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Reliability of buildings in service limit state for maximum horizontal displacements

Confiabilidade de edifícios no estado limite de serviço para deslocamentos horizontais máximos



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Abstract

Brazilian design code ABNT NBR6118:2003 - Design of Concrete Structures - Procedures - [1] proposes the use of simplified models for the consideration of non-linear material behavior in the evaluation of horizontal displacements in buildings. These models penalize stiffness of columns and beams, representing the effects of concrete cracking and avoiding costly physical non-linear analyses. The objectives of the present paper are to investigate the accuracy and uncertainty of these simplified models, as well as to evaluate the reliabilities of structures designed following ABNT NBR6118:2003 [1] in the service limit state for horizontal displacements. Model error statistics are obtained from 42 representative plane frames. The reliabilities of three typical (4, 8 and 12 floor) buildings are evaluated, using the simplified models and a rigorous, physical and geometrical non-linear analysis. Results show that the 70/70 (column/beam stiffness reduction) model is more accurate and less conservative than the 80/40 model. Results also show that ABNT NBR6118:2003 [1] design criteria for horizontal displacement limit states (masonry damage according to ACI 435.3R-68(1984) [10]) are conservative, and result in reliability indexes which are larger than those recommended in EUROCODE [2] for irreversible service limit states.

Keywords: reinforced concrete, physical nonlinearity, structural reliability, plane frame structures, service limit state.

Resumo

A norma ABNT NBR6118:2003 - Projeto de Estruturas de Concreto - Procedimento - [1] propõe o uso de modelos simplificados para a consideração da não-linearidade física na avaliação de deslocamentos em estruturas de concreto armado. Estes modelos penalizam a rigidez de pilares e vigas, representando efeitos de fissuração do concreto e dispensando a realização de análises não-lineares físicas de material. O presente trabalho tem por objetivos investigar a incerteza dos modelos simplificados propostos nesta norma, bem como determinar a confiabilidade de estruturas de edifícios projetadas segundo esta norma nos estados limites de serviço para deslocamentos horizontais. Estatísticas de erro de modelo são obtidas através da análise de 42 pórticos planos representativos. A confiabilidade de três edifícios típicos (de 4, 8 e 12 andares) é analisada, utilizando-se os modelos simplificados e a análise não-linear física dita rigorosa. Os resultados mostram que o modelo 70/70 (penalização de rigidez pilar/viga) é menos conservador e mais preciso do que o modelo 80/40. Os resultados mostram ainda que os critérios de verificação da norma ABNT NBR6118:2003 [1] para estado limite de serviço de deslocamentos horizontais (tendo em vista fissuração da alvenaria pelas prescrições da ACI 435.3R-68(1984) [10]) são conservadores, e resultam em índices de confiabilidade superiores aqueles sugeridos no EUROCODE [2] para estados limites de serviço irreversíveis.

Palavras-chave: concreto armado, não-linearidade física, confiabilidade estrutural, pórtico plano, estado limite de serviço.

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1. Introduction

In the design of reinforced concrete structures, it is common practice to use simplified models which penalize the stiffness of structural elements, in order to avoid non-linear material analysis. A lot of research is dedicated to improve these simplified models. However, it is hard to find research works addressing the precision or errors of the simplified models. The objective of the present article is to investigate the precision of simplified stiffness-reducing models recommended in Brazilian code ABNT NBR6118:2003 [1] in the evaluation of horizontal displacements of plane reinforced concrete frames. The investigation is based on a comparison, for a set of representative frames, of the displacements obtained using simplified models and rigorous physical (material) non-linear analysis. This article also investigates the reliability, with respect to serviceability limit states for horizontal displacements, of plane frames representing usual reinforced concrete buildings. Reliability analyses are performed using rigorous non-linear material analysis and using the simplified models recommended in ABNT NBR6118:2003 [1]. Geometrical non-linearities are treated in a consistent way in all the analyses. Reliability analyses performed herein consider uncertainties in loads and in the structural strengths, as well as the uncertainties originated in the use of the simplified stiffness reduction models.

Non-linear structural analyses are performed using a finite element code developed by the authors (CORELHANO [3]). Reliability analyses are performed using the StRAnD software (BECK [4]).

2. Non-linear analyses in reinforced concrete

2.1 Non-linear geometrical analysis

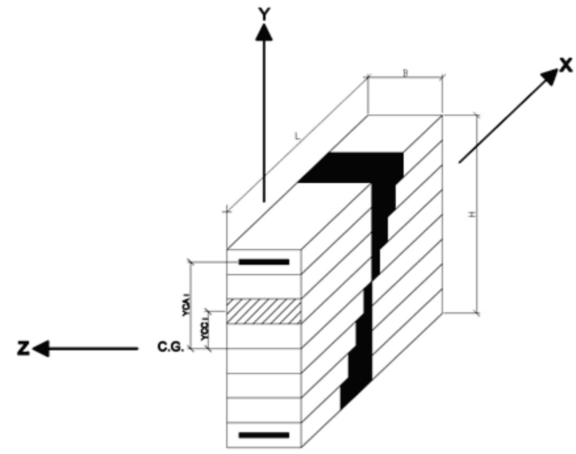
A formulation is considered based on second-order Piola Kirchhoff tensors, developed by WEN & RAMIZADEH [5]. The deformation tensor and deformation energy are given, respectively, by:

$$\varepsilon_x = u_0' - Y.v_0'' + \frac{1}{L}\int_0^L (v_0')^2.dx \qquad (1)$$

$$U = \frac{1}{2}\int_V \left[(u_0' - Y.v_0) + \frac{1}{L}\int_0^L (v_0')^2.dx \right]^2 E.dV \qquad (2)$$

where:
ε_x : longitudinal strains;
u_0 and v_0 : axial and transversal displacements;
Y : distance from a given fiber to the sessions gravity center (C.G.);
L : lenght of the element;
E : Young´s modulus;
U : internal strain energy.
Details of the formulation can be found in CORRÊA [6].

Figure 1 – Details of a 2D beam element formed by layers of steel and concrete



2.2 Rigorous material non-linear analysis

In this article, material non-linearities are considered by the method of layers, which allows independent constitutive models to be considered for each layer. The element cross-section is divided in steel and concrete slices, and the sum of the contribution of each layer defines the behavior of the cross-section (Figure 1).

Properties of the cross-section (stiffness EA and EI_z) are evaluated from the sum of the contribution of each layer, at the integration points at the extremes of each element:

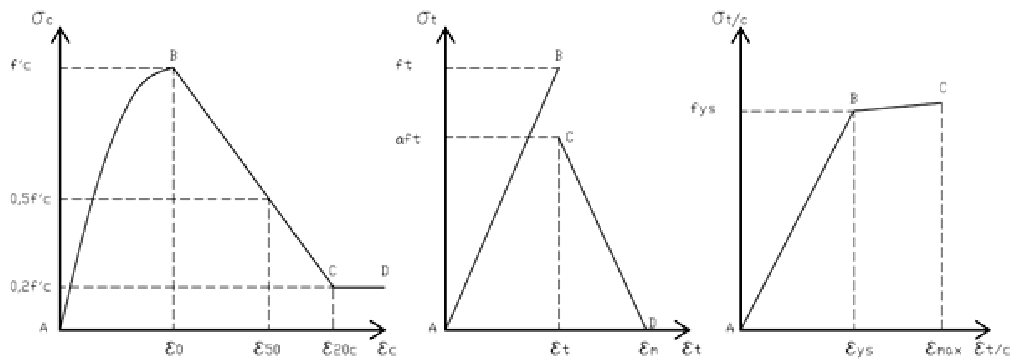
$$EA = \sum E_i.A_i \qquad (3)$$

$$EI_Z = \sum E_i.I_{Zi} \qquad (4)$$

where:
A_i: area of the i[th] layer;
E_i: Young's modulus of the i[th] layer;
Iz_i: inertia of the i[th] layer w.r.t. Z axis.
For the compressed concrete, the constitutive model of KENT & PARK[7] is adopted, following Figure 2. Segment AB of this model is described by:

$$\sigma = f_c'\left[\frac{2\varepsilon_c}{\varepsilon_0} - \left(\frac{\varepsilon_c}{\varepsilon_0}\right)^2\right] \qquad (5)$$

Figure 2 – Constitutive model for compressed (left), tensioned (centre) concretes and for steel (right)



$$f_c^{'} = f_{ck} + 3.5\text{MPa} \quad (6)$$

where:
$f_c{}'$: maximum compressive strength of concrete;
ε_0 : specific deformation of concrete corresponding to maximum tension;
ε_c : specific deformation of concrete ;
σ : tension in concrete.
Segment BC is a line defined by the point of maximum compressive strength and by the point corresponding to 50% of the maximum compressive strength. In segment CD it is admitted that the compressed concrete retains a tension corresponding to 20% of peak tension indefinitely. For the tensioned concrete, the constitutive model of FIGUEIRAS [8] is adopted, following Figure 2 (centre). For the reinforcing steel, an elasto-plastic model with hardening is considered (Figure 2, right). Details of the adopted constitutive models and of the strategies used to solve the non-linear problem are presented in CORELHANO [3].

2.3 Simplified material non-linear analysis

Brazilian code ABNT NBR6118:2003 [1] proposes two alternatives for the simplified material non-linear analysis of reinforced concrete structures. These models penalize stiffness of the cross-section, in order to take into account, in a simplified way, the effects of concrete cracking. In the first model, bending stiffness of columns

Figure 3 – Geometry of the studied frames: 4 floors (left), 8 floors (centre), and 12 floors (right), all measures in cm

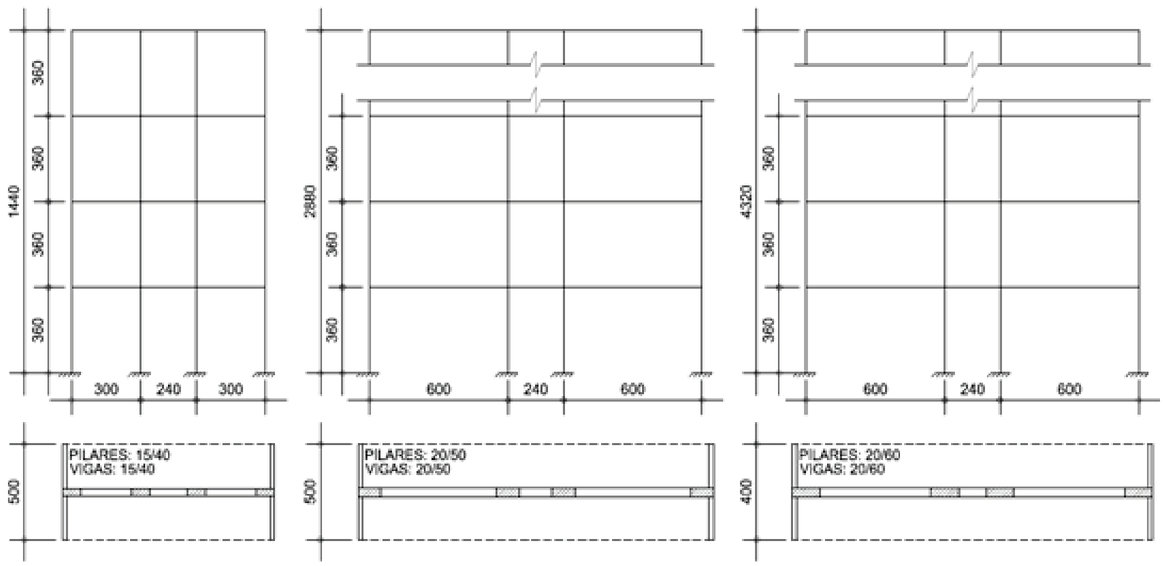
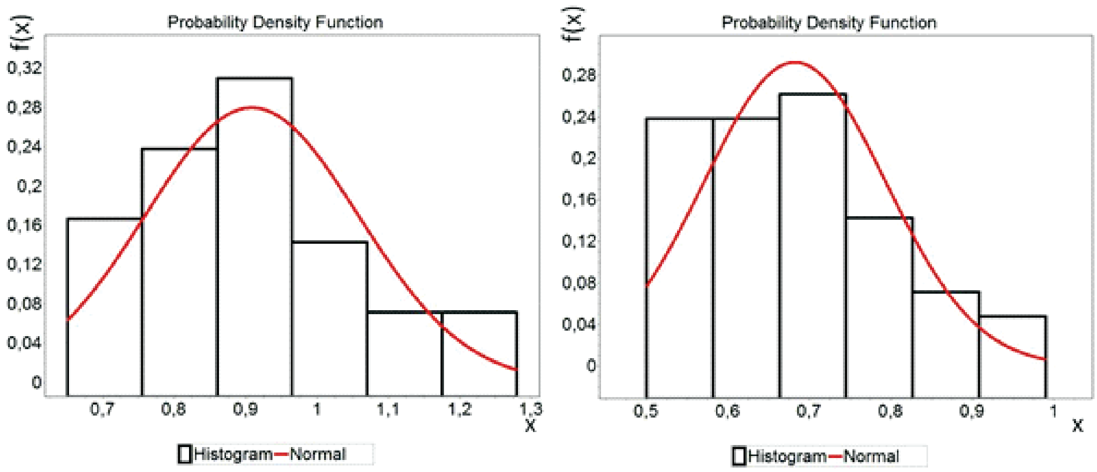


Table 1 – Evaluated horizontal displacements and model error samples

	Frame	fck (MPa)	Reinforcement ratio	Analysis			Model error	
				70/70 u (cm)	80/40 u (cm)	Rigorous u (cm)	70/70 $u^{rigorous}/$	80/40 $u^{simplifyed}$
1	4 floors / 1 bay	30	High	1.51	1.94	1.35	0.89	0.70
2	4 floors / 1 bay	30	High	2.13	2.74	2.61	1.23	0.95
3	4 floors / 1 bay	35	Medium	1.39	1.79	1.27	0.91	0.71
4	4 floors / 1 bay	35	Medium	1.96	2.53	2.51	1.28	0.99
5	4 floors / 1 bay	40	Low	1.3	1.67	1.22	0.94	0.73
6	4 floors / 1 bay	40	Low	1.84	2.36	2.13	1.16	0.90
7	8 floors / 3 bays	23	Low	1.84	2.57	1.53	0.83	0.60
8	8 floors / 3 bays	23	Low	2.62	3.68	2.56	0.98	0.70
9	8 floors / 3 bays	23	Medium	4.07	5.26	3.05	0.75	0.58
10	8 floors / 3 bays	23	Medium	5.91	7.73	5.06	0.86	0.65
11	8 floors / 3 bays	23	High	6.16	7.86	4.19	0.68	0.53
12	8 floors / 3 bays	23	High	9.15	11.85	6.74	0.74	0.57
13	8 floors / 3 bays	30	Low	1.62	2.26	1.43	0.88	0.63
14	8 floors / 3 bays	30	Low	2.31	3.23	2.44	1.06	0.76
15	8 floors / 3 bays	30	Medium	3.56	4.6	3.12	0.88	0.68
16	8 floors / 3 bays	30	Medium	5.15	6.71	5.2	1.01	0.77
17	8 floors / 3 bays	30	High	5.36	6.82	4.42	0.82	0.65
18	8 floors / 3 bays	30	High	7.9	10.1	7.2	0.91	0.71
19	8 floors / 3 bays	40	Low	1.4	1.94	1.08	0.77	0.56
20	8 floors / 3 bays	40	Low	1.98	2.77	1.87	0.94	0.68
21	8 floors / 3 bays	40	Medium	3.05	3.93	2.5	0.82	0.64
22	8 floors / 3 bays	40	Medium	4.39	5.7	4.29	0.98	0.75
23	8 floors / 3 bays	40	High	4.57	5.79	3.71	0.81	0.64
24	8 floors / 3 bays	40	High	6.68	8.54	6.14	0.92	0.72
25	12 floors / 3 bays	22	Medium	4.67	6.34	4.07	0.87	0.64
26	12 floors / 3 bays	22	Medium	6.7	9.18	7.07	1.06	0.77
27	12 floors / 3 bays	22	High	5.52	7.15	4.12	0.75	0.58
28	12 floors / 3 bays	22	High	7.97	10.45	7.03	0.88	0.67
29	12 floors / 3 bays	22	Low	4.33	5.97	3.92	0.91	0.66
30	12 floors / 3 bays	22	Low	6.2	8.63	6.8	1.10	0.79
31	12 floors / 3 bays	30	High	4.28	5.3	3.54	0.83	0.67
32	12 floors / 3 bays	30	High	5.6	7.63	6.35	1.13	0.83
33	12 floors / 3 bays	30	Medium	3.92	5.98	3.72	0.95	0.62
34	12 floors / 3 bays	30	Medium	6.4	8.65	6.58	1.03	0.76
35	12 floors / 3 bays	30	Low	3.63	4.99	3.4	0.94	0.68
36	12 floors / 3 bays	30	Low	5.16	7.18	6.07	1.18	0.85
37	12 floors / 3 bays	40	Medium	3.37	4.55	2.64	0.78	0.58
38	12 floors / 3 bays	40	Medium	4.8	6.52	3.56	0.74	0.55
39	12 floors / 3 bays	40	High	3.97	5.13	2.88	0.73	0.56
40	12 floors / 3 bays	40	High	5.68	7.38	3.72	0.65	0.50
41	12 floors / 3 bays	40	Low	3.12	4.29	2.52	0.81	0.59
42	12 floors / 3 bays	40	Low	4.45	6.14	3.38	0.76	0.55

Figure 4 – Histograms and probability distribution functions of model error variables: 70/70 stiffness reduction (left), 80/40 column/beam stiffness reduction (right)



and beams are multiplied by 0,70. In the second model, equivalent stiffness are obtained by multiplying the stiffness of columns and beams by 0,80 and 0,40, respectively. In this article, these models are referred to as 70/70 and 80/40, respectively.
The secant Young´s modulus of concrete is:

$$E_{sec} = 0.85 \cdot 5600\sqrt{fck} \quad (7)$$

where:
E_{sec}: secant Young´s modulus;
f_{ck}: characteristic concrete resistance at 28 days.

3. Model errors

The simplified stiffness reducing models proposed in ABNT NBR6118:2003 [1] are, naturally, approximations of reality. A variable that measures the accuracy or precision of these, called model error, is obtained by dividing the displacements obtained via a rigorous material non-linear analysis by the displacements obtained using the simplified model (OLIVEIRA et al. [9]):

$$E_M = \frac{u_{\text{rigorous}}}{u_{\text{simplifyed}}} \quad (8)$$

This is a random variable as, for different structures, the simplified model can be more or less precise. One sample (set of observations) of the model error random variable is obtained by evaluating equation (8) for a set of different structural configurations. In this article, a sample of the two model error random variables is obtained by evaluating 42 representative plane frames of different geometries, materials and reinforcement ratios. Frames of four, eight and twelve floors are considered, with one to three bays. The studied frames are variations from the frames represented in Figure 3. Concrete resistances varied from 20 to 40 MPa. Three reinforcement ratios were considered: low, medium and high. The low reinforcement ratio is close to the lower limit, medium is around 2% and high is close to the upper limit (3 to 4%) allowed in ABNT NBR6118:2003 [1]. Vertical loading was determined based on the process of influence areas (slabs, beams, columns, walls and coverings). Accidental load was adopted as 1,5 kN/m² in the influence area. Details of the studied frames are presented in Table 1. The table also presents the model error observations obtained for these frames.
Figure 4 shows the histograms that were obtained from the model error samples, as well as the probability distribution functions that were adjusted to the data. For the simplified model with 70/70 stiffness reduction, a Normal distribution was obtained with parameters:

$$E_M^{70/70} \sim N(\mu=0.908,\ \sigma=0.150) \quad (9)$$

For the simplified model with 80/40 column/beam stiffness reduction, a Normal distribution was obtained with parameters:

$$E_M^{80/40} \sim N(\mu=0.682,\ \sigma=0.111) \quad (10)$$

The coefficient of variation (c.o.v) is similar for both models error variables ($\sigma/\mu=0.16$).
The Normal distribution resulted in a good fit for both variables, as indicated by the statistics shown in Table 2. For both cases, the Normal distribution passed the Kolmogorov-Smirnov, Anderson Darling and Chi-square goodness-of-fit tests.

Table 2 – Statistics of goodness-of-fit tests for the Normal distribution

Model error	Distribution	Goodness-of-fit test		
		Kolmogorov-Smirnov	Anderson Darling	Chi Squared
70/70	Normal	0.10675	0.53353	1.7787
80/40	Normal	0.10290	0.58495	1.4824

The studied model errors (Eq. 8) compare displacements which represent load effects. Hence, model error values smaller than unity indicate a conservative model, as $u^{simplified} > u^{rigorous}$. It can be observed that both models are conservative, on average, as the means are smaller than unity (μ < 1). The 70/70 is moderately conservative, with mean just below unity, whereas the 80/40 model is very conservative, with mean equal to 0.682. Since the c.o.v. is the same for both variables (σ/μ=0.16), one concludes that the 70/70 model is more precise.

4. Structural reliability analysis

4.1 Design and verification of the frames

For the reliability analysis with respect to maximum displacements limit state, three frames were designed: with four, eight and twelve floors (following Figure 3). Design of the frames followed guidelines of ABNT NBR6118:2003 [1] for ultimate limit states. Once the proportioning of the frames was complete (for ultimate limit states), their flexibility was increased until they reached the maximum horizontal displacement allowed by ABNT NBR6118:2003 [1]. The verification with respect to horizontal displacements was made for frequent load combination, with maximum displacement of H/1700, where H is the total height of the building. For the frequent load combination, one has:

$$F_{ser} = \sum F_{giK} + \psi_1 F_{q1K} + \sum \psi_2 F_{qjK} \quad (11)$$

where:
F_{giK} : permanent actions;
F_{q1K} : main variable action;
F_{qjK} : secondary variable action;
ψ_1 : combination coefficient for the main variable action;
ψ_2 : combination coefficient for the secondary variable action.
For the studied buildings, only one equation is obtained, as the combination factor for wind, when considered the secondary action, is null. Hence, one obtains:

Table 3 – Characteristic and nominal resistance and load values considered in design

Variable	Symbol	Characteristic or nominal values		
		4 floors	8 floors	12 floors
Concrete strength	f_{ck}	25 MPa	25 MPa	30 MPa
Dead load	D_n	24 kN/m	25.5 kN/m	22 kN/m
Life load	L_n	7.5 kN/m	7.5 kN/m	6 kN/m
Wind load at the floors	W_n	13.5 kN	11.4 kN	13.5 kN

Table 4 – Horizontal displacements at the top of the buildings

Floors	Height H (m)	Displacement at top (mm)		
		70/70	80/40	Limit displacement (H/1700)
4	14.4	6.62	8.10	8.47
8	28.8	12.09	16.50	16.94
12	43.2	18.29	25.05	25.41

$$F_{ser} = D_n + 0.3W_n + 0.3L_n \quad (12)$$

where:
F_{ser} : combined action value for service limit states;
D_n : nominal value of dead load;
W_n : nominal value of wind action;
L_n : nominal value of life action.
Table 3 summarizes the characteristic values (f_{ck}) and nominal load values (D_n, L_n, W_n) used to verify the frames in the service limit state. Table 4 shows the results obtained, in terms of the horizontal displacements at the top of the studied buildings. It can be observed in this table that the representative frames were designed for maximum flexibility.

4.2 Data for reliability analysis

For the service limit state related to horizontal displacements, the "failure" condition is given by a displacement at the top of the building larger than H/500. This displacement is related to damage of the masonry. This limit, indicated by ACI 435.3R-68(1984) [10], is virtually equivalent to the H/1700 limit considered in the Brazilian code [1], when the load combination factor for wind load is 0.3 and the structural response is linear. Hence, the (service) limit state equation for horizontal displacements is:

$$g\left(E_M, f_c, D, L, W\right) = E_M \cdot u^{\text{evaluated}}\left(f_c, D, L, W\right) - H/500 \quad (13)$$

where E_M, f_c, D, L e W are the random variables of the problem, described in Table 5. The parameters and probability distribution functions of actions (D, L e W) are evaluated as indicated in Table 5, using the nominal values shown in Table 3.

The reduced loads for service verification (Eq. 12) correspond to frequent load combinations, to which the structure will be exposed during its design life. In principle, the reliability analysis for service limit state could be performed for frequent loads, by combining the arbitrary-point-in-time life load with the annual maximum of the wind load. This analysis would result in an annual failure probability, which would have to be compared with the annual target for irreversible service limit state (β_{target}=2.9 following the EUROCODE [2]). Alternatively, the 50-year maximum of these actions can be considered, in order to evaluate reliability for the same period (design life of the structure). In this case, the target reliability following the EUROCODE [2] is β_{target}=1.5 (for irreversible service limit states). In the first case, the probability being evaluated is the probability that the limit state will occur any year during the structure´s life. In the second situation, one calculates the probability of the limit state occurring at least once during the building›s design life. In this article, the second situation is adopted, as it is considered to be more representative of the desired situation for a building (no damage to masonry during the structures lifetime)
Reliability analyses, considering extreme actions, are made for two load combinations: the first considers the 50-year extreme of the life load, combined with the annual maximum of the wind load; the second considers the 50-year extreme wind combined with the arbitrary-point-in-time of the life load (the value at any point in time). These combinations are usual, when time-dependent reliability problems are converted in time-independent problems (ELLINGWOOD et al.[11], BECK & SOUZA JR, [12]). The parameters and probability distributions of: the 50 year extreme live and wind loads, the annual maximum wind loads and the arbitrary-point-in-time life load are presented in Table 5.
Following the First-Order Reliability Method (FORM), failure probabilities are evaluated by:

$$P_f = \int_{g(\mathbf{x})<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \approx \Phi(-\beta) \quad (14)$$

where **X** is the vector of random variables, $g(\mathbf{x})$ is the limit state

Table 5 – Random variables, their parameters and distributions

Random varaible	Distrib.	Média	Desvio-padrão	C.V.	Fonte
70/70 model error	Normal	0.908	0.150	0.165	this work
80/40 model error	Normal	0.682	0.111	0.162	this work
f_c	Normal	fck + 1.65.σ	4.00 MPa	0.150	MELCHERS (13)
Dead load	Normal	1.05 D_n	0.105 D_n	0.100	ELLINGWOOD et al.(11)
Arbitrary-point-in-time life load	Gamma	0.25 L_n	0.148 L_n	0.55	ELLINGWOOD et al.(11)
50-year extreme life load	Gumbel	1.00 L_n	0.250 L_n	0.25	ELLINGWOOD et al.(11)
Annual extreme wind load	Gumbel	0.33 W_n	0.155 W_n	0.47	BECK & SOUZA JR. (12)
50-year extreme wind load	Gumbel	0.90 W_n	0.306 W_n	0.34	BECK & SOUZA JR. (12)

Table 6 – Results for 50-year extreme live load combined with annual maximum wind load

N. Floors	Model	β_{aprox}	P_f	Sensitivity coefficients E_M	f_c	D	L	W
4	70/70	4.019	2.92 E-5	0.305	0.025	0.0	0.0	-0.669
	80/40	4.265	9.97 E-6	0.301	0.027	0.0	0.0	-0.672
8	70/70	4.292	8.84 E-6	0.344	0.026	0.0	0.0	-0.630
	80/40	4.331	7.43 E-6	0.308	0.030	0.0	0.0	-0.662
12	70/70	4.116	1.92 E-5	0.302	0.023	0.0	0.0	-0.675
	80/40	4.159	1.60 E-5	0.292	0.020	0.0	0.0	-0.688

equation (Eq. 13), Φ( ) is the cumulative standard Gaussian distribution function and β is the reliability index. In this article, equation (14) is solved by the FORM method (MELCHERS, [13]), using the StRAnD software (BECK [4]). In the FORM method, the original problem is transformed to the standard normal space, and solved as a restricted optimization problem: the reliability index is the smallest distance between the limit state equation and the origin of the standard normal space. The reliability index is related to the failure probability by means of:

$$\beta = -\Phi^{-1}(P_f) \tag{15}$$

4.3 Reliability analyses using simplified stiffness reducing models: results

Tables 6 and 7 show results of the reliability analysis, for the 50-year extreme live load combined with annual maximum wind load (Table 6) and using the 50-year extreme wind load combined with the arbitrary-point-in-time live load (Table 7). Results refer to reliability analysis using the simplified models with stiffness reduction and non-linear geometrical analysis. It can be observed that the load combination involving the 50-year extreme wind load (Table 7) leads to larger "failure" probabilities than the combination involving the 50-year extreme live load. This is to be expected, as the wind load acts directly in the direction of the calculated displacements.

The term "failure", in this context, is used between quotes, as it represents failure in respecting the constraint of maximum displacement (H/500) which, in theory, corresponds to a state of masonry damage. This damage is an irreversible limit state. As a reference, annex C of the EUROCODE [2] suggests a target reliability index of β_{target}=1.5 for irreversible limit states and for 50 years reference. The reliability indexes found in this article, which correspond to very flexible structures designed following ABNT NBR6118:2003 [1], are slightly larger that this target value. Hence, failure probabilities can be considered acceptable. These results show that the horizontal displacement verifications of Brazilian code ABNT NBR6118:2003 [1] (Eq. 13), together with the maximum allowed displacement of H/1700 (for frequent load combinations) are conservative.

Table 7 – Results for 50-year extreme wind load combined with arbitrary-point-in-time life load

N. Floors	Model	β_{aprox}	P_f	Sensitivity coefficients E_M	f_c	D	L	W
4	70/70	2.127	1.60 E-2	0.226	0.029	0.0	0.0	-0.745
	80/40	2.369	8.90 E-3	0.216	0.030	0.0	0.0	-0.754
8	70/70	2.441	7.33 E-3	0.238	0.030	0.0	0.0	-0.732
	80/40	2.453	7.08 E-3	0.218	0.032	0.0	0.0	-0.750
12	70/70	2.235	1.27 E-2	0.232	0.022	0.0	0.0	-0.746
	80/40	2.253	1.21 E-2	0.214	0.023	0.0	0.0	-0.763

Table 8 – Results for 50-year extreme live load combined with annual maximum wind load

N. Floors	$\beta_{rigorous}$	P_f	f_c	Sensitivity coefficients		
				D	L	W
4	4.957	3.58E-07	0.079	0.000	0.000	-0.921
8	5.016	2.64E-07	0.267	-0.004	-0.002	-0.727
12	5.129	1.46E-07	0.050	0.000	0.000	-0.950

Tables 6 and 7 also show sensitivity coefficients of the problems random variables. These coefficients show the relative importance of each random variable towards failure. As expected, the horizontal wind action has the largest contribution towards this displacement failure. Uncertainty in concrete strength, which by way of Eq. (7) affects the Young´s modulus, has minimal contribution. The model error random variables have a significant contribution (from 21 to 34%) in the evaluated failure probabilities.
It is significant to note that reliability indexes obtained using the simplified stiffness-reducing models 70/70 and 80/40 are similar. This result is, in part, consequence of incorporating model error random variables in the analysis. In the next section, it is verified whether these reliability indexes agree with results of a rigorous physical and geometrical non-linear analysis.

4.4 Reliability analysis using rigorous physical and geometrical non-linear analysis: results

Tables 8 and 9 show results of the rigorous reliability analysis, using the 50-year extreme live load (Table 8) and the 50-year extreme wind load (Table 9). Results in both tables correspond to reliability analysis performed using rigorous physical and geometrical non-linear analysis.
As in the simplified analysis, sensitivity coefficients show the same behavior, with the wind load being the most important variable for this horizontal displacement failure mode.
It is observed that reliability indexes obtained with the rigorous analysis are significantly larger than those using the simplified stiffness-reducing models. For the combination involving 50-year extreme live loads (less relevant), reliability indexes obtained in the rigorous analysis were larger than for the simplified analysis. For the combination involving 50-year extreme wind loads, different results were obtained for the three frames studied. For the four and twelve floor buildings, larger reliability indexes were obtained. For the eight floor building, a smaller reliability index was obtained in the rigorous analysis. This result may be a particularity of the frames studied. However, since the rigorous physical analysis is more precise, one can conclude that the stiffness-reducing simplified models can be used for design, but cannot be used for reliability analysis (even if model errors are considered).
Since reliability indexes found in the rigorous analysis are all larger than β=1.5, one concludes that the design criteria of ABNT NBR6118:2003 [1] with respect to the service limit state for horizontal displacements (masonry damage) are conservative.

5. Concluding remarks

This article presented a study of model errors for the simplified stiffness-reducing models proposed in ABNT NBR6118:2003 [1] for evaluation of horizontal displacements of reinforced concrete plane frames. A limited analysis composed of 42 plane frames of four, eight and twelve floors has shown that the 70/70 model is more precise than the 80/40 (column/beam stiffness reduction) model.
Reliability analyses for service limit state of horizontal displacements (masonry damage) were made using the simplified stiffness-reducing models and using rigorous physical non-linear analysis. It was observed that the simplified models are appropriate for a verification of the structural design, but are not suitable for reliability analyses (even if model errors are considered).
It was found that the load combination involving the 50-year extreme live load is not relevant for the limit state of horizontal displacements, even when geometrical non-linear effects are considered. The combination involving the 50-year extreme wind and the arbitrary-point-in-time live load is more critical and leads to smaller reliability indexes. Since these reliability indexes are larger than EUROCODE-recommended values, it is concluded that the design and verification criteria of Brazilian code ABNT NBR6118:2003 [1] for horizontal displacements (Eq. 13 and the maximum allowed

Table 9 – Results for 50-year extreme wind load combined with arbitrary-point-in-time life load

N. Floors	$\beta_{rigorous}$	P_f	f_c	Sensitivity coefficients		
				D	L	W
4	2.747	3.00E-03	0.088	0.000	0.000	-0.912
8	2.293	1.09E-02	0.000	0.000	0.000	-1.000
12	2.955	1.56E-03	0.057	0.000	0.000	-0.943

displacement limit of H/1700) are conservative, and result in acceptable reliability indexes for the irreversible limit state of masonry damage.

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