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Damage localization and quantification of tower structures based on the super-element method

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Abstract

Tower structures are sensitive to hurricanes or earthquakes, whereupon they are easily damaged due to large deflection and dynamic responses. Herein, a method is proposed to accurately identify the location and extent of damage in tower structures. Firstly, a tower structure is divided into several sections along its height, and each section is regarded as a super element. Based on the finite element method (FEM), the displacement, mass, and stiffness matrices of a super element are constructed to establish the free motion equation of tower structures. Secondly, the stiffness of each component of the tower structure is included in a coefficient as the damage parameter. The first-order partial derivative of the frequencies and mode shapes of the structure for the damage parameters is obtained through Taylor expansion to construct over-determined linear equations with the damage parameters as unknown. The values of the damage parameters can be obtained by solving the equations, and the locations and extent of damages of the structure can be obtained according to the number and values of the parameters. Furthermore, to greatly improve the accuracy of the damage identification, the modification of modal truncation error is proposed. Finally, the numerical simulation of a 12-story steel TV tower verifies the feasibility and effectiveness of the proposed method.

Keywords: damage detection; tower structures; super elements; dynamic parameters; modal truncation error.

1. Introduction

Tower structures are a type of slender and lofty structures, whose altitude is much greater than its width, often by a significant margin. Since the cross section of tower structures is a relatively small lateral load, it plays an important role in the structural dynamic responses [Haoxiang, He Xin Xie, and Wentao Wang, 2017]. Because of the tall and beautiful form of tower structures, they are widely used in the field as communication facilities, power facilities, as well as in chemical engineering, and so forth. Compared with other common building structures, tower structures have weak horizontal stiffness, so they are sensitive to hurricanes and earthquakes, and prone to generate large static deflection and even damage. Fig.1 shows the wind-induced collapse accident of the 5237 liner with 500 kV transmission tower, and 10 transmission towers were broken once in Jiangsu, China. Therefore, the research on damage diagnosis of tower structures has received increasing attention.

Figure 1 - Collapsed tower in Jiangsu.

Hui Hou *et al.* [2019] proposed an assessment method for damage probability of a transmission line-tower system suffering typhoons. Taking into account the difference between actual and in-design wind loads, they established a physical model to calculate the damage probability of the transmission line and power tower. And then, the damage probability of the system was obtained by being combined with the above two parts. However, this method cannot determine the location and extent of damages. Zhou Ling *et al.* [2016] forwarded a new damage detection index with clear physical signification for the damage detection of transmission towers. They carried out a damage detection test on an elastic model of a transmission tower with 1/35 scale, and the results showed that the proposed index is feasible and effective. Hermes Carvalho *et al.* [2016] experimented on a suspension tower of a 138kV transmission line in use and verified that the geometric nonlinear has a great influence on transmission towers. Zhuoqun Zhang *et al.* [2013] established two finite element models for the single tower and tower-line system to simulate the wind-induced progressive collapse with the birth-to-death element technique in ABAQUS /Explicit. The numerical simulation results demonstrated that the collapse mechanism of the transmission tower-line system depended on the number, position, and last deformation of damaged elements.

Due to numerous components of tower structures, improving calculation efficiency plays an important role in analyzing them. The super element method (SEMD) largely reduces degrees of freedom and greatly improves calculation efficiency [Ma Hongliang, Jia Haitao and Liu Wei, 2009]. Liu *et al.* [1995] applied SEMD to complicated special frame structures for the first time. Their work indicates that it is a very practical and effective method with few degrees of freedom and much less computational work. Cao *et al.* [2000] used SEMD to successfully perform nonlinear analysis of complex structures. A new super-element with twelve degrees of freedom to be used in finite element modeling of latticed columns was presented by A. Fooladi *et al.* [2015]. To show the calculation accuracy of the super-element, they performed both linear and nonlinear analysis on a three-dimensional frame. The outcome revealed that the currently practiced model for latticed columns suffers from some major shortcomings that are resolved to some extent by the proposed super-element. M.P. Galanin *et al.* [2014] presented different variants of SEMD for the simulation of media with small inclusions and implemented the algorithm for the construction of SEMD. A numerical method for determining the characteristics of arbitrary super elements was developed by Alexander Tsybenko *et al.* [2012]. They built simulation models with two-node super elements and demonstrated the efficacy of the method in the structural analysis of elastic systems. Cao Zhiyuan *et al.* [1994] applied SEMD to the computation of a tower structure with sixteen layers, and compared the calculation results of SEMD with FEM. The results of their work showed that SEMD was more effective.

As can be seen from the above, although many scholars have studied and analyzed the damage of tower structures from many aspects, they cannot determine the location and degree of the damage, which is the greatest concerned. SEMD is widely used in the analysis of complex structures with its high computational efficiency. Based on the two aspects, a method is proposed to identify damages of tower structures in the article, which not only accurately diagnose the damage location of the structures, but also precisely determine the degree of damage. The idea is: Firstly, a tower structure is divided into several sections along its height, and each section is regarded as a super element with eight nodes. Based on FEM, the free motion equation of the tower structure is established. Secondly, the damage of the tower structure will inevitably cause a change in the stiffness of each component, so the stiffness of each component is included in a coefficient as the damage parameter. And then, the frequency and mode of the structure are regarded as a function of the damage parameters, and the first-order partial derivative of the frequency and mode to the damage parameters is obtained through Taylor expansion. Finally, the overdetermined linear equations with the damage parameters as unknown is constructed, which is solved to obtain the values of the damage parameters. If the value of the damage parameter is not zero, the corresponding component is the damaged one, otherwise, it is undamaged, that is, by querying the number of damaged components and the values of their damage parameters, you can know the location and degree of damages for the entire structure. When constructing the equations, only the first few modes and frequencies of the structure are used, so there is a mode truncation error. In order to improve the accuracy of damage identification, a model error correction technique is proposed. Numerical simulation of a 12-layer steel TV tower verifies the effectiveness and feasibility of the proposed method.

2. Super elements

Tower structures are characterized in that their component members are rods that only bear axial forces, whose height is much larger than their width. Therefore, we can divide one tower structure into several segments along its height, and a segment represents a super element.

2.1 Displacement modes of super element

A spatial 8-node element is adopted as a super element, see Fig.2.

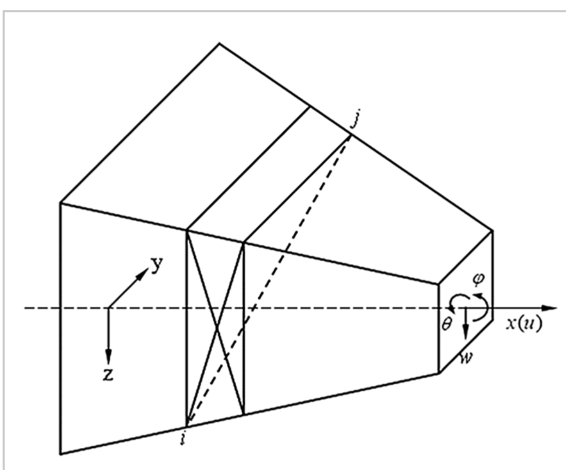

Figure 2 - Super element.

For the super element, there are four deformations in the plane such as tensile deformation, bending deformation, shear deformation and torsional deformation, whereby four independent variables along the axis x are taken as follows [Cao Zhiyuan, Liu Yongren and Zhou Hanbin, 1994; Cao Zhiyuan and Fu Zhiping, 2000; Liu Yongren, Chao Zhiyuan, 1995].

$$u = \sum_{k=1}^{8} N_k u_k , \qquad w = \sum_{k=1}^{8} N_k w_k , \qquad \varphi = \sum_{k=1}^{8} N_k \varphi_k , \qquad \theta = \sum_{k=1}^{8} N_k \theta_k \tag{1}$$

where u_k, w_k, φ_k and θ_k are the displacements of the k th node; N_k are the interpolation functions, which can be calculated by the following equations [Wang Xucheng, 2003].

$$N_k = \frac{1}{8}(1+x_0)(1+y_0)(1+z_0) \qquad (k = 1, \text{L}, 8)$$
$$x_0 = x_k x \qquad y_0 = y_k y \qquad z_0 = z_k z \tag{2}$$

where x_k, y_k and z_k are the coordinate values of the k th node. Eq. (1) can also be expressed in matrix form as

$$\{U\} = \{u\, w\, \varphi\, \theta\} = [N]\{\delta\}^e = [N_1 I_4 N_2 I_4 \text{L}\, N_8 I_4][u_1 w_1 \varphi_1 \theta_1 \text{L}\, u_8 w_8 \varphi_8 \theta_8]^T \tag{3}$$

where I_4 is a fourth-order unit matrix.

2.2 Finite element analysis of components

When analyzed by the finite element method, every component of the tower structure is treated as a rod with three degrees of freedom at each end. The node displacement vector of the component ij is

$$\{\delta\}_b = [u_i\ v_i\ w_i\ u_j\ v_j\ w_j]^T \tag{4}$$

The stiffness matrix and mass matrix of the component ij are

$$[K^b] = \int_\Gamma [B^b]^T [D^b][B^b]\, dV \tag{5}$$

$$[M^b] = \int_\Gamma [N^b]^T [\rho^b][N^b]\, dV \tag{6}$$

where $[B^b]$, $[D^b]$ and $[N^b]$ are the strain matrix, elastic matrix and shape function matrix, respectively.

2.3 Conversion of operational matrix for rod elements

The node displacements of the component can be also expressed

$$\begin{cases} u_i = u - \varphi z_i, & v_i = \theta z_i, \quad w_i = w - \theta y_i \\ u_j = u - \varphi z_j, & v_j = \theta z_j, \quad w_j = w - \theta y_j \end{cases} \tag{7}$$

The values in the global coordinate system of the node i and j are $(x_i\ y_i\ z_i)$ and $(x_j\ y_j\ z_j)$ and , respectively. From Eq. (7), the displacements of the node i and j can be expressed by the displacements of the eight nodes of the super-element

$$\{\delta\}_b = [u_i v_i w_i u_j v_j w_j]^T = [E]_b \{\delta\}^e \tag{8}$$

where $[E]_b$ is the transformation matrix. The stiffness matrix and mass matrix of the component ij are respectively

$$\begin{cases} [K_b]^e = [E]_b^T [K_b][E]_b \\ [M_b]^e = [E]_b^T [M_b][E]_b \end{cases} \tag{9}$$

where $[K_b]^e$ and $[M_b]^e$ are all thirty-second order matrixes.

2.4 The stiffness matrix and mass matrix of the super element

According to the variation principle, the stiffness matrix and mass matrix of the super element are formed by simply superposing the corresponding operation matrices of all its components

$$[K]^e = \sum_e [K_b]^e$$
$$[M]^e = \sum_e [M_b]^e \tag{10}$$

3. Basic equations of damage diagnose

When the damage parameters regarded as the stiffness coefficients of each rod, the tensile-compressive stiffness of the i th component of the j th super element is

$$(EA)_{ij} = (EA)\,(1\text{-}D_{ij}) \tag{11}$$

where D_{ij} is the damage parameter of the i th component of the j th super element and used to describe the damage severity. The range of its values is from 1 to 0, when D_{ij} is equal to 0, it means that the component is intact; when D_{ij} is equal to 1, it means that the component is completely damaged. The larger the value of D_{ij}, the more serious the damage. EA is the tensile-compressive stiffness of the undamaged rod.

The elastic behavior of a system can be expressed either in terms of the stiffness or the flexibility. We write the equations of motion for the normal mode vibration in terms of the stiffness:

$$M\ddot{U} + K_d U = \{\,0\,\} \tag{12}$$

where M and K_d are the mass and stiffness matrix of the damaged structure, respectively. We assume that damage is not accompanied by a change in the mass matrix; U is the n -dimension displacement vector. The general solution to Eq. (12) is $U = \Phi\sin\,(\,\omega t + \theta)$, where Φ and ω are the amplitude vector and the natural frequency, respectively. Substituting into Eq. (12) gives

$$K_d\,\Phi_r = \omega_r^2 M\Phi_r \tag{13}$$

where ω_r and Φ_r are the r th natural frequency and mode shape, respectively. The following equation defines the orthogonal character of the normal modes

$$\begin{cases}\Phi_r^T M\Phi_s = \delta_{rs}\\ \Phi_r^T K\Phi_s = \omega_r^2\delta_{rs}\end{cases} \tag{14}$$

where $\delta_{rs} = \begin{cases}1\,, & r = s\\ 0\,, & r \neq s\end{cases}$. With the frequency ω_r being regarded as the function of the damage parameter D_{ij}, the function is expanded as the Tailor series given by

$$\omega_r = \omega_r^0 + \sum_{j=1}^{q}\sum_{i=1}^{p}\left(\partial\omega_r/\partial D_{ij}\right)D_{ij} + \frac{1}{2}\sum_{j=1}^{q}\sum_{i=1}^{p}\left(\partial^2\omega_r/\partial D_{ij}^2\right)D_{ij}^2 + \cdots \tag{15}$$

where ω_r^0 is r th th natural frequency of the intact structure, p is the number of components of the j^{th} super element, and q is the number of super elements. With the item including the first power of D_{ij} in Eq. (15) being only reserved, the difference of frequency between the damaged structure and the intact is:

$$\Delta\omega_r = \omega_r - \omega_r^0 = \sum_{j=1}^{q}\sum_{i=1}^{p}\left(\partial\omega_r/\partial D_{ij}\right)D_{ij} \quad \left(r = 1, 2, \cdots, N\right) \tag{16}$$

where $\partial\omega_r/\partial D_{ij}$ is the constant term and solved later, and N is the total number of components. In the same way, the difference of mode shape of the damaged structure and the intact can be obtained

$$\Delta\Phi_r = \Phi_r - \Phi_r^0 = \sum_{j=1}^{q}\sum_{i=1}^{p}\left(\partial\Phi_r/\partial D_{ij}\right)D_{ij} \quad \left(r = 1, 2, \cdots, N\right) \tag{17}$$

where $\partial\Phi_r/\partial D_{ij}$ is the constant term too and solved later. Due to the incomplete measurement, only the first n-order mode shape can be taken ($n \le N$). When r changes from 1 to n, the linear equations for damage parameters as unknowns can be constructed. The whole damage parameters can be found by solving the following algebraic equations, that is, the damaged components are identified.

$$\Delta = SD \tag{18}$$

where $\Delta = \{\,\Delta\omega_1, \cdots, \Delta\phi_{11}, \cdots, \Delta\phi_{n1}, \cdots, \Delta\phi_{1n}, \cdots, \Delta\phi_{nn}\}$ is the array of difference of both frequency and mode shape; $D = \{\,D_{11}, \cdots, D_{ij}, \cdots, D_{pq}\}^T$ is the array of damage parameters to be sought; S is the matrix of first-order partial derivative of frequency and mode shape to damage parameters, whose expression is shown as

$$S=\left\{\begin{array}{ccccc}\dfrac{\partial\omega_1}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\omega_1}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\omega_1}{\partial D_{pq}}\\ \vdots & & \vdots & & \vdots\\ \dfrac{\partial\omega_n}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\omega_n}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\omega_n}{\partial D_{pq}}\\ \dfrac{\partial\phi_{11}}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\phi_{11}}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\phi_{11}}{\partial D_{pq}}\\ \vdots & & \vdots & & \vdots\\ \dfrac{\partial\phi_{n1}}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\phi_{n1}}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\phi_{n1}}{\partial D_{pq}}\\ \vdots & & \vdots & & \vdots\\ \dfrac{\partial\phi_{1n}}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\phi_{1n}}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\phi_{1n}}{\partial D_{pq}}\\ \vdots & & \vdots & & \vdots\\ \dfrac{\partial\phi_{nn}}{\partial D_{11}} & \cdots\cdots & \dfrac{\partial\phi_{nn}}{\partial D_{ij}} & \cdots\cdots & \dfrac{\partial\phi_{nn}}{\partial D_{pq}}\end{array}\right\} \tag{19}$$

3.1 Calculating formulas of $\partial\omega_r/\partial D_{ij}$

The calculating formulas of $\partial\omega_r/\partial D_{ij}$ is deduced as follows: By differentiating Eq. (13) with respect to ∂D_{ij}, we can write

$$\frac{\partial K_d}{\partial D_{ij}}\Phi_r-\frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}}M\Phi_r=\omega_r^2M\frac{\partial\Phi_r}{\partial D_{ij}}-K_d\frac{\partial\Phi_r}{\partial D_{ij}} \tag{20}$$

Premultiplying the above equation by Φ_r^T and taking note of the orthogonal property of Φ_r, we obtain

$$\frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}}=\Phi_r^T\frac{\partial K_d}{\partial D_{ij}}\Phi_r \tag{21}$$

When $\partial K_d/\partial D_{ij}$ is expressed by the first-order partial derivative of the stiffness of each component to D_{ij}, the above equation becomes

$\dfrac{\partial\left(\omega_r^2\right)}{\partial D_{ij}}=\Phi_r^T\Phi_r\sum_{g=1}^{q}\sum_{a=1}^{32}\sum_{c=1}^{32}\dfrac{\partial K_d}{\partial K_d^g}\dfrac{\partial K_d^g}{\partial k_{ac}}\dfrac{\partial k_{ac}}{\partial D_{ij}}$, therefore, we obtain the result

$$\frac{\partial\omega_r}{\partial D_{ij}}=\frac{\Phi_r^T\Phi_r}{2\omega_r}\sum_{g=1}^{q}\sum_{a=1}^{32}\sum_{c=1}^{32}\frac{\partial K_d}{\partial K_d^g}\frac{\partial K_d^g}{\partial k_{ac}}\frac{\partial k_{ac}}{\partial D_{ij}} \tag{22}$$

where k_{ac} is the element of the stiffness matrix K_d^g.

3.2 Calculating formulas of $\partial\Phi_r/\partial D_{ij}$

Premultiplying Eq. (20) by Φ_s^T then leads to the following equation

$$\Phi_s^T\frac{\partial K_d}{\partial D_{ij}}\Phi_r+\Phi_s^TK_d\frac{\partial\Phi_r}{\partial D_{ij}}-\Phi_s^T\frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}}M\Phi_r-\omega_r^2\Phi_s^TM\frac{\partial\Phi_r}{\partial D_{ij}}=0 \tag{23}$$

Then, the above equation is discussed in two cases.

When $s \neq r$, taking note of the orthogonal property of Φ_r, we obtain

$$\left(\omega_s^2-\omega_r^2\right)\Phi_s^T M \frac{\partial \Phi_r}{\partial D_{ij}} + \Phi_s^T \frac{\partial K_d}{\partial D_{ij}} \Phi_r = 0 \tag{24}$$

When $\partial\Phi_r/\partial D_{ij}$ can be expressed as a linear combination of feature vectors, we have

$$\frac{\partial \Phi_r}{\partial D_{ij}} = \sum_{s=1}^{n} a_s \Phi_s \tag{25}$$

where a_s is a constant coefficient.

Premultiplying Eq. (23) by Φ_r^T, and considering the orthogonal property of Φ_r, we have $\Phi_s^T M \frac{\partial \Phi_r}{\partial D_{ij}} = a_s$.

Substituting into Eq. (24), we can obtain

$$a_s = \frac{\Phi_r^T \Phi_r}{\omega_r^2-\omega_s^2} \sum_{g=1}^{q} \sum_{a=1}^{32} \sum_{c=1}^{32} \frac{\partial K_d}{\partial K_d^g} \frac{\partial K_d^g}{\partial k_{ac}} \frac{\partial k_{ac}}{\partial D_{ij}} \tag{26}$$

When $s=r$, there is $\Phi_r^T M \Phi_r = 1$. Differentiating the equation with respect to D_{ij} gives $\frac{\partial \mathrm{K}_r^T}{\partial D_i} M \Phi_r + \Phi_r^T M \frac{\partial \Phi_r}{\partial D_i} = 0$.

Because M is a symmetric matrix, we can obtain $\Phi_r^T M \frac{\partial \Phi_r}{\partial D_{ij}} = 0$.

Therefore, the coefficient $a_s = 0$.

Combining the above two situations, we arrive at the result

$$\frac{\partial \Phi_r}{\partial D_{ij}} = \sum_{s=1}^{n} a_s \Phi_s, \qquad a_s = \begin{cases} \dfrac{\Phi_r^T \Phi_r}{\omega_r^2-\omega_s^2} \displaystyle\sum_{g=1}^{q} \sum_{a=1}^{32} \sum_{c=1}^{32} \frac{\partial K_d}{\partial K_d^g} \frac{\partial K_d^g}{\partial k_{ac}} \frac{\partial k_{ac}}{\partial D_{ij}} & s \neq r \\ 0 & s = r \end{cases} \tag{27}$$

4. Modal truncation error

Considering the incompleteness of measured structural mode shapes, the practical complete modal space theory is used to eliminate the influence of modal truncation error.

Let the right side of Eq. (20) be equal to g_{ij}, we can obtain

$$\frac{\partial K_d}{\partial D_{ij}} \Phi_r - \frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}} M \Phi_r = g_{ij} \tag{28}$$

$\partial\Phi_r/\partial D_{ij}$ can be expressed as

$$\frac{\partial \Phi_r}{\partial D_{ij}} = \sum_{s=1}^{n} a_s \Phi_s + \frac{\partial \Phi_r^{(0)}}{\partial D_{ij}} \tag{29}$$

In the above formula, Wang B. P. (1996) points out that $\partial\Phi_r^{(0)}/\partial D_{ij}$ is solved by the following equation

$$K_d \frac{\partial \Phi_r^{(0)}}{\partial D_{ij}} = g_{ij} \tag{30}$$

Substituting Eq. (30) into Eq. (28) gives

$$\frac{\partial \Phi_r^{(0)}}{\partial D_{ij}} = K_d^{-1} \frac{\partial K_d}{\partial D_{ij}} \Phi_r - K_d^{-1} \frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}} M \Phi_r \tag{31}$$

Substituting Eq. (31) into Eq. (29), we can obtain

$$\frac{\partial \Phi_r}{\partial D_{ij}} = K_d^{-1} \frac{\partial K_d}{\partial D_{ij}} \Phi_r - K_d^{-1} \frac{\partial\left(\omega_r^2\right)}{\partial D_{ij}} M \Phi_r + \sum_{s=1}^{n} a_s \Phi_s \tag{32}$$

5. Flow chart of the implementation of the proposed method

The flow chart of the implementation of the proposed method is shown in Fig. 3.

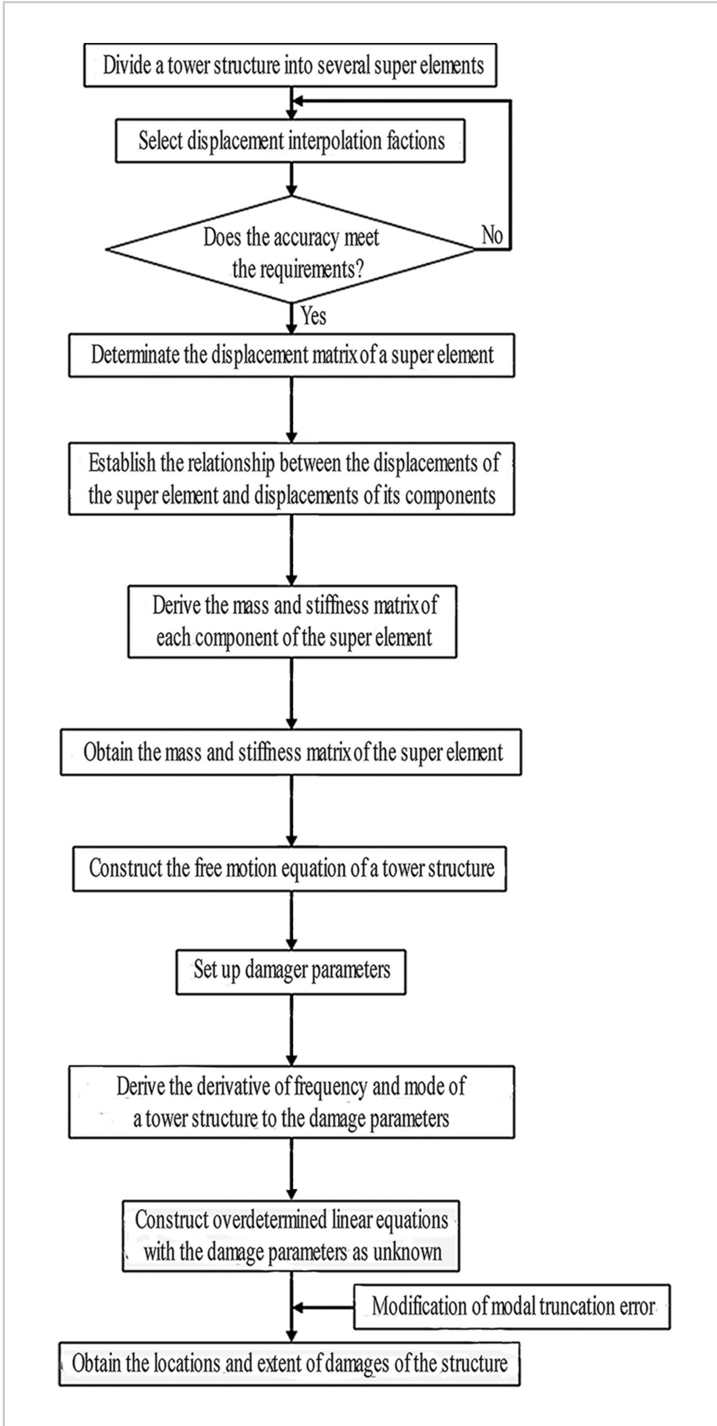

Figure 3 - Flow chart of the implementation of the proposed method.

6. Numerical example

To verify the effect of the method presented herein, a twelve-layer steel tube structure TV Tower is chosen as an analysis example. As shown in Fig. 4(a), the total height of the television tower is 64.26 , whose top view is a regular hexagon shown in Fig. 4(b). Component Parameters of TV tower are shown in Table 1. The material constants of the tower are: the modulus of elasticity E = 2.06 x 10^5 Mpa, the mass density ρ = 7800kg/m^3 and Poisson's ratio ν = 0.30. It is assumed that the stiffness of components ①, ② and ③ are reduced by 25, 20 and 15 percent, respectively, with their locations shown in Fig. 4(a). Component 1, 2 and 3 are No. 24 element of the first layer, No. 14 element of the fourth layer and No. 13 element of the fifth layer, and the corresponding damage parameters are $D_{24\ 1}$, $D_{14\ 4}$ and $D_{13\ 5}$, respectively. Now, the proposed method is used to analyze the structure and see whether the three components are accurately diagnosed for damages and whether their damage values are equal to or close to the attenuation value of their stiffness.

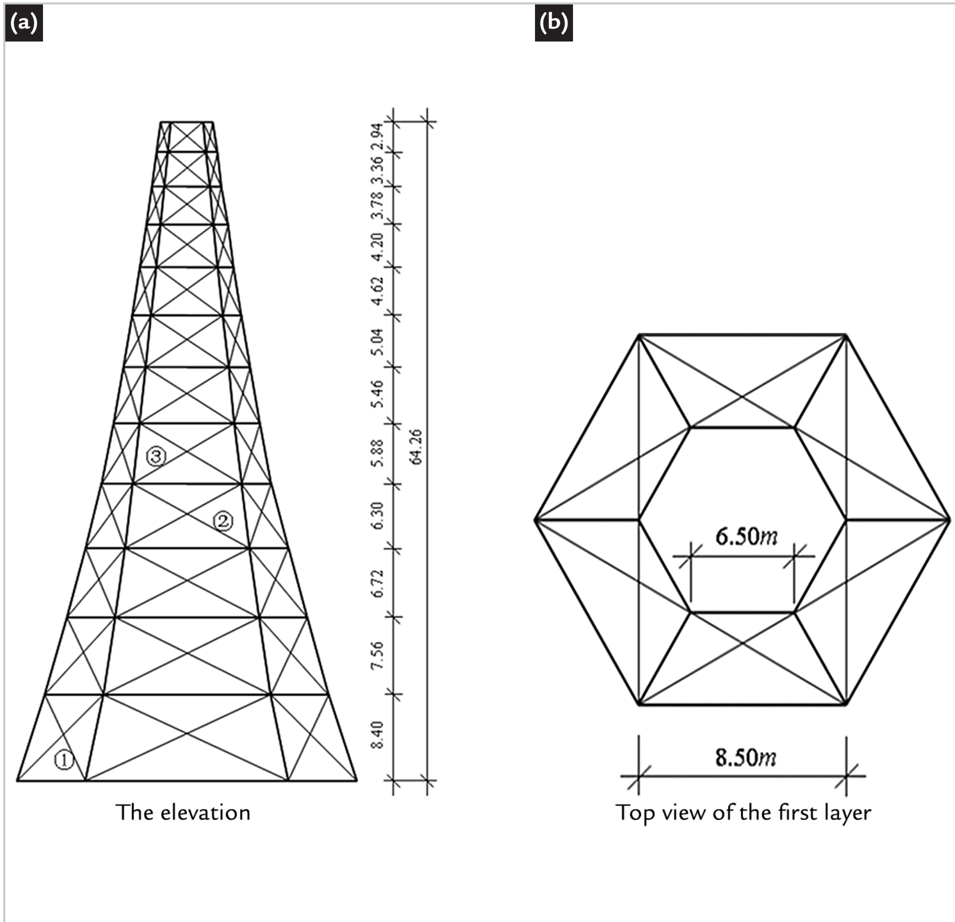

Figure 4 - Finite element model of TV Tower.

Table 1 - Component Parameters of TV tower.

Number of tower layer	Tower columns		Diagonal rods		Horizontal rods	
	Length(m)	Model	Length(m)	Model	Length(m)	Model
12	2.945	Φ245/12	4.204	Φ32	3.102	Φ133/6
11	3.368	Φ245/12	4.791	Φ32	3.408	Φ133/6
10	3.786	Φ299/12	5.260	Φ36	3.652	Φ146/6
9	4.342	Φ299/12	5.862	Φ36	3.938	Φ146/6
8	4.796	Φ351/12	6.250	Φ36	4.007	Φ168/6
7	5.213	Φ351/12	6.849	Φ36	4.442	Φ168/6
6	5.682	Φ426/12	7.324	Φ40	4.621	Φ194/6
5	6.016	Φ426/12	7.743	Φ40	4.875	Φ194/6
4	6.536	Φ478/12	8.300	Φ40	5.116	Φ219/6
3	6.962	Φ478/12	9.819	Φ40	5.413	Φ219/6
2	8.231	Φ478/12	12.095	Φ45	5.845	Φ273/6
1	9.235	Φ478/12	14.296	Φ45	6.505	Φ377/8

Before calculation, the TV tower is divided into 12 sections along its height to form 12 super elements, that is, each layer is a super element. Eq. (13) is solved in two cases (Case 1, the structure is intact; Case 2, No. 1, 2 and 3 components are damaged) to obtain the natural frequencies and mode shapes of the structure. The first 10-order frequencies and mode shapes of both the intact and damaged structure are selected to construct Eq. (18). In actual damage diagnosis, the measured frequencies and mode shapes before and after damage should be used. By solving the Eq. (18) and using the method of eliminating modal truncation error, the values of the damage parameters shown in Table 2 were obtained. Due to limited space, Table 2 only lists the calculation results of the damage parameters for No. 1, 4, and 5 super elements.

Table 2 - Calculated values and real values of damage parameters.

Component number ($i(j)$)	The calculated damage of without eliminating modal truncation error	The calculated damage with eliminating modal truncation error	Real damages
1(1)	0.132	0.038	0
2(1)	0.065	0.006	0
3(1)	0.207	0.028	0
4(1)	0.078	0.005	0
5(1)	0.115	0.045	0
6(1)	0.002	0.022	0
7(1)	0.054	0.028	0
8(1)	0.008	0.000	0
9(1)	0.097	0.041	0
10(1)	0.078	0.039	0
11(1)	0.108	0.046	0
12(1)	0.005	0.013	0
13(1)	0.074	0.016	0
14(1)	0.045	0.007	0
15(1)	0.107	0.005	0
16(1)	0.009	0.012	0
17(1)	0.078	0.014	0
18(1)	0.000	0.001	0
19(1)	0.019	0.007	0
20(1)	0.112	0.043	0
21(1)	0.004	0.004	0
22(1)	0.062	0.008	0
23(1)	0.009	0.027	0
24(1)	0.154	0.234	0.250
1(4)	0.038	0.013	0
2(4)	0.051	0.006	0
3(4)	0.001	0.000	0
4(4)	0.120	0.048	0
5(4)	0.084	0.043	0
6(4)	0.008	0.004	0
7(4)	0.057	0.021	0
8(4)	0.000	0.000	0
9(4)	0.081	0.026	0
10(4)	0.065	0.012	0
11(4)	0.109	0.035	0
12(4)	0.068	0.000	0
13(4)	0.098	0.005	0
14(4)	0.195	0.212	0.200
15(4)	0.035	0.003	0
16(4)	0.018	0.000	0
17(4)	0.007	0.003	0
18(4)	0.018	0.024	0
19(4)	0.094	0.016	0
20(4)	0.057	0.005	0
21(4)	0.000	0.000	0
22(4)	0.000	0.000	0
23(4)	0.059	0.023	0
24(4)	0.000	0.001	0
1(5)	0.073	0.012	0
2(5)	0.150	0.022	0
3(5)	0.007	0.000	0
4(5)	0.000	0.005	0
5(5)	0.065	0.003	0
6(5)	0.018	0.042	0
7(5)	0.001	0.014	0
8(5)	0.000	0.010	0
9(5)	0.069	0.012	0
10(5)	0.000	0.000	0
11(5)	0.086	0.017	0
12(5)	0.012	0.000	0
13(5)	0.301	0.161	0.150
14(5)	0.009	0.028	0
15(5)	0.000	0.007	0
16(5)	0.000	0.024	0
17(5)	0.000	0.004	0
18(5)	0.053	0.000	0
19(5)	0.019	0.020	0
20(5)	0.110	0.004	0
21(5)	0.092	0.000	0
22(5)	0.007	0.001	0
23(5)	0.064	0.041	0
24(5)	0.000	0.010	0

When using the presented method to construct Equation 12, only 12 elements are used. Taking each component as an element, there are a total of 288 elements when using FEM, which is much more computational than SEMD. Solving Eq. 18 gets the damage values of all components. It can be seen from Table 2 that the calculation results are basically consistent with the real values, and only a few values are slightly larger or smaller than the real values, such as Components 5(1), 10(1), 11(1), 4(4), 6(5) and 23(5). The errors are also within 5%, which fully meets the requirements for damage detection in practical engineering. The severity of the damages of the components is determined by the calculated values of the damage parameters, and the locations of the damages are diagnosed according to the number of the components. This example fully verifies the feasibility and accuracy of the presented method, which can simultaneously identify the number, location and severity of damages of tower structures.

7. Conclusions

A new method is proposed herein, which can accurately identify the locations and severity of damages in tower structures. While taking full advantage of super elements, it also greatly improves the computing efficiency. The modification technology of modal truncation error is proposed to greatly improve the accuracy of the damage identification. By using only the measured frequencies and mode shapes as input data that have easy testability and high precision, the method is easily implemented on computers and so can be applied into practice. It can be seen from the implementation process of the method that if there are more damaged components in a super element, the damages cannot be accurately diagnosed.

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References

FOOLADI, A.; BANAN, M. R. A super-element based on finite element method for latticed columns. ***International Journal of Civil Engineering,*** v. 13, n. 2, p. 202-212, 2015.

CAO, Z.; LIU, Y.; ZHOU, H. Super finite element method and its application in structural engineering analysis. ***Computational Structural Mechanics and Applications,*** v. 11, n. 4, p. 454-460, 1994.

CAO, Z.; FU, Z. Super element method for the destruction and collapse analyses of complicated systems. ***Journal of Tongji University,*** v.28, n. 3, p. 253-256, 2000.

CARVALHO, H.; QUEIROZ, G.; FAKURY, R. H. Experimental evaluation of the wind effects on an operating power transmission tower. ***Mechanic and Energy,*** v. 69, n. 3, p. 325-331, 2016.

CHENG, W. X. ***Finite Element Method [M].*** Beijing: Tsinghua University Press, 2003. 120p.

GALANIN, M. P.; LAZAREVA, S. A. The finite super element method and its application for solving Science and technology problems. ***Mathematical Models and Computer Simulations,*** v. 6, n. 1, p. 19-24, 2014.

HE, H.; XIE, X.; WANG, W. Vibration control of tower structure with multiple cardan gyroscopes. ***Shock and Vibration,*** v. 2017, p. 1-11, 2017.

HOU, H.; GENG, H.; HUANG, Y.; WU, H.; WU, X.; YU, S. Damage probability assessment of transmission line-tower system under typhoon disaster, based on model-driven and data-driven views. ***Energies,*** v. 12, n. 8, p. 1-17, 2019.

LING, Z.; YING-TAO, L.; CHAO-SHAN, Y.; ZHI-PING, D.; JIN, C. Research on dynamic similarity model test of damage detection for transmission tower. ***The Civil Engineering Journal,*** v. 3, p. 1-9, 2016.

MA, H.; JIA, H.; LIU, W; WU, Q. A Study of the key problem in the application of super element. ***Computer Simulation,*** v. 26, n. 5, p. 48-51, 2009.

TSYBENKO, A.; KONYUKHOV, A.; TSYBENKO, H. Numerical method for determining stiffness characteristics of an arbitrary form super element. ***Applied Computer Systems,*** v. 18, n. 1, p. 52-56, 2015.

WANG, B. P. An improved approximate method for calculation of eigenvector derivatives. *In*: STRUCTURES, STRUCTURAL DYNAMICS, AND MATERIALS CONFERENCE, 26th, Orlando, Florida. ***Proceedings*** [...]. AIAA/ASME/ASCE/AHS, 1996. p. 117-123.

YONGREN, L.; ZHIYUAN, C. The static analysis of special frames in architectural structures by the use of super--element method. ***Shang Hai Journal of Mechanics,*** v. 16, n. 4, p. 282-289, 1995.

ZHANG, Z.; LI, H.; LI, G.; WANG, W.; TIAN, L. The numerical analysis of transmission tower-line system wind-induced collapsed performance. ***Mathematical Problems in Engineering,*** v. 2013, p. 1-11, 2013.

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