental results for this data set of columns, a correlation coefficient that varies from 0.88 to 0.92 is obtained (Figure 11). The best correlation is observed from the combination of the fib Model Code 2010 [5] and Spoelstra and Monti's model [15], that is, combination number 36 (Figure 11 (j)). Moreover, the ratio between the axial strength predicted by the eleven models and the experimental results of the columns in the data set ranged from 0.93 to 1.00.

The proposal of Machado [9] overestimated the resistance of the dataset columns, on average, by 6%. When it was analyzed statistically, this proposal was significantly different from the experimental results at a significance level of 10%. Therefore, the combinations of strengthening systems shown in Table 12 were considered more efficient than Machado's proposal [9] in the prediction of the resistance of columns strengthened by transverse reinforcement and CFRP sheet wrapping.

4. Conclusions

In this paper, some of main models used for strengthening of concrete columns with concrete jacket or wrapping with CFRP sheets were analyzed. They were applied to a database with 135 columns tested in the laboratory to evaluate the effectiveness of the design models and the results were statistically analyzed. The main conclusions are as follows:

- For strengthening with a concrete jacket, the design models proposed by Cusson and Paultre [2], Saatcioglu and Razvi [3], Frangou *et al.* [4], and the fib Model Code 2010 [5] showed good correlation with the experimental results. However, when they were applied to columns confined with transverse reinforcement (Subsection 3.2.2), only Saatcioglu and Razvi's model [3] was efficient, showing a correlation coefficient of 0.98 with the database columns;
- For columns wrapped exclusively with CFRP sheets, the models of Samaan *et al.* [10], Saafi *et al.* [14], and Spoelstra and Monti [15] predicted the experimental results best. Spoelstra and Monti's model [15] showed a better correlation with the strength of the columns in the database;

Table 9
Database of reinforced columns confined with FRP and transverse reinforcement (part 1)

Reference	Model	Dimensions of column and concrete strength				FRP					Longitudinal reinforcement			Transverse reinforcement			
		D (mm)	H (mm)	c (cm)	f_c (MPa)	Fiber type	t_f (mm)	ξ_f (‰)	E_f (MPa)	n	ϕ_l (cm)	n°	$f_{y,l}$ (MPa)	Type	ϕ_t (cm)	s (mm)	$f_{y,t}$ (MPa)
Carrazedo [6]	C1S50	190	570	1.5	26.16	CFRP	0.130	11.00	218950	1	0.8	6	554.8	Spirals	0.50	50	756
	C2S50	190	570	1.5	26.16	CFRP	0.130	8.78	218950	2	0.8	6	554.8	Spirals	0.50	50	756
	C1S25	190	570	1.5	28.86	CFRP	0.130	10.63	218950	1	0.8	6	554.8	Spirals	0.50	25	756
	C2S25	190	570	1.5	28.86	CFRP	0.130	10.65	218950	2	0.8	6	554.8	Spirals	0.50	25	756
Eid, Roy, and Paultre [20]	A5NP2C	303	1200	2.5	29.40	CFRP	0.381	13.40	78000	2	1.6	6	423	Stirrups	0.95	150	602
	A3NP2C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	2	1.6	6	550	Stirrups	0.95	70	602
	A1NP2C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	2	1.6	6	486.5	Stirrups	0.95	45	602
	C4NP2C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	2	1.6	6	423	Stirrups	1.13	100	456
	C4N1P2C	303	1200	2.5	36.00	CFRP	0.381	13.40	78000	2	1.6	6	423	Spirals	1.13	100	456
	C4NP4C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	4	1.6	6	423	Spirals	1.13	100	456
	B4NP2C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	2	1.6	6	550	Stirrups	1.13	100	456
	C4MP2C	303	1200	2.5	50.80	CFRP	0.381	13.40	78000	2	1.6	6	423	Spirals	1.13	100	456
	C2NP2C	303	1200	2.5	31.70	CFRP	0.381	13.40	78000	2	1.6	6	423	Spirals	1.13	65	456
	C2N1P2C	303	1200	2.5	36.00	CFRP	0.381	13.40	78000	2	1.6	6	423	Spirals	1.13	65	456
	C2N1P4C	303	1200	2.5	36.00	CFRP	0.381	13.40	78000	4	1.6	6	423	Spirals	1.13	65	456
	C2MP2C	303	1200	2.5	50.80	CFRP	0.381	13.40	78000	2	1.6	6	423	Spirals	1.13	65	456
	C2MP4C	303	1200	2.5	50.80	CFRP	0.381	13.40	78000	4	1.6	6	423	Spirals	1.13	65	456
Huang *et al.* [23]	P1S1	150	300	0	30.04	GFRP	0.436	16.00	60800	1	–	–	–	Spirals	0.80	25	356
	P2S1	150	300	0	30.04	GFRP	0.436	16.00	60800	2	–	–	–	Spirals	0.80	25	356
	P3S1	150	300	0	30.04	GFRP	0.436	16.00	60800	3	–	–	–	Spirals	0.80	25	356
	P1S2	150	300	0	30.04	GFRP	0.436	16.00	60800	1	–	–	–	Spirals	0.80	50	356
	P2S2	150	300	0	30.04	GFRP	0.436	16.00	60800	2	–	–	–	Spirals	0.80	50	356
	P3S2	150	300	0	30.04	GFRP	0.436	16.00	60800	3	–	–	–	Spirals	0.80	50	356

- From the 41 combinations of the reinforced column strengthened with FRP and transverse reinforcement analyzed, only eleven combinations predicted a resistance that did not differ statistically from the resistance of columns evaluated in the data set (Table 12). The best correlation was obtained for the combination of the fib Model Code 2010 [5] and Spoelstra and Monti [15],
- For column strengthening with FRP and transverse reinforcement, it was shown that there is simultaneous confinement due to both materials. Moreover, the hypothesis that the confined concrete strength can be obtained by adding the parcels of resistance due to CFRP sheet wrapping and due to transverse reinforcement separately was more effective than the hypothesis that the confined concrete resistance would be obtained from the sum of the lateral pressure due to the CFRP and transverse reinforcement.

Table 9
Database of reinforced columns confined with FRP and transverse reinforcement (part 2)

Reference	Model	Dimensions of column and concrete strength				FRP					Longitudinal reinforcement			Transverse reinforcement			
		D (mm)	H (mm)	c (cm)	f_c (MPa)	Fiber type	t_f (mm)	ξ_f (‰)	E_f (MPa)	n	ϕ_l (cm)	n°	$f_{y,l}$ (MPa)	Type	ϕ_t (cm)	s (mm)	$f_{y,t}$ (MPa)
Lee *et al.* [22]	S6F1	150	300	0.725	36.20	CFRP	0.110	18.04	250000	1	–	–	–	Spirals	0.55	60	569.6
	S6F2	150	300	0.725	36.20	CFRP	0.110	18.04	250000	2	–	–	–	Spirals	0.55	60	569.6
	S6F4	150	300	0.725	36.20	CFRP	0.110	18.04	250000	4	–	–	–	Spirals	0.55	60	569.6
	S6F5	150	300	0.725	36.20	CFRP	0.110	18.04	250000	5	–	–	–	Spirals	0.55	60	569.6
	S4F1	150	300	0.725	36.20	CFRP	0.110	18.04	250000	1	–	–	–	Spirals	0.55	40	569.6
	S4F2	150	300	0.725	36.20	CFRP	0.110	18.04	250000	2	–	–	–	Spirals	0.55	40	569.6
	S4F3	150	300	0.725	36.20	CFRP	0.110	18.04	250000	3	–	–	–	Spirals	0.55	40	569.6
	S4F4	150	300	0.725	36.20	CFRP	0.110	18.04	250000	4	–	–	–	Spirals	0.55	40	569.6
	S4F5	150	300	0.725	36.20	CFRP	0.110	18.04	250000	5	–	–	–	Spirals	0.55	40	569.6
	S2F1	150	300	0.725	36.20	CFRP	0.110	18.04	250000	1	–	–	–	Spirals	0.55	20	569.6
	S2F2	150	300	0.725	36.20	CFRP	0.110	18.04	250000	2	–	–	–	Spirals	0.55	20	569.6
	S2F3	150	300	0.725	36.20	CFRP	0.110	18.04	250000	3	–	–	–	Spirals	0.55	20	569.6
	S2F4	150	300	0.725	36.20	CFRP	0.110	18.04	250000	4	–	–	–	Spirals	0.55	20	569.6
	S2F5	150	300	0.725	36.20	CFRP	0.110	18.04	250000	5	–	–	–	Spirals	0.55	20	569.6
Yin *et al.* [24]	1L-50-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	50	335
	1L-50-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	50	335
	1L-50-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	50	335
	1L-25-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	25	335
	1L-25-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	25	335
	1L-25-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	1	–	–	–	Spirals	0.60	25	335
	2L-50-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	–	Spirals	0.60	50	335
	2L-50-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	–	Spirals	0.60	50	335
	2L-50-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	–	Spirals	0.60	50	335
	2L-25-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	–	Spirals	0.60	25	335
	2L-25-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	-	Spirals	0.60	25	335
	2L-25-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	2	–	–	–	Spirals	0.60	25	335
	3L-50-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	50	335
	3L-50-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	50	335
	3L-50-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	50	335
	3L-25-N1	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	25	335
	3L-25-N2	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	25	335
	3L-25-N3	150	300	0.5	30.60	CFRP	0.167	15.00	213000	3	–	–	–	Spirals	0.60	25	335
Wang *et al.* [21]	C1H1L1M	305	915	2.1	24.50	CFRP	0.167	17.79	244000	1	1.2	8	340.0	Stirrups	0.60	80	397
	C1H1L2M	305	915	2.1	24.50	CFRP	0.167	17.79	244000	2	1.2	8	340.0	Stirrups	0.60	80	397
	C1H2L1M	305	915	2.1	24.50	CFRP	0.167	17.79	244000	1	1.2	8	340.0	Stirrups	0.60	40	397
	C1H2L2M	305	915	2.1	24.50	CFRP	0.167	17.79	244000	2	1.2	8	340.0	Stirrups	0.60	40	397
	C2H1L1M	204	612	1.5	24.50	CFRP	0.167	17.79	244000	1	1	6	312.0	Stirrups	0.60	120	397
	C2H1L2M	204	612	1.5	24.50	CFRP	0.167	17.79	244000	2	1	6	312.0	Stirrups	0.60	120	397
	C2H2L1M	204	612	1.5	24.50	CFRP	0.167	17.79	244000	1	1	6	312.0	Stirrups	0.60	60	397
	C2H2L2M	204	612	1.5	24.50	CFRP	0.167	17.79	244000	2	1	6	312.0	Stirrups	0.60	60	397

5. Notation

AFRP Aramid Fiber Reinforced Polymers,
$A_{c,n}$ Area of core of section within center lines of transverse reinforcement,
A_g Gross area of column,
$A_{s,tx}$ Total area of transverse reinforcement parallel to y-axis,
$A_{s,ty}$ Total area of transverse reinforcement parallel to x-axis,
$A_{s,l}$ Total area of longitudinal reinforcement of the column,
$A_{s,t}$ Area of the transverse reinforcement,
b Rectangular column width,
b_c Distance between centers of longitudinal bars,
c Concrete cover,
c_x Core dimension of the column perpendicular to the x direction, measured between centers of transverse reinforcement,
c_y Core dimension of the column perpendicular to the y direction, measured between centers of transverse reinforcement,
CFRP Carbon Fiber Reinforced Polymers,
d_i Diameter of circular stirrups or spiral between bar centers,

Table 10
Comparison between the theoretical and experimental results for columns strengthened with FRP and transverse reinforcement

Comb.	References	Mean	CV
1	Cusson and Paultre [2] and Samaan *et al.* [10]	0.97	0.22
2	Cusson and Paultre [2] and Miyauchi *et al.* [11]	1.05	0.20
3	Cusson and Paultre [2] and Kono *et al.* [12]	0.84	0.24
4	Cusson and Paultre [2] and Toutanji [13]	1.13	0.20
5	Cusson and Paultre [2] and Saafi *et al.* [14]	0.93	0.21
6	Cusson and Paultre [2] and Spoelstra and Monti [15]	0.93	0.20
7	Cusson and Paultre [2] and Fardis and Khalili [16]	0.86	0.21
8	Cusson and Paultre [2] and Karbhari and Eckel [17]	0.91	0.21
9	Cusson and Paultre [2] and Mirmiran and Shahawy [18]	0.80	0.26
10	Cusson and Paultre [2] and Shehata, Carneiro, and Shehata [19]	0.86	0.21
11	Saatcioglu and Razvi [3] and Samaan *et al.* [10]	1.06	0.21
12	Saatcioglu and Razvi [3] and Miyauchi *et al.* [11]	1.15	0.20
13	Saatcioglu and Razvi [3] and Kono *et al.* [12]	0.93	0.24
14	Saatcioglu and Razvi [3] and Toutanji [13]	1.22	0.20
15	Saatcioglu and Razvi [3] and Saafi *et al.* [14]	1.02	0.21
16	Saatcioglu and Razvi [3] and Spoelstra and Monti [15]	1.02	0.20
17	Saatcioglu and Razvi [3] and Fardis and Khalili [16]	0.95	0.21
18	Saatcioglu and Razvi [3] and Karbhari and Eckel [17]	1.00	0.21
19	Saatcioglu and Razvi [3] and Mirmiran and Shahawy [18]	0.89	0.26
20	Saatcioglu and Razvi [3] and Shehata, Carneiro, and Shehata [19]	0.95	0.21
21	Frangou *et al.* [4] and Samaan *et al.* [10]	0.96	0.22
22	Frangou *et al.* [4] and Miyauchi *et al.* [11]	1.04	0.21
23	Frangou *et al.* [4] and Kono *et al.* [12]	0.83	0.24
24	Frangou *et al.* [4] and Toutanji [13]	1.12	0.20
25	Frangou *et al.* [4] and Saafi *et al.* [14]	0.92	0.22
26	Frangou *et al.* [4] and Spoelstra and Monti [15]	0.92	0.20
27	Frangou *et al.* [4] and Fardis and Khalili [16]	0.85	0.22
28	Frangou *et al.* [4] and Karbhari and Eckel [17]	0.90	0.22
29	Frangou *et al.* [4] and Mirmiran and Shahawy [18]	0.79	0.26
30	Frangou *et al.* [4] and Shehata, Carneiro, and Shehata [19]	0.85	0.22
31	*fib* Model Code 2010 [5] and Samaan *et al.* [10]	1.00	0.21
32	*fib* Model Code 2010 [5] and Miyauchi *et al.* [11]	1.09	0.19
33	*fib* Model Code 2010 [5] and Kono *et al.* [12]	0.87	0.23
34	*fib* Model Code 2010 [5] and Toutanji [13]	1.16	0.19
35	*fib* Model Code 2010 [5] and Saafi *et al.* [14]	0.97	0.21
36	*fib* Model Code 2010 [5] and Spoelstra and Monti [15]	0.97	0.19
37	*fib* Model Code 2010 [5] and Fardis and Khalili [16]	0.90	0.21
38	*fib* Model Code 2010 [5] and Karbhari and Eckel [17]	0.94	0.21
39	*fib* Model Code 2010 [5] and Mirmiran and Shahawy [18]	0.83	0.25
40	*fib* Model Code 2010 [5] and Shehata, Carneiro, and Shehata [19]	0.89	0.21
41	Machado [9]	1.06	0.23

d	Diameter of the concrete section confined by the stirrups,
D	Diameter of circular columns,
E_f	Modulus of elasticity of FRP,
f_c	Compressive strength of concrete of the column,
f_{cc}	Compressive strength of confined concrete,
$f_{cc,e}$	Compressive strength of confined concrete with transverse reinforcement,

Table 11
Results of Student's t-test for columns strengthened with FRP and transverse reinforcement.
t-critical = 1.99897

References	t
Cusson and Paultre [2] and Samaan *et al.* [10]	-0.12386
Cusson and Paultre [2] and Miyauchi *et al.* [11]	-3.05512
Cusson and Paultre [2] and Kono *et al.* [12]	5.66179
Cusson and Paultre [2] and Toutanji [13]	-4.95605
Cusson and Paultre [2] and Saafi *et al.* [14]	1.58771
Cusson and Paultre [2] and Spoelstra and Monti [15]	2.15434
Cusson and Paultre [2] and Fardis and Khalili [16]	5.33989
Cusson and Paultre [2] and Karbahari and Eckel [17]	2.77478
Cusson and Paultre [2] and Mirmiran and Shahawy [18]	6.74943
Cusson and Paultre [2] and Shehata, Carneiro and Shehata [19]	5.66797
Saatcioglu and Razvi [3] and Samaan *et al.* [10]	-2.85229
Saatcioglu and Razvi [3] and Miyauchi *et al.* [11]	-5.52444
Saatcioglu and Razvi [3] and Kono *et al.* [12]	1.99897
Saatcioglu and Razvi [3] and Toutanji [13]	-6.67271
Saatcioglu and Razvi [3] and Saafi *et al.* [14]	-1.78451
Saatcioglu and Razvi [3] and Spoelstra and Monti [15]	-1.72622
Saatcioglu and Razvi [3] and Fardis and Khalili [16]	0.73222
Saatcioglu and Razvi [3] and Karbahari and Eckel [17]	-0.96384
Saatcioglu and Razvi [3] and Mirmiran and Shahawy [18]	2.56853
Saatcioglu and Razvi [3] and Shehata, Carneiro and Shehata [19]	0.98920
Frangou *et al.* [4] and Samaan *et al.* [10]	0.23657
Frangou *et al.* [4] and Miyauchi *et al.* [11]	-2.70569
Frangou *et al.* [4] and Kono *et al.* [12]	6.19093
Frangou *et al.* [4] and Toutanji [13]	-4.71205
Frangou *et al.* [4] and Saafi *et al.* [14]	2.02394
Frangou *et al.* [4] and Spoelstra and Monti [15]	2.64534
Frangou *et al.* [4] and Fardis and Khalili [16]	5.87421
Frangou *et al.* [4] and Karbahari and Eckel [17]	3.24570
Frangou *et al.* [4] and Mirmiran and Shahawy [18]	7.20314
Frangou *et al.* [4] and Shehata, Carneiro, and Shehata [19]	6.20487
fib Model Code 2010 [5] and Samaan *et al.* [10]	-1.35751
fib Model Code 2010 [5] and Miyauchi *et al.* [11]	-4.28415
fib Model Code 2010 [5] and Kono *et al.* [12]	3.95378
fib Model Code 2010 [5] and Toutanji [13]	-5.88707
fib Model Code 2010 [5] and Saafi *et al.* [14]	0.13277
fib Model Code 2010 [5] and Spoelstra and Monti [15]	0.48751
fib Model Code 2010 [5] and Fardis and Khalili [16]	3.50500
fib Model Code 2010 [5] and Karbahari and Eckel [17]	1.20664
fib Model Code 2010 [5] and Mirmiran and Shahawy [18]	5.19730
fib Model Code 2010 [5] and Shehata, Carneiro and Shehata [19]	3.81781
Machado [9]	-2.85789

$f_{cc,exp}$	Experimental compressive strength of confined concrete,
$f_{cc,f}$	Compressive strength of confined concrete with RFP,
f_f	Tensile strength of FRP,
f_l	Lateral pressure,
f_{le}	Nominal lateral pressure,
$f_{l,e}$	Lateral pressure of transverse reinforcement,
$f_{l,f}$	Lateral pressure of CFRP,
$f_{l,f(b)}$	Lateral pressure applied on side b of cross-section,
$f_{l,f(h)}$	Lateral pressure applied on side h of cross-section,
$f_{y,t}$	Yield strength of transverse steel,
$f_{y,l}$	Yield strength of longitudinal steel,
FRP	Fiber Reinforced Polymers,
$F_{u,exp}$	Experimental ultimate load of the column,
$F_{u,theor}$	Theoretical ultimate load of the column,
GFRP	Glass Fiber Reinforced Polymers,
h	Cross-section height of the rectangular column,
H	Column height,
k_a, k_b, k_2, K_e	Coefficient of confinement effectiveness,
n	Number of CFRP sheets,
nº	Number of longitudinal reinforcements,
r'	Ratio of the column dimensions,
s	Center-to-center spacing between stirrups,
s'	Clear spacing of stirrups,
t_f	Thickness of the CFRP sheet,
w_i	Clear spacing between adjacent longitudinal steel bars,
α	Level of significance for Student's t-test,
α_n, α_s, α'	Reduction factor,
θ	Angle between the transverse reinforcement and b_c,
λ	Slenderness index,
ξ_f	CFRP deformation,
ρ_l	Longitudinal reinforcement ratio in the core section,
ρ_t	Transverse reinforcement ratio in the core section,
ϕ_l	Longitudinal reinforcement diameter,
ϕ_t	Transverse reinforcement diameter,
ω_c, ω_w	Confinement rate.

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Table 12
Design models with the best predictions of column strengthening

Comb.	References	Mean	CV
1	Cusson and Paultre [2] and Samaan *et al.* [10]	0.97	0.22
5	Cusson and Paultre [2] and Saafi *et al.* [14]	0.93	0.22
13	Saatcioglu and Razvi [3] and Kono *et al.* [12]	0.93	0.24
17	Saatcioglu and Razvi [3] and Fardis and Khalili [16]	0.95	0.22
18	Saatcioglu and Razvi [3] and Karbhari and Eckel [17]	1.00	0.22
20	Saatcioglu and Razvi [3] and Shehata, Carneiro, and Shehata [19]	0.95	0.22
21	Frangou *et al.* [4] and Samaan *et al.* [10]	0.96	0.22
31	*fib* Model Code 2010 [5] and Samaan *et al.* [10]	1.00	0.21
35	*fib* Model Code 2010 [5] and Saafi *et al.* [14]	0.97	0.21
36	*fib* Model Code 2010 [5] and Spoelstra and Monti [15]	0.97	0.20
38	*fib* Model Code 2010 [5] and Karbhari and Eckel [17]	0.94	0.21

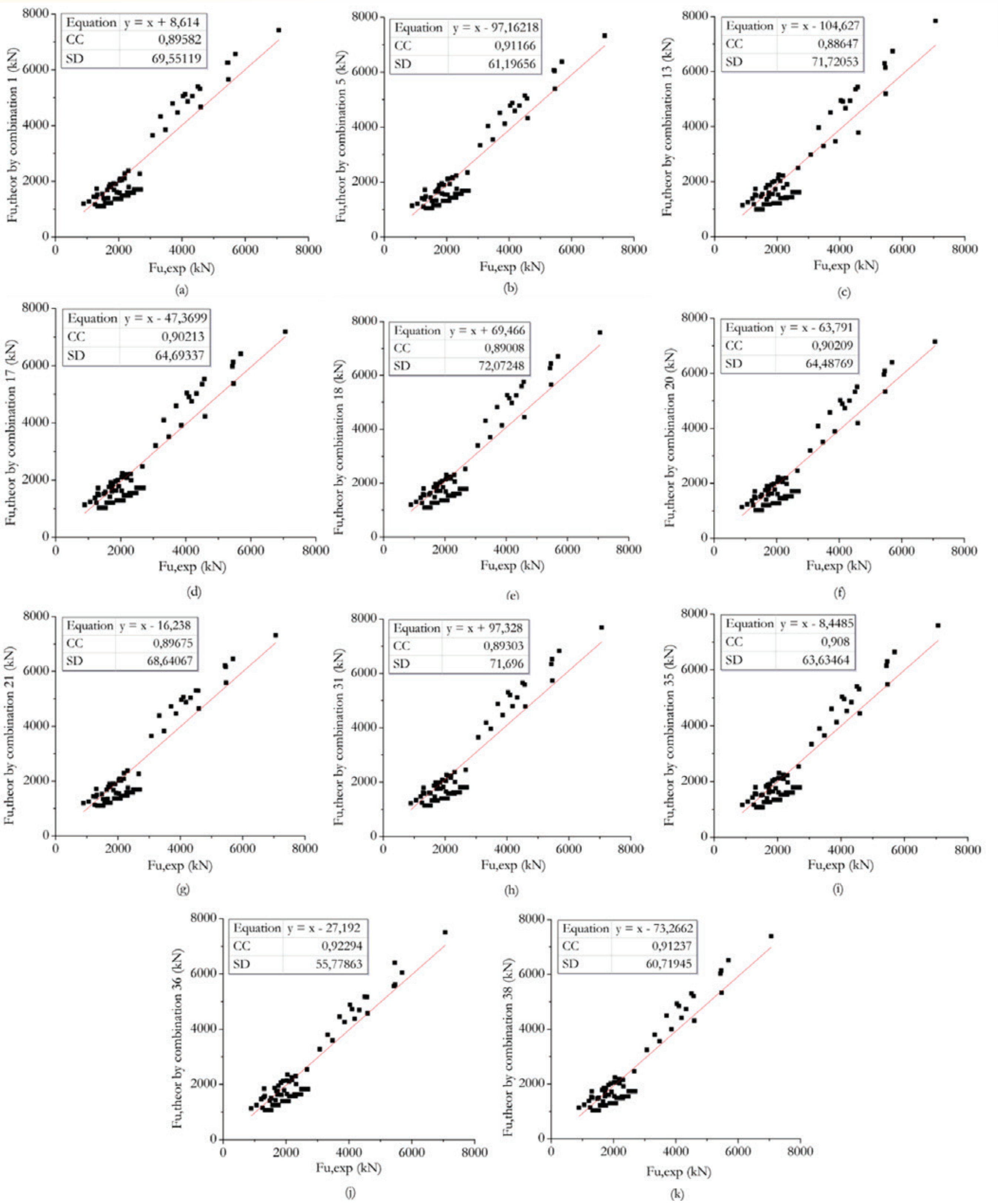

Figure 11

Comparison of theoretical and experimental results of ultimate load of the column (F_{RuR}) strengthened with FRP and transverse reinforcement. (a) Combination 1; (b) Combination 5; (c) Combination 13; (d) Combination 17; (e) Combination 18; (f) Combination 20; (g) Combination 21; (h) Combination 31; (i) Combination 35; (j) Combination 36; (k) Combination 38. CC: Correlation coefficient; SD: Standard deviation.