

DOI: <http://dx.doi.org/10.1590/1807-1929/agriambi.v25n7p439-445>

New explicit correlation to compute the friction factor under turbulent flow in pipes¹

Nova correlação explícita para calcular o fator de atrito sob fluxo turbulento em tubos

Alan Olivares Gallardo^{2*} , Rodrigo Guerra Rojas²  & Marco Alfaro Guerra³ ¹ Research developed at La Serena, Región de Coquimbo, Chile² Universidad de La Serena/Facultad de Ingeniería/Departamento de Ingeniería en Obras Civiles, La Serena, Región de Coquimbo, Chile³ Universidad de La Serena/Instituto de Investigación Multidisciplinario en Ciencia y Tecnología, La Serena, Región de Coquimbo, Chile

HIGHLIGHTS:

*The correlation facilitates the calculation of head losses in hydraulic systems.**The correlation was design for regimes of high and low turbulence.**The best performance of the correlation is obtained for a range of roughness that goes from 10^{-2} to 5×10^{-3} .*

ABSTRACT: Colebrook's implicit equation has been widely used to estimate the friction factor in pipes under turbulent flow. This friction factor is used to calculate pressure drops to solve different problems and applications of Engineering such as the design of water distribution systems. Many researchers have proposed explicit approximations to estimate the friction factor without carrying out iterative calculations. In this study, a new explicit correlation is given to determine the friction factor in cylindrical pipes under turbulent flow. This study allows evaluating the characteristics of the new explicit approximation of the friction factor and compare them with the value obtained using Colebrook's equation and with other explicit approximations developed by several authors. This has allowed finding a new equation of simple structure and of few mathematical operations that approximates the value of the friction factor with a maximum relative error of 1.60% with respect to the value solved for Colebrook's equation.

Key words: Colebrook's equation, pressure drop in pipes, maximum relative error

RESUMO: A equação implícita de Colebrook tem sido amplamente utilizada para estimar o fator de atrito em tubulações sob fluxo turbulento. Este fator de atrito é usado para calcular quedas de pressão para resolver diferentes problemas e aplicações da engenharia, como o projeto de sistemas de distribuição de água. Muitos pesquisadores propuseram aproximações explícitas para estimar o fator de atrito sem realizar cálculos iterativos. Neste estudo, uma nova correlação explícita é dada para determinar o fator de atrito em tubos cilíndricos sob fluxo turbulento. Este estudo permite avaliar as características da nova aproximação explícita do fator de atrito e compará-las com o valor obtido pela equação de Colebrook e com outras aproximações explícitas desenvolvidas por vários autores. Isso permitiu encontrar uma nova equação de estrutura simples e com poucas operações matemáticas que se aproxima ao valor do fator de atrito com um erro relativo máximo de 1,60% em relação ao valor resolvido para a equação de Colebrook.

Palavras-chave: equação de Colebrook, perda de carga em tubulações, erro relativo máximo



INTRODUCTION

Colebrook’s equation is an implicit expression that requires an approximate numerical solution (Augusto et al., 2016) or a solution of exact analytical type to estimate the friction factor (Mikata & Walczak, 2017). Mikata & Walczak (2017) proposed that there are three types of solutions for Colebrook’s equation. The first generation solutions correspond to the approximations based on curves adjustments with data obtained from Colebrook’s equation, for example the proposals developed by Filonenko (1954), Papaevangelou et al. (2010), Buzzelli (2008), among others, which allow determining the friction factor (f) of the pipes.. The second generation solutions correspond to the ones presented by Brkić (2011c) based on the formal solution of Lambert’s W function; the solution determined by the use of Boyd’s displaced function (Boyd, 2017); there is also the solution proposed by Barry et al. (2000). Finally it is presented the solution proposed by Winitzki (2003). Then there are the third generation solutions that correspond to the estimations based on the exact analytic solutions of Colebrook’s equation (Mikata & Walczak, 2017).

Colebrook’s equation was developed experimentally and a large number of the explicit approximations are based on fits of numerical data obtained from said equation. This numerical work has been a contribution to predict the value of the friction factor using conventional methods. However, in recent times authors such as Najafzadeh have made relevant contributions using Gene-Expression Programming (GEP), Evolutionary Polynomial Regression (EPR), Model Tree (MT), Neuro-Fuzzy GMDH to predict the behavior of fluids in different media and with very good results (Najafzadeh & Barani, 2011; Najafzadeh & Sattar, 2015; Najafzadeh & Kargar, 2019; Najafzadeh, 2019).

The aim of this study is to propose a new correlation to determine the friction factor in pipes and its comparison with recent explicit approximations.

MATERIAL AND METHODS

Colebrook and White published in 1937, in the Journal “The Royal Society”, the article “Experiments with Fluid Friction in Roughened Pipes” citing the 1933 works of Nikuradse and Prandtl (Colebrook & White, 1937). In 1939, C. F. Colebrook published the work called “Turbulent Flow in Pipes, with particular reference to the Transition Region between the Smooth and Rough Pipe Laws.” in the Institution Journal and containing as its main part “A New Theoretical Formula for Flow in the Transition Region”. This theoretical formula for flows in the Transition Region has endured over time and has been studied and cited by countless authors, and is accepted and validated as the most accurate value in calculating the friction factor (f) of hydraulically smooth and rough pipes for transition and turbulent flows, as observed below (Eq. 1) (Colebrook, 1939):

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}}\right] \tag{1}$$

where:

f - friction factor, dimensionless;

Re - Reynolds number, dimensionless;
 ε - pipe roughness;
 D - pipe diameter; and,
 ε/D - relative pipe roughness.

Several studies have been conducted with a revision of these great number of explicit proposals and its benefits (Anaya-Durand et al., 2014). That is why, below it is proposed an explicit correlation for the calculation of Colebrook’s friction factor and at the same time a revision is made of the samples of the explicit correlations that have been published recently, Swamee and Jain’s explicit correlation (Swamee & Jain, 1976) is also included, since it has a similar structure proposed by the authors of this study.

The proposal of explicit approximation of friction factor (f) in pipes under turbulent flow is:

$$f = \left[-2\log\left(\frac{4.859}{Re^{-0.888}} + \frac{\varepsilon/D}{3.7}\right)\right]^{-2} \tag{2}$$

The Eq. 2 was obtained from the authors work in the study of the behavior of Colebrook’s equation for different values of Re y ε/D with the purpose of comparing the estimation of its value.

In the following sessions, it will be shown that the values for the friction factor obtained from Colebrook’s equation (Eq. 1) and equation (Eq. 2) match adequately for the values of Re that go from 10⁴ to 10⁸ and the values of ε/D that go from 10⁻¹ to 10⁻⁶.

To develop a comparison in the estimate of Colebrook’s friction factor, the following explicit approximations proposed by several authors will be used, which approximate the result of the friction factor (f) with great precision and simplify calculation, some of them also present a similar structure with respect to this proposal:

- Swamee and Jain (Swamee & Jain, 1976)

$$f = \left[-2\log\left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{Re^{-0.90}}\right)\right]^{-2} \tag{3}$$

- Manadilli (Manadili & Silverberg, 1997)

$$f = \left[-2\log\left(\frac{\varepsilon/D}{3.7} + \frac{95}{Re^{0.983}} - \frac{96.82}{Re}\right)\right]^{-2} \tag{4}$$

- Romeo et al. (Romeo et al., 2002)

$$f = \left\{-2\log\left[\frac{\varepsilon/D}{3.7065} - \frac{5.0272}{Re} \log\left(\frac{\varepsilon/D}{3.827} - \frac{4.567}{Re}\right) \cdot \log\left(\left(\frac{\varepsilon/D}{7.79}\right)^{0.9924} + \left(\frac{5.3326}{208.82 + Re}\right)^{0.9345}\right)\right]\right\}^{-2} \tag{5}$$

- Fang et al. (Fang et al., 2011)

$$f = 1.613 \left[\ln\left\{0.234(\varepsilon/D)^{1.1007} - \frac{60.525}{Re^{1.1105}} + \frac{56.291}{Re^{1.0712}}\right\}\right]^{-2} \tag{6}$$

- Brkić I (Brkić, 2011a)

$$f = \left[-2 \log \left(10^{-0.4343\beta} + \frac{\varepsilon/D}{3.71} \right) \right]^{-2} \quad (7)$$

- Brkić II (Brkić, 2011a)

$$f = \left[-2 \log \left(\frac{2.18\beta}{Re} + \frac{\varepsilon/D}{3.71} \right) \right]^{-2} \quad (8)$$

where:

$$\beta = \ln \left[\frac{Re}{1.816 \ln \left\{ \frac{1.1Re}{\ln(1+1.1Re)} \right\}} \right] \quad (9)$$

Colebrook's equation (Eq. 1) is widely used to determine the friction factor for turbulent flows in pipes. Clearly this equation is implicit for the determination of the friction factor and the numerical solution can be obtained in different ways. In this study, a numerical solution is used based on Newton's method for the implicit Eq. 1 for f . The numerical solution of Newton's method is based on the following algorithm:

$$x_{i+1} = x_i - \frac{g(x_i)}{g'(x_i)} \quad (10)$$

In Newton's method, the Eq. 10 is used to calculate the friction factor from a known initial value and it allows estimating the following interaction value to find the solution for the friction factor's equation (Eq. 1). Colebrook's equation (Eq. 1) can be reorganized by replacing the value of $F=1/f$ and be expressed through a new relation $g(F)$ as shown in the Eq. 11.

$$g(F) = F + 2 \log \left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{Re} \cdot F \right] \quad (11)$$

If the Eq. 11 is assumed as a function that is always continuous and differentiable, the derivative of the function can be obtained in respect to F . The Eq. 11 has the capability for rapid convergence, especially if there is an adequate estimation of the initial value of the friction factor (Augusto et al., 2016) (Najafzadeh et al., 2018).

In the validation of the new explicit approximation (Eq. 2) the maximum percentage of the relative error has been determined considering 21 values of ε/D , from 10^{-1} to 10^6 , and 39,997 Re values, between 10^4 and 10^8 , which generates a matrix of analysis for the friction factor that has 839,937 values.

The validation of explicit correlations or approximations requires the handling of a large amount of data and traditionally statistical parameters are used to determine the precision of the fit between the estimated data and the observed data. In the works of Brkić (2011b) and Najafzadeh (2019), they used the error of the estimated data with respect to the observed data as a way of optimizing the fit. This method generally

involves the least squares methodology to determine different statistical parameters.

The correlation of the friction factor results obtained from the explicit approximation proposed and other explicit approximations are shown in this section. To evaluate explicit approximations performance in terms of statistical indices: maximum relative error (RE^+ , RE^-), mean relative error (MRE), coefficient of determination (R^2), root mean square error (RMSE), scatter index (SI), Akaike information criterion (AIC), BIAS and index of agreement (IOA) can be defined as follows (Shaikh et al., 2015; Najafzadeh, 2019):

- Maximum positive relative error (%)

The maximum positive relative error (RE^+) percentage can be obtained for each correlation that approximates the value of Colebrook's friction factor (f_{cw}). This parameter is defined by:

$$RE^+ = \left(\frac{[f_{cw}] - [f_{estimated}]}{[f_{cw}]} \right) \times 100 \quad (12)$$

- Maximum negative relative error (%)

To obtain the maximum negative relative error (RE^-) percentage for each correlation that approximates the value of Colebrook's friction factor, Eq. 13 must be applied:

$$RE^- = \left(\frac{[f_{estimated}] - [f_{cw}]}{[f_{cw}]} \right) \times 100 \quad (13)$$

- Mean relative error (%)

If n is the total number of elements of the estimation matrix of the friction factor for different values of ε/D and Re , the mean relative error (MRE) value can be obtained, in percentage, using Eq. 14:

$$MRE = \mu_{RE} = \frac{1}{n} \sum_{i=1}^n \left(\frac{[f_{cw}] - [f_{estimated}]}{[f_{cw}]} \right) \times 100 \quad (14)$$

- Coefficient of determination

Through the coefficient of determination (R^2), it is possible to establish the consistency of the estimated values for the friction factor ($f_{estimated}$) in comparison to the value of Colebrook's friction factor.

$$R^2 = 1 - \frac{\sum_{i=1}^n ([f_{cw}] - [f_{estimated}])^2}{\sum_{i=1}^n ([f_{cw}] - [\overline{f_{cw}}])^2} \quad (15)$$

- Root mean square error

The root mean square error (RMSE) to describe average model-performance error

$$RMSE = \left[\frac{\sum_{i=1}^n ([f_{estimated}] - [f_{cw}])^2}{n} \right]^{1/2} \quad (16)$$

- Scatter index

$$SI = \frac{\sqrt{\frac{1}{n} \sum_{i=1}^n \left(\left([f_{estimated}] - [\overline{f_{estimated}}] \right) - \left([f_{CW}] - [\overline{f_{CW}}] \right) \right)^2}}{\frac{1}{n} \sum_{i=1}^n [f_{CW}]} \quad (17)$$

- Akaike information criterion

The Akaike information criterion (AIC) has the capability of evaluating relative quality of statistical performances for a given dataset. Negative values of AIC indicate better performance of the proposed model in comparison with positive ones.

$$AIC = n \ln(nRMSE^2) + 2NOV \quad (18)$$

where:

- n - total number of elements; and,
- NOV - number of independent variables.

- BIAS

$$BIAS = \frac{\sum_{i=1}^n \left([f_{estimated}] - [f_{CW}] \right)}{n} \quad (19)$$

- Index of agreement

The Index of agreement (IOA) is a standardized criterion for evaluation of the proposed model prediction error ranging from 0 to 1. A value of 0 shows that the proposed model stands at lowest level of accuracy without an agreement between observed values and predicted ones

$$IOA = 1 - \frac{\sum_{i=1}^n \left([f_{estimated}] - [f_{CW}] \right)^2}{\sum_{i=1}^n \left(\left([f_{estimated}] - [\overline{f_{estimated}}] \right) + \left([f_{CW}] - [\overline{f_{CW}}] \right) \right)^2} \quad (20)$$

where:

- $[f_{estimated}]$ - matrix of values for the friction factor for each of the explicit correlations studied;
- $[f_{CW}]$ - matrix of values of Colebrook's friction factor resulting from the numerical solution of the Eq. 11 in each node of the matrix of Re and ϵ/D values;
- $[f_{CW}]$ - arithmetic mean of Colebrook's friction factors, dimensionless; and,
- $[f_{estimated}]$ - arithmetic mean of explicit approximations friction factors, dimensionless.

RESULTS AND DISCUSSION

The Eq. 1 is implicit given that the term $1/\sqrt{f}$ is presented in both sides of the formula and is part of the logarithm argument that characterizes this equation. This logarithm has as an argument two summands, the first one has a relation with the term that corresponds to ϵ/D relative roughness and the other term is:

$$\frac{2.51}{Re\sqrt{f}} \quad (21)$$

This term depends on Re and in an implicit way on the friction factor value f. Such term for each value of ϵ/D has a behavior of decreasing potential type as observed in Figure 1.

Because of this, for each ϵ/D value, it is possible to apply a potential regression to the values obtained for the friction factor and thus estimate the value of the constants A and B so that the term can be expressed as an approximate explicit equation. The previous can be seen in Eq. 22.

$$\frac{2.51}{Re\sqrt{f}} = \frac{A}{Re^B} \quad (22)$$

For each regression applied in the range of Reynolds Number that goes from 4,000 to 94,000, it is possible to calculate the values of the constants A and B that minimize the value of RE (%) maximum relative error. The curves adjusted for each ϵ/D value define a value of R^2 that allows concluding that they are a good representation of the friction factor. Nevertheless, it is possible to highlight that the maximum relative error is minimized in the range of values for A between 6.3116 and 4.7721, obtaining values for RE^+ between 1.862 and 2.762%.

In a more delimited analysis, it is possible to define the value of the constant B, and with it calculate the value of constant A that minimizes RE (%) value. It is possible to observe for the proposed correlation that the minimum relative error can be obtained for a value of the constants $A = 4.859$ and $B = -0.888$ and that are part of the authors proposal.

Several authors have carried out extensive researches or revisions of the characteristics of the implicit correlations published until today. One of the studies carried out by Winning and Coole (Winning & Coole, 2013) concluded that Fang and Romeo's correlations have great precision in the determination of the value of the friction factor in an explicit way. In Table 1, it is shown the results of the statistical parameters for the different correlations analyzed in this study.

To conduct an appropriate comparison test, with the aim of analyzing the performance of the explicit approximation

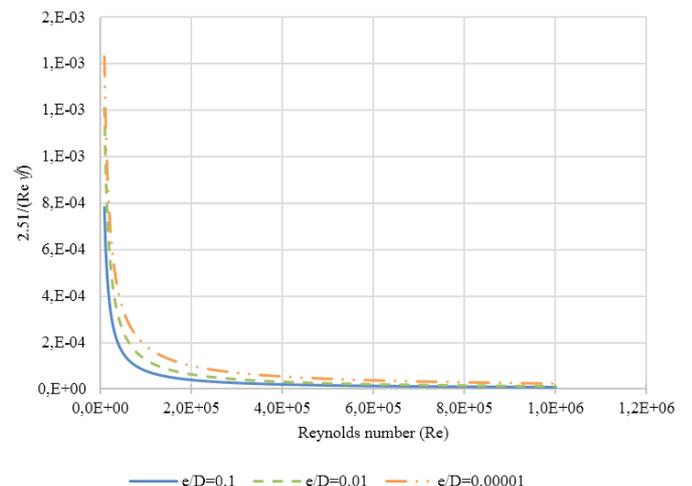


Figure 1. Relationship between the implicit term of the Colebrook equation and Reynolds number

Table 1. Results of statistical indices for each correlation analyzed

	Swamee	Manadilli	Romeo	Fang	Brkić I	Brkić II	Guerra
Max RE ⁺	0.704	0.003	0.098	0.425	3.156	0.149	1.594
Max RE ⁻	2.122	2.000	0.135	0.309	1.096	2.141	1.599
MRE	0.247	0.156	0.044	0.129	0.200	0.169	0.320
R ²	0.999998	0.999999	0.999999	0.999996	0.999995	0.999996	0.999997
RMSE	3.511E-05	2.495E-05	3.254E-05	5.237E-05	6.268E-05	5.344E-05	4.523E-05
AIC	-5772805	-6346818	-5900454	-5101132	-4799159	-5067065	-5347327
SI	0.0009364	0.00067518	0.00096360	0.00184287	0.00178626	0.0018383	0.0012313
BIAS	2.306E-05	1.606E-05	-1.780E-05	-5.303E-06	-3.713E-05	-1.245E-05	2.888E-05
IOA	0,99999958	0,99999979	0,99999964	0,99999906	0,99999865	0,99999902	0,9999993

proposed and the six explicit correlations of recent formulation, it was analyzed 839,937 nodes of the calculation matrix. The results for the maximum positive and negative values, average values of relative error, root mean square error, scatter index, Akaike information criterion, BIAS and index of agreement for each ϵ/D value that varies from 10^{-6} to 10^{-1} , and for Re values in 10^4 and 10^8 interval, are shown in Table 1. In the explicit approximations studied, the statistical indices R², RMSE, SI, AIC, BIAS and IOA present a good fit of the analyzed models. The most unfavorable index turns out to be the relative error. The maximum relative errors for the approximations of the friction factor and for the explicit

approximation proposed (Eq. 2) for the mesh points defined, are shown in Figure 2.

As shown in Table 1, the proposed correlation and the six explicit approximations that include Swamee and Jain (Swamee & Jain, 1976), Manadilli (Manadili & Silverberg, 1997), Romeo et al. (Romeo et al., 2002), Fang et al. (Fang et al., 2011), Brkić I (Brkić, 2011a), Brkić II (Brkić, 2011a), in general showed results in different ranges of maximum value of the relative error (%). All the correlations gave results for this error that are less than 3.20%. This absolute maximum value of relative error varies between 0.135 and 3.156% for the proposals analyzed. Romeo and Fang correlations are the

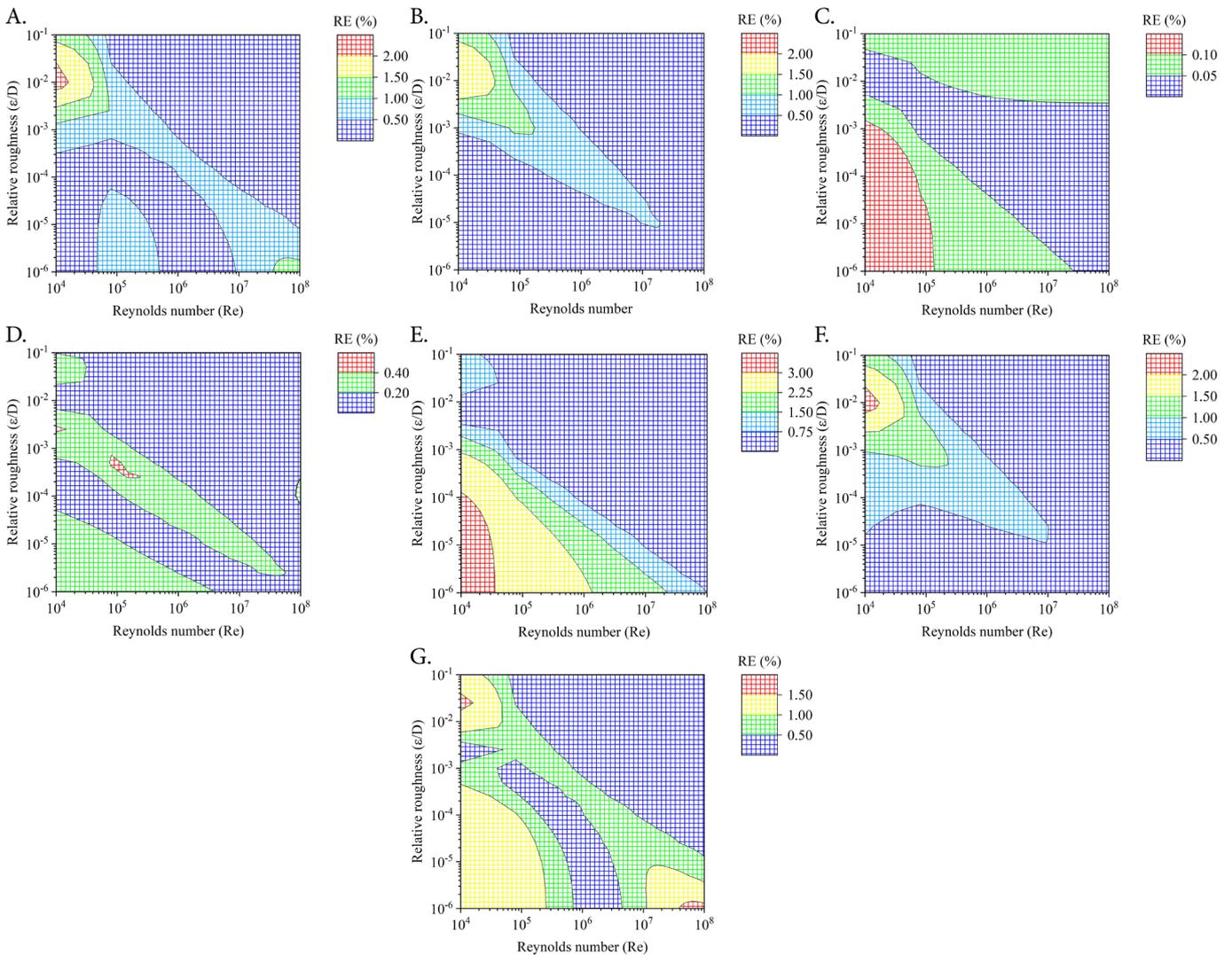


Figure 2. Distribution of the relative error estimate (RE (%)), as function of Reynolds number (Re) and relative roughness (ϵ/D), produced by the equations of (A) Swamee (1976), (B) Manadilli (1997), (C) Romeo (2002), (D) Fang (2011), (E) Brkić I (2011), (F) Brkić II (2011), (G) Guerra (Eq. 2), when compared to the Colebrook equation (Eq. 1)

most precise ones and have a maximum relative error of 0.135 and 0.425%, respectively.

The values of the coefficient of determination (R^2) and MRE showed a similar behavior in all the correlations. All the explicit correlations analyzed are equations that predict properly the value of Colebrook's friction factor, since these equations have a value of R^2 that goes from 0.999998 to 0.999995 and have lower values for MRE that are between 0.044 and 0.320.

The correlations studied showed maximum relative errors (RE) whose distribution is shown in Figure 2.

When analyzing the results of the correlation of Swamee and Jain (Swamee & Jain, 1976) in Figure 2, it can be observed that they show results with errors lower than 0.50% when Re is greater than 10^6 and roughness lower than 10^3 , situation that is commonly produced in systems of galvanized steel water pipe with diameters greater than 600 mm, where the fluid can reach the speed from 2.0 to 3.0 m s^{-1} . This correlation presents at the same time results with an error lower than 1.0% when Re is small and the pipe has a smaller diameter to calculate its relative roughness.

When analyzing the results of Manadilli's correlation (Manadili & Silverberg, 1997), shown in Figure 2, for highly turbulent regimes and rough pipe, this is subject to relative errors close to 1.50%. Nevertheless, the method has a better precision of 0.50% in the different water system areas where it is found less rugged pipes with flows that go from relative low speeds to a runoff speed that generates turbulent flows.

In literature it is reported that the explicit approximation of Romeo et al. (Romeo et al., 2002) is among the most precise ones (Özger & Yildirim, 2009; Winning & Coole, 2013; Brkić, 2011b) and presents minor relative errors in comparison to the other explicit proposals, shown in Figure 2. The error is constant for all Re and ϵ/D value. Nevertheless, it is a correlation that requires many mathematical operations and is complex to remember or to process. At the same time, it is worthy mentioning that together with the approximations of Romeo et al., in the explicit correlation of Fang et al (Fang et al., 2011), the distribution of errors is quite low (Figure 2), for most of the relative roughness of the pipes and specially Reynolds' numbers.

For the correlation proposed by Brkić I (Brkić, 2011a), represented in Figure 2, it presents a better result in those systems that have larger diameter steel pipes and that move water at an important speed (magnitude of Re); on the other hand for the correlation represented in Figure 2, it shows better results for rough pipes such as PVC independently of the runoff speed of the water inside.

The correlation proposed (Eq. 2) was designed to approach the value of Colebrook's friction factor for all the range of pipes, both rough and smooth as well as for regimes of high and low turbulence.

The best performance of the correlation proposed by the authors, represented in Figure 2, is obtained for a range of roughness that goes from 10^{-2} to 5×10^{-3} , with maximum relative errors lower than 1.50% for all the range of Reynolds values, which make possible to conclude that if someone is reviewing the pressure loss in a water system, this correlation can be perfectly used when the system presents steel pipes or PVC in its path, for commercial diameters that are between 100 and 600 mm, since in reviewing the behavior of the correlation for steel pipes, the relative error does not exceed 1.00% and in PVC pipes does not exceed 1.50%.

The matrix of values of the friction factors calculated with the proposed approximation (Eq. 2) correlates well with a value of 99%, with the matrix of values of Colebrook's numerical solution.

The Eq. 2 proposed presents a simple structure, easy to remember and with a few mathematical operations for its solution. This correlation has a maximum relative error of 1.60% and an acceptable medium relative error within the orders of magnitude of most of the proposals analyzed. As a result, the correlation proposed is located immediately after the most precise correlations such as Romeo et al. (Romeo et al., 2002) and the correlation proposed by Fang et al. (Fang et al., 2011).

The evolution of the maximum value of the relative error (%) for each value of ϵ/D considered in the analysis allows studying the behavior of such parameters that in some of the correlations stays approximately constant for all values of ϵ/D and in other correlations presents a better adjustment to certain ϵ/D values. This comparison of the maximum relative error was carried out for the values of ϵ/D from 10^{-1} to 10^{-6} and for all Re values between 10^4 and 10^8 .

The general analysis and the values shown in Table 1, indicate that to calculate the friction factor in pipes under turbulent flow, the new explicit approximation proposed in this study can be used with a low error percentage and with the simplicity of its operations.

CONCLUSION

Through the proposed new first generation explicit correlation equation, the friction factor value can be estimated by a simple structured formula, mathematically easy to be solved with few operations and with a relative low error percentage (1.60%) in comparison to the value given by Colebrook's equation.

LITERATURE CITED

- Anaya-Durand, A. I.; Cauich-Segovia, G. I.; Funabazama-Bárceñas, O.; Gracia-Medrano-Bravo, V. A. Evaluación de ecuaciones de factor de fricción explícito para tuberías. *Educacion Quimica*, v.25, p.128-134, 2014. [https://doi.org/10.1016/S0187-893X\(14\)70535-X](https://doi.org/10.1016/S0187-893X(14)70535-X)
- Augusto, G. L.; Culaba, A. B.; Tanhueco, R. M. T. Pipe sizing of District cooling distribution network using implicit Colebrook-White equation. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, v.20, p.76-83, 2016. <https://doi.org/10.20965/jaciii.2016.p0076>
- Barry, D. A.; Parlange, J. Y.; Li, L.; Prommer, H.; Cunningham, C. J.; Stagnitti, F. Analytical approximations for real values of the Lambert W-function. *Mathematics and Computers in Simulation*, v.53, p.95-103, 2000. [https://doi.org/10.1016/S0378-4754\(00\)00172-5](https://doi.org/10.1016/S0378-4754(00)00172-5)
- Boyd, J. P. New approximations to the principal real-valued branch of the Lambert W-function. *Advances in Computational Mathematics*, v.43, p.1403-1436, 2017. <https://doi.org/10.1007/s10444-017-9530-3>
- Brkić, D. An explicit approximation of Colebrook's equation for fluid flow friction factor. *Petroleum Science and Technology*, v.29, p.1596-1602, 2011a. <https://doi.org/10.1080/10916461003620453>
- Brkić, D. Review of explicit approximations to the Colebrook relation for flow friction. *Journal of Petroleum Science and Engineering*, v.77, p.34-48, 2011b. <https://doi.org/10.1016/j.petrol.2011.02.006>

- Brkić, D. W solutions of the CW equation for flow friction. *Applied Mathematics Letters*, v.24 p.1379-1383, 2011c. <https://doi.org/10.1016/j.aml.2011.03.014>
- Buzzelli, D. Calculating friction in one step. *Machine Design*, v.80, p.54-55, 2008.
- Colebrook, C. F. Turbulent flow in pipes, with particular reference to the transition region between the smooth and rough pipe laws. *Journal of the Institution of Civil Engineers*, v.12, p.393-422, 1939. <https://doi.org/10.1680/ijoti.1939.14509>
- Colebrook, C. F.; White, C. M. Experiments with fluid friction in roughened pipes. *Proceedings of the Royal Society A: Mathematical. Physical and Engineering Sciences*, v.161, p.367-381, 1937. <https://doi.org/10.1098/rspa.1937.0150>
- Fang, X.; Xu, Y.; Zhou, Z. New correlations of single-phase friction factor for turbulent pipe flow and evaluation of existing single-phase friction factor correlations. *Nuclear Engineering and Design*, v.241, p.897-902, 2011. <https://doi.org/10.1016/j.nucengdes.2010.12.019>
- Filonenko, G. Hydraulic resistance in pipes. *Teplotenergetika*, v.1, p.40-44, 1954.
- Manadili, G.; Silverberg, P. M. Replace implicit equations with signomial functions. *Chemical Engineering*, v.104, p.129-130, 1997.
- Mikata, Y.; Walczak, W. S. Closure to "exact analytical solutions of the Colebrook-White equation". *Journal of Hydraulic Engineering*, v.143, p.1-2, 2017. [https://doi.org/10.1061/\(ASCE\)HY.1943-7900.0001340](https://doi.org/10.1061/(ASCE)HY.1943-7900.0001340)
- Najafzadeh, M. Evaluation of conjugate depths of hydraulic jump in circular pipes using evolutionary computing. *Soft Computing*, v.23, p.13375-13391, 2019. <https://doi.org/10.1007/s00500-019-03877-9>
- Najafzadeh, M.; Barani, G. A. Comparison of group method of data handling based genetic programming and back propagation systems to predict scour depth around bridge piers. *Scientia Iranica*, v.18, p.1207-1213, 2011. <https://doi.org/10.1016/j.scient.2011.11.017>
- Najafzadeh, M.; Kargar, A. R. Gene-expression programming, evolutionary polynomial regression, and model tree to evaluate local scour depth at culvert outlets. *Journal of Pipeline Systems Engineering and Practice*, v.10, p.1-12, 2019. [https://doi.org/10.1061/\(ASCE\)PS.1949-1204.0000376](https://doi.org/10.1061/(ASCE)PS.1949-1204.0000376)
- Najafzadeh, M.; Sattar, A. M. A. Neuro-Fuzzy GMDH approach to predict longitudinal dispersion in water networks. *Water Resources Management*, v.29, p.2205-2219, 2015. <https://doi.org/10.1007/s11269-015-0936-8>
- Najafzadeh, M.; Shiri, J., Sadeghi, G.; Ghaemi, A. Prediction of the friction factor in pipes using model tree. *ISH Journal of Hydraulic Engineering*, v.24, p.9-15, 2018. <https://doi.org/10.1080/09715010.2017.1333926>
- Özger, M.; Yildirim, G. Determining turbulent flow friction coefficient using adaptive neuro-fuzzy computing technique. *Advances in Engineering Software*, v.40, p.281-287, 2009. <https://doi.org/10.1016/j.advengsoft.2008.04.006>
- Papaevangeliou, G.; Evangelides, C.; Tzimopoulos, C. A new explicit relation for friction coefficient f in the Darcy-Weisbach equation. *Protection and Restoration of the Environment*, v.5/9, p.166-173, 2010.
- Romeo, E.; Royo, C.; Monzón, A. Improved explicit equations for estimation of the friction factor in rough and smooth pipes. *Chemical Engineering Journal*, v.86, p.369-374, 2002. [https://doi.org/10.1016/S1385-8947\(01\)00254-6](https://doi.org/10.1016/S1385-8947(01)00254-6)
- Shaikh, M. M.; Massan, S. U. R.; Wagan, A. I. A new explicit approximation to Colebrook's friction factor in rough pipes under highly turbulent cases. *International Journal of Heat and Mass Transfer*, v.88, p.538-543, 2015. <https://doi.org/10.1016/j.ijheatmasstransfer.2015.05.006>
- Swamee, P. K.; Jain, A. K. Explicit equations for pipe-flow problems. *Journal of the Hydraulics Division-Asce*, v.102, p.657-664, 1976. <https://doi.org/10.1061/JYCEAJ.0004542>
- Winitzki, S. Uniform approximations for transcendental functions. In: *International Conference on Computational Science and Its Applications*. Berlin: Springer, 2003. p.780-789. https://doi.org/10.1007/3-540-44839-X_82
- Winning, H. K.; Coole, T. Explicit friction factor accuracy and computational efficiency for turbulent flow in pipes. *Flow, Turbulence and Combustion*, v.90, p.1-27, 2013. <https://doi.org/10.1007/s10494-012-9419-7>

ERRATA

Na página 440, **Onde se lia:**

$$f = \left[-2 \log \left(\frac{4.859}{\text{Re}^{-0.888}} + \frac{\varepsilon/D}{3.7} \right) \right]^{-2} \quad (2)$$

Leia-se:

$$f = \left[-2 \log \left(\frac{4.859}{\text{Re}^{0.888}} + \frac{\varepsilon/D}{3.7} \right) \right]^{-2} \quad (2)$$

Na página 440, **Onde se lia:**

$$f = \left[-2 \log \left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{\text{Re}^{0.90}} \right) \right]^{-2} \quad (3)$$

Leia-se:

$$f = \left[-2 \log \left(\frac{\varepsilon/D}{3.7} + \frac{5.74}{\text{Re}^{0.90}} \right) \right]^{-2} \quad (3)$$

Na página 442, **Onde se lia:**

In a more delimited analysis, it is possible to define the value of the constant B, and with it calculate the value of constant A that minimizes RE (%) value. It is possible to observe for the proposed correlation that the minimum relative error can be obtained for a value of the constants A = 4.859 and B = - 0.888 and that are part of the authors proposal.

Leia-se:

In a more delimited analysis, it is possible to define the value of the constant B, and with it calculate the value of constant A that minimizes RE (%) value. It is possible to observe for the proposed correlation that the minimum relative error can be obtained for a value of the constants A = 4.859 and B = 0.888 and that are part of the authors proposal.

Revista Brasileira de Engenharia Agrícola e Ambiental
(2023) 27:79-79.

