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Testing the consumption-based CAPM using the stochastic discount factor*

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Contents

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rule of thumb, consumption models, stochastic discount factor, CCAPM

JEL Codes
C32, E21

Abstract · Resumo

This article investigates the problem of optimal intertemporal consumption in the CCAPM setup from a new empirical perspective. The econometric analysis is based on use of the equality between the stochastic discount factor (SDF) and the marginal rate of intertemporal substitution of consumption, which in the CCAPM is equivalent to the Euler equation resulting from the intertemporal optimization problem of the representative individual. We start from an asset pricing equation to find the estimators of the SDF, without the need to make a parametric assumption about preferences, and then estimate the parameters of the consumption models. In our empirical exercise, the dataset covers income, aggregate consumption and return on financial assets in the quarterly period from 1996:1 to 2016:4. We also consider the existence of a portion of rule-of-thumb consumers and the utility functions CRRA and habit formation in consumer preferences. The empirical results suggest that the preferences that exhibit the formation of consumption habits combined with the stochastic discount factor originating from the hypotheses of Brownian motion are those that most closely correspond to the hypotheses related to the behavior of aggregate consumption.

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1. Introduction

Aggregate consumption is one of the most important macroeconomic components of GDP and links the stock market and production in most economies. This relationship is based on the wealth effect as the traditional channel for transferring risks and assets. The risk of macroeconomic variables is considered an important factor in investor decisions. It is on this basis that the consumption-based capital asset pricing models – CCAPMs (Rubinstein, 1976; Lucas, 1978; and Breeden, 1979) gained relevance in the literature.

Several authors have used the CCAPM setup to estimate and test aggregated nonlinear rational expectation models directly from stochastic Euler equations. Some examples are Hansen and Jagannathan (1991), Hansen and Singleton (1982, 1984), and Epstein and Zin (1991). Other extensions of these works include different functional forms of constant relative risk aversion (CRRA) for consumption preferences. For instance, Campbell and Cochrane (1999) and Constantinides (1990) used a model with habit formation of utility, and Epstein and Zin (1989, 1991) modeled the Kreps and Porteus (1978) recursive utility function. In the Brazilian literature, several studies have followed this line of research; we can mention the works of Cavalcanti (1993), Reis, Issler, Blanco, and Carvalho (1998), Issler and Rocha (2000), Issler and Piqueira (2000), Gomes and Paz (2004), Gomes (2004, 2010), Costa and Carrasco-Gutierrez (2015), and Silva and Carrasco-Gutierrez (2019).

This paper investigates the problem of optimal intertemporal consumption in the CCAPM setup from a new empirical perspective. Unlike the procedures presented by Hansen and Jagannathan (1991) and Hansen and Singleton (1982, 1984), we start from an asset pricing equation to find the estimators of the stochastic discount factor (SDF). This procedure estimates the SDF with the available information about asset returns of an economy and without the need to make a parametric assumption about preferences. We demonstrate this empirical approach in an example with Brazilian data in the quarterly period from 1996:1 to 2016:4. Besides the traditional utility function of the CRRA type, we also include a function whose preferences reveal the existence of consumption habits. Additionally, in line with the evidence presented by Cavalcanti (1993), Reis et al. (1998), Gomes and Paz (2004), Issler and Rocha (2000), Gomes and Paz (2004), Gomes (2004), Costa and Carrasco-Gutierrez (2015) and Oliveira and Carrasco-Gutierrez (2016), we incorporate in the model the existence of a portion of consumers with rule-of-thumb behavior.[1] Finally, we

[1] Flavin (1981) argued that consumption is sensitive to current income and is greater than that predicted by the permanent income hypothesis. This conclusion was widely interpreted as evidence of the existence of a liquidity constraint, which is one of the main reasons why it is difficult to observe consumption smoothing in the data. For this reason, Campbell and Mankiw (1989, 1990) suggested that aggregated data on consumption would be better characterized if there were two types of consumers: optimizers and the rule-of-thumb type.

consider three types of SDF models:[2] the SDF with minimum variance, as presented by Hansen and Jagannathan (1991); the SDF model based on the hypothesis of Brownian motion of prices; and the SDF model resting on the hypothesis of the CAPM.

This work contributes to the literature by presenting a new empirical approach that allows estimating and testing the implications of aggregate consumption models by using the SDF model. In addition, we present an example with Brazilian data. The empirical results show that the consumption habit preferences and the stochastic discount factor model based on the Brownian motion hypothesis best fit the hypotheses related to the implications of aggregate consumption behavior.

This paper is organized into five sections including this introduction. Section 2 presents the empirical approach using the CCAPM framework; section 3 describes the asset pricing models; section 4 presents the database and econometric results; and section 5 presents our conclusions.

2. Consumption-Based Capital Asset Pricing Model (CCAPM)

The CCAPM is a stochastic dynamic equilibrium model in an economy in which a representative agent chooses how much to consume and how much to invest to maximize the expected present value of his future utility function, constrained by the evolution of his stock of wealth. The optimal choice problem of this agent for separable utility is represented as follows:

$$\begin{gathered}\max_{[C_{2t+s},\theta_{t+s}]_{s=0}^{\infty}} \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s u_{t+s}\left(C_{2,t+s}\right)\right] \\ C_{2,t}+\theta_{t+1}P_t=\theta_t P_t+\theta_t d_t+Y_t; \quad C_{2t},\theta_{t+1}\geq 0,\forall t,\end{gathered} \tag{1}$$

where $u_t(\cdot)$ is the instantaneous utility function at t; β is the intertemporal discount coefficient; j is an index that refers to each asset available in the market; $C_{2,t}$ is the aggregate consumption of households that have optimizing behavior; θ_t is the vector of assets; P_t is the vector of asset prices in each period; d_t is the vector of dividends paid by the assets; and Y_t is the exogenous income received in each period by agents. The Euler condition for this problem results in:

$$1=\mathbb{E}_t\left[\beta\frac{\left(\partial u_{t+1}/\partial C_{2,t+1}\right)}{\left(\partial u_t/\partial C_{2,t}\right)}\left(R_{j,t+1}\right)\right], \tag{2}$$

where the gross domestic product is calculated as $R_{j,t+1} = \left(P_{j,t+1}+d_{j,t+1}\right)/P_{j,t}$. Therefore, by defining the marginal rate of intertemporal substitution of consumption

[2]Certainly, other SDF models could be considered in this study, but we leave this as a suggestion for future works.

as

$$m_{t+1} = \beta \frac{\left(\partial u_{t+1}/\partial C_{2,t+1}\right)}{\left(\partial u_t/\partial C_{2,t}\right)},$$

equation (2) results in $1 = \mathbb{E}_t\left[m_{t+1}R_{j,t+1}\right]$, which is the pricing equation established by Harrison and Kreps (1979), Hansen and Richard (1987), and Hansen and Jagannathan (1991, 1997). The procedure adopted by Weber (2002) incorporates the portion of the population (defined by λ) that consume with all their current income. Thus, considering that $C_t = C_{1,t} + C_{2,t}$, and knowing that $C_{1,t} = \lambda Y_t$, we can represent the portion of the population that optimizes their consumption in function of total consumption and income as $C_{2,t} = C_t - \lambda Y_t$. The Euler equation related to the problem of the CCAPM, equation (2), will be valid to guarantee optimization of consumption of representative agents that consume the portion $C_{2,t}$. By substituting the consumption $C_{2,t}$ in equation (2), we obtain equation (3), which considers the portion of individuals that consume with all their current income:

$$\mathbb{E}_t\left[\beta \frac{u'\left(C_{t+1} - \lambda Y_{t+1}\right)}{u'(C_t - \lambda Y_t)}\left(R_{j,t+1}\right)\right] = 1; \qquad j = 1, 2, \dots, N. \tag{3}$$

Therefore, the stochastic discount factor is defined as

$$m_{t+1} = \beta \frac{u'\left(C_{t+1} - \lambda Y_{t+1}\right)}{u'(C_t - \lambda Y_t)}. \tag{4}$$

We use this relation as the starting point to estimate and test the aggregate consumption models by using the stochastic discount factor. In particular, we consider the CRRA and external habits utilities (Abel, 1990). Table 1 shows the functional form of these functional preferences and their stochastic discount factor as a result of the optimization process.

Table 1. Utility functions and stochastic discount factors of the models

Model	Utility function	Stochastic discount factor
CRRA	$u(C_{2,t}) = \frac{\left((C_{2,t})^{1-\gamma} - 1\right)}{(1-\gamma)}$	$m_{t+1} = \left[\beta\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$
Habit Formation	$u(C_{2t}, C_{2,t-1}) = \frac{1}{1-\gamma}\left(\frac{C_{2,t}}{C_{2,t-1}^k}\right)^{1-\gamma}$	$m_{t+1} = \left[\beta\left(\frac{C_t - \lambda Y_t}{C_{t-1} - \lambda Y_{t-1}}\right)^{k(\gamma-1)}\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$
		In logarithmic terms
CRRA		$\ln m_{t+1} = \ln\beta - \gamma\Delta\left[\ln(C_{t+1} - \lambda Y_{t+1})\right]$
Habit Formation		$\ln m_{t+1} = \ln\beta + k(\gamma-1)\,\Delta\left[\ln(C_t - \lambda Y_t)\right] - \gamma\Delta\left[\ln(C_{t+1} - \lambda Y_{t+1})\right]$

Note: The parameter γ is the relative risk aversion coefficient and k is the parameter that governs the temporal separability of the utility function.

3. Stochastic Discount Factor Model (SDF)

The asset pricing structure described by Harrison and Kreps (1979) and Hansen and Richard (1987) is based on a price equation:

$$p_t = \mathbb{E}_t\left[m_{t+1}x_{j,t+1}\right], \tag{5}$$

where $\mathbb{E}_t$ denotes the expectation conditional on the information available at time t; p_t is the price of the representative asset; m_{t+1} is the stochastic discount factor; and $x_{j,t+1}$ is the payoff of asset j at time $t+1$. Cochrane (2001) defined the following model for the SDF that provides a general structure for pricing assets:

$$m_{t+1} = f\left(data, parameters\right), \tag{6}$$

where $f(\cdot)$ and the pricing equation (5) can lead to different predictions stated in terms of returns. We present three SDF models used in the empirical application: the SDF of Hansen and Jagannathan; the SDF that uses Brownian motion; and the SDF from the CAPM.

3.1 Hansen and Jagannathan (SDF-HJ)

Hansen and Jagannathan (1991) proposed a nonparametric form to estimate the stochastic discount factor, providing a lower bound of its variance. Although not estimating it directly, the authors demonstrated that the simulated SDF (m^*_{t+1}) is directly related to the minimum conditional variance of the asset portfolio. Furthermore, they exploited the fact that it is always possible to project the SDF for a space of returns (payoffs) by directly expressing the simulation of the portfolio as a function of the observed variables, as below:

$$m^*_{t+1} = i'_N\left[\mathbb{E}_t\left(R_{t+1}R'_{t+1}\right)\right]^{-1}R_{t+1}. \tag{7}$$

Equation (7) provides a nonparametric way to estimate the SDF in function of the return on assets, where i_N is a vector of ones with dimension $N \times 1$; and R_{t+1} is a vector with dimension $N \times 1$ containing the returns of all assets.

3.2 Brownian Motion (SDF-BM)

According to Ross (2014), the calculation of the SDF according to the pricing model based on Brownian motion is valid under the basic assumptions of Black and Scholes (1973) and causes the asset price dynamic to be a stochastic process. Thus, the assumption that the vector of prices follows geometric Brownian motion (GBM) is given by

$$\frac{\mathrm{d}P}{P} = \left(R^f + \mu\right)\mathrm{d}t + \Sigma^{1/2}\mathrm{d}B,$$

where $dP/P = \left(dP_1/P_1, \dots, dP_N/P_N\right)'$; $\mu = (\mu_1, \dots, \mu_N)'$; Σ is a defined and positive $N \times N$ matrix; P_j is the price of asset j; μ is the risk premium vector; R^f is the risk-free rate of return; and B is a GBM process with dimension N. Oksendal (2002) showed it is possible to use Itô's theorem to demonstrate that

$$R^j_{t+\Delta t} = \frac{P^j_{t+\Delta t}}{P^j_t} = \exp\left\{ \left(R^f + \mu - \frac{1}{2}\Sigma_{j,j}\right)\Delta t + \sqrt{\Delta t}\left(\Sigma_j^{1/2}\right)' Z_t \right\},$$

where Z_t is a vector of N independent variables with Gaussian distribution that causes the SDF associated with Brownian motion to be computed as

$$m_{t+\Delta t} = \exp\left\{ -\left(R^f + \frac{1}{2}\mu'\Sigma^{-1}\mu\right)\Delta t - \sqrt{\Delta t}\mu\left(\Sigma^{1/2}\right)' Z_t \right\}. \tag{8}$$

Hence, the SDF is given by

$$\widehat{m}_t = \exp\left\{ -\left(R^f + \frac{1}{2}\widehat{\mu}'\widehat{\Sigma}^{-1}\widehat{\mu}\right)\Delta t - \widehat{\mu}'\widehat{\Sigma}^{-1}\left(R_t - \bar{R}\right) \right\}, \tag{9}$$

where $R_t = \left(R^1_t, \dots, R^N_t\right)'$, $\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$, and $\widehat{\mu}$, $\widehat{R}$ and $\widehat{\Sigma}$ are estimated by

$$\widehat{\mu} = \frac{\bar{R} - R^f}{\Delta t} \quad \text{and} \quad \widehat{\Sigma} = \frac{1}{\Delta t}\frac{1}{T}\sum_{t=1}^{T}\left(R_t - \bar{R}\right)\left(R_t - \bar{R}\right)'.$$

3.3 Capital Asset Pricing Model (SDF-CAPM)

According to Cochrane (2001), a relationship of equivalence exists between the representation of the model $\mathbb{E}\left(mR^i\right) = \gamma + \lambda'\beta_i$ and the linear model for the stochastic discount factor $m = a + b'f$, where f is a vector of factors and b is a vector that contains the parameters associated with these factors. The model is based on the following equation:

$$m = a + b'f, \qquad 1 = \mathbb{E}\left(mR^j\right), \tag{10}$$

where $\mathbb{E}(f) = 0$. We can find γ and λ such that

$$\mathbb{E}\left(R^j\right) = \gamma + \lambda'\beta_j, \qquad \gamma \equiv \frac{1}{\mathbb{E}(m)} = \frac{1}{a}, \qquad \text{and} \qquad \lambda \equiv -\frac{1}{a}\text{cov}(f, f')b.$$

Forthe market risk factor $R_{w,t+1}$, the SDF result is:

$$m_{t+1} = a + bR_{w,t+1}. \tag{11}$$

4. Results

4.1 Data

The dataset used covers the period from the first quarter of 1996 to the fourth quarter of 2016. The series on final household consumption and national gross income were obtained from Ipeadata. The market index was the Ibovespa and the financial assets were common and preferred shares listed for trading on the B3 (BM&Fbovespa). The financial returns were obtained from the Economatica database.[3] The interbank deposit rate (CDI), obtained from the Central Bank of Brazil, was used as the risk-free rate of return. The data on consumption and income were deflated by the Comprehensive Consumer Price Index – IPCA (mean 2000= 100). We treated the seasonality of the series by the Census X-12 method and the figures were expressed in per capita, utilizing data on the Brazilian population from 1996 to 2016.[4] Finally, the financial asset returns were deflated by the IPCA.

Figure 1 presents the series related to real consumption and income of Brazilians (data in millions of reais–R$). As of 2009, there has been a more pronounced difference between the consumption and income series. Among some events that occurred after that date, we can mention the consequences of the subprime crisis in 2008–2009 and increased household indebtedness due to increased use of credit between 2010 and 2012. On the other hand, in 2015 the economy entered a recession, with deterioration of the labor market due to the reduction of the occupied population. Another factor is the generally high unemployment rate in Brazil, which rose from about 8 % in December 2014 to 11.80 % in the third quarter of 2016. Table 2 reports the descriptive statistics and the correlation matrix of the SDFs estimated.[5] In terms of the sample mean, the SDF-CAPM was the highest and the SDF-BM was

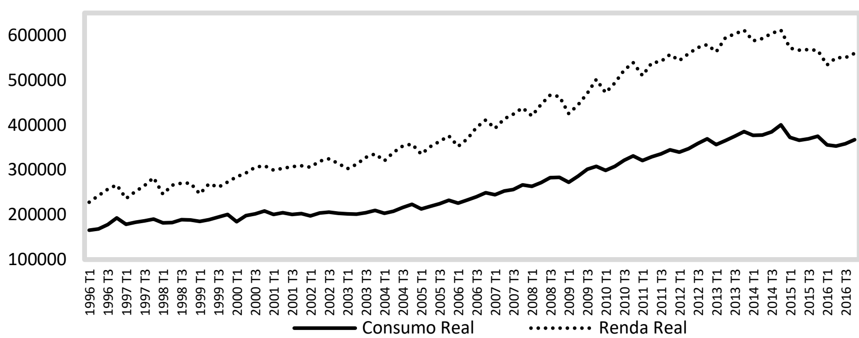

Figure 1. Real consumption and real income series

[3]The returns on financial assets are obtained from the BM&FBovespa database. We use 69 asset returns for common (ON) and preferred (PN) shares to estimate the SDFs.

[4]The annual series of the Brazilian population were converted into quarterly series by the formula: $I = \left(P_{t+n}/P_t\right)^{1/n} - 1$, where P_t is the population at the start of the period; P_{t+n} is the population for the year; and $(t + n)$ and (t) are the time intervals between the two periods. Source: http://www.ripsa.org.br

[5]To deal with problems of correlation of the returns in estimating the SDFs, we use common factor analysis, as described by Carrasco-Gutierrez and Issler (2015). We construct three common factor returns from all the returns available in the sample.

Table 2. Descriptive statistics related to the SDFs generated

	SDF-BM	SDF-HJ	SDF-CAPM	Correlation	SDF-BM	SDF-HJ	SDF-CAPM
Mean	0.78	0.94	0.98	SDF-BM	1.000		
Median	0.75	0.91	0.98	SDF-HJ	-0.377	1.000	
Maximum	1.36	1.54	1.32	SDF-CAPM	0.839	-0.574	1.000
Minimum	0.22	0.62	0.61				
Std. Dev.	0.23	0.19	0.14				

Note: SDF-B, SDF-HJ and SDF-CAPM are the three stochastic discount factors generated with the market data. SD is the standard deviation.

most volatile. There also was a high positive correlation between the SDF-BM and SDF-CAPM metrics. Figure 2 contains the graph of the temporal dimension of the estimated SDFs.

Before performing estimations of the models presented in Table 1, it was necessary to calibrate the parameter λ. Evidence about this parameter for the Brazilian case was previously presented. Cavalcanti (1993) found $\lambda = 0.32$; Reis et al. (1998) reported evidence for λ approximately equal to 0.80; Gomes and Paz (2004) found $\lambda = 0.61$; Issler and Rocha (2000) and Gomes (2004) obtained average values of 0.74 and 0.85 respectively; and Costa and Carrasco-Gutierrez (2015) found $\lambda = 0.72$. We used $\lambda = 0.3$ and $\lambda = 0.6$. These values are close to those reported in the literature mentioned and were chosen considering the restrictions of the proposed approach[6] Besides this, we studied the case without rule-of-thumb consumers, i.e., when $\lambda = 0$. In all cases, we estimated the equation using the nonlinear least squares (NLLS) method. The results of these estimates are presented in tables 3, 4 and 5.

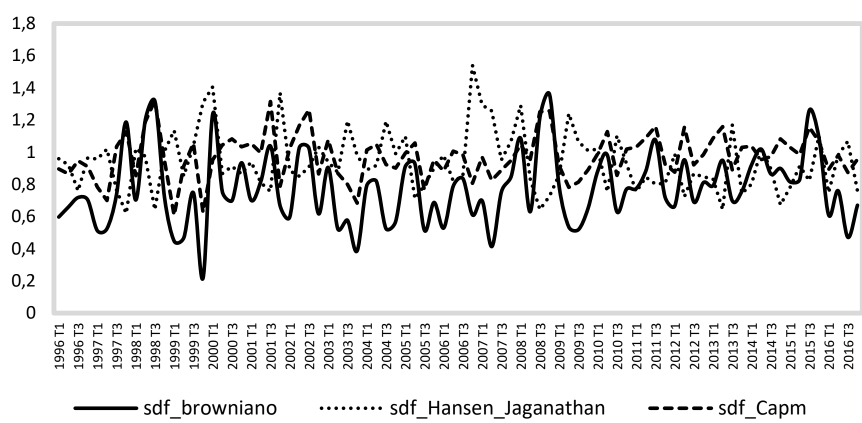

Figure 2. Stochastic discount factors: SDF-BM, SDF-HJ and SDF-CAPM

[6]This choice, besides being in accordance with the literature, also is in line with the constraints given by the theoretical approach and by the behavior of the consumption and income series treated in this paper. The aggregate consumption series of the optimizing individuals, $C_{2,t}$, obeys the constraint imposed by the model $C_{2,t} = \left(C_{t+1} - \lambda Y_{t+1}\right)$. Since $C_{2,t}$ cannot be negative, the values of λ were limited. Empirically we observed that this restriction was satisfied for values of λ lower than 0.65.

Table 3: Estimation of the parameters for the consumption models – SDF with Brownian motion

	CRRA: $u(c_{2,t}) = \frac{(c_{2,t})^{1-\gamma}-1}{1-\gamma}$						**External Habit:** $u(C_{2t}, C_{2,t-1}) = \frac{1}{1-\gamma}\left(\frac{C_{2,t}}{C_{2,t-1}^{k}}\right)^{1-\gamma}$					
	$m_{t+1} = \left[\beta\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + -\gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$			$m_{t+1} = \left[\beta\left(\frac{C_t - \lambda Y_t}{C_{t-1} - \lambda Y_{t-1}}\right)^{k(\gamma-1)}\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + k(\gamma-1)\,\Delta[\ln(C_t - \lambda Y_t)] - \gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$		
	(I)			(II)			(I)			(II)		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$
β	0.7931*** (0.0258)	0.7851*** (0.0255)	0.7810*** (0.0255)	0.7586*** (0.0272)	0.7506*** (0.0263)	0.7467*** (0.0260)	0.8032*** (0.0267)	0.7954*** (0.0253)	0.7809*** (0.02570)	0.7694*** (0.0288)	0.7641*** (0.0267)	0.7466*** (0.0261)
γ	3.1869* (1.7895)	1.6370* (0.9275)	-0.0218 (0.1060)	2.9965 (1.9378)	1.372 (1.1132)	-0.028 (0.1123)	3.3722* (1.7914)	2.3789** (0.9032)	-0.0096 (0.1081)	3.0711 (1.9314)	2.4658** (1.1759)	-0.0138 (0.1149)
k		-	-	-	-	-	-0.9352 (1.0063)	-1.3013 (0.9311)	0.0519 (0.1071)	-1.1701 (1.4090)	-1.5301 (1.2117)	0.0712 (0.1153)
ARCH Test[1]	8.5496*	5.5882	8.4329*	1.8771	1.7209	2.1605	7.691585	4.446435	7.4668	1.9296	1.7548	2.0991
LM Test[2]	2.3064	3.5597	3.703	0.8984*	1.8165	1.4511	3.123008	4.735853	3.496	1.202	2.638	1.4032
Jarque-Bera[3]	1.2215	1.3451	2.1766	16.477***	16.123***	16.653***	1.207257	1.526151	2.2992	17.209***	11.170***	15.192***

Notes: *, ** and *** refer to rejection of the null hypothesis at the levels 10%, 5% and 1%. Steps of the Gauss-Newton/Marquardt method. (1) ARCH test, least squares method; (2) Breusch-Godfrey serial correlation LM Test; (3) Jarque-Bera test of normality of the residuals. The shaded columns represent the valid models as defined by the diagnostic tests at 5% significance. i) m_{t+1} is the estimator of the stochastic discount factor (SDF), according to the hypothesis proposed by Hansen and Jagannathan (1991) for asset pricing. ii) Specification test: (4 lags)-ARCH Test: with null hypothesis of constant variance. Test of the autoregressive effect of the variance of the errors; iii) Specification test: (4 lags)- LM Test: with null hypothesis of absence of autocorrelation of the errors. Test of the presence of serial correlation in the model; iv) Specification test: (4 lags)- Jarque-Bera, with the null hypothesis of normality of the errors. Tests whether the errors are constant, as in a normal distribution. v) C_{t+1} represents the aggregate consumption; Y_{t+1} represents the aggregate income; and $C_{2,t}$ represents the consumption of individuals that optimize (rational expectations), where: $C_{2,t+1} = C_{t+1} - \lambda Y_{t+1}$. vi) The data of aggregate consumption and aggregate income of the individuals utilized in the model are per capita; vii) Considering for the term (λ) the values 0; 0.3; and 0.6, with minimum value (zero) and maximum value (0.6) set so that consumption cannot be negative. viii) Model I (CRRA) estimates results in level; as well as model I (External Habit). ix) Model II (CRRA) estimated results in logarithms, as well as model II (External Habit).

Table 4: Estimation of the parameters for the consumption models – SDF with Hansen and Jagannathan (HJ)

	CRRA: $u(c_{2,t}) = \frac{(c_{2,t})^{1-\gamma}-1}{1-\gamma}$						**External Habit:** $u(C_{2t}, C_{2,t-1}) = \frac{1}{1-\gamma}\left(\frac{C_{2,t}}{C_{2,t-1}^{k}}\right)^{1-\gamma}$					
	$m_{t+1} = \left[\beta\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + -\gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$			$m_{t+1} = \left[\beta\left(\frac{C_t - \lambda Y_t}{C_{t-1} - \lambda Y_{t-1}}\right)^{k(\gamma-1)}\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + k(\gamma-1)\,\Delta[\ln(C_t - \lambda Y_t)]$ $-\gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$		
	(I)			(II)			(I)			(II)		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$
β	0.9368*** (0.2195)	0.9424*** (0.0213)	0.9405*** (0.0273)	0.9202*** (0.0206)	0.9244*** (0.0202)	0.9237*** (0.0197)	0.9338*** (0.0231)	0.9418*** (0.0220)	0.9399*** (0.0206)	0.9171*** (0.0216)	0.9228*** (0.0209)	0.9236*** (0.0197)
γ	-0.8871 (1.2567)	0.4752 (0.6975)	0.1090 (0.0686)	-0.7591 (1.2133)	0.3238 (0.6940)	0.0884 (0.0687)	-0.9120 (1.2620)	0.4265 (0.7626)	0.1292* (0.0687)	-0.7765 (1.2200)	0.2087 (0.7609)	0.0996 (0.0702)
k	-	-	-	-	-	-	-0.2731 (0.6763)	-0.2447 (1.3002)	0.1053 (0.0848)	-0.3187 (0.7093)	-0.3713 (0.8886)	0.0630 (0.0792)
ARCH Test[1]	3.1252	2.7246	4.2501	3.1773	1.9352	3.0199	2.8937	3.0035	2.5674	3.1802	2.4359	2.2058
LM Test[2]	3.7501	4.0376	2.9404	3.9872	4.3067	3.3979	3.8774	3.8813	3.3203	4.1887	4.0752	3.5833
Jarque-Bera[3]	10.048***	8.6162**	6.2302**	1.0365	1.2140	0.8823	10.210**	8.4676**	3.9070	1.1911	1.4771	0.4304

Notes: *, ** and *** refer to rejection of the null hypothesis at the levels 10%, 5% and 1%. Steps of the Gauss-Newton/Marquardt method. (1) ARCH test, least squares method; (2) Breusch-Godfrey serial correlation LM Test; (3) Jarque-Bera test of normality of the residuals. The shaded columns represent the valid models as defined by the diagnostic tests at 5% significance. i) m_{t+1} is the estimator of the stochastic discount factor (SDF), according to the hypothesis proposed by Hansen and Jagannathan (1991) for asset pricing. ii) Specification test: (4 lags)-ARCH Test: with null hypothesis of constant variance. Test of the autoregressive effect of the variance of the errors; iii) Specification test: (4 lags)- LM Test: with null hypothesis of absence of autocorrelation of the errors. Test of the presence of serial correlation in the model; iv) Specification test: (4 lags)- Jarque-Bera, with the null hypothesis of normality of the errors. Tests whether the errors are constant, as in a normal distribution. v) C_{t+1} represents the aggregate consumption; Y_{t+1} represents the aggregate income; and $C_{2,t}$ represents the consumption of individuals that optimize (rational expectations), where: $C_{2,t+1} = C_{t+1} - \lambda Y_{t+1}$. vi) The data of aggregate consumption and aggregate income of the individuals utilized in the model are per capita; vii) Considering for the term (λ) the values 0; 0.3; and 0.6, with minimum value (zero) and maximum value (0.6) set so that consumption cannot be negative. viii) Model I (CRRA) estimates results in level; as well as model I (External Habit). ix) Model II (CRRA) estimated results in logarithms, as well as model II (External Habit).

Table 5: Estimation of the parameters for the consumption models – SDF with CAPM

	CRRA: $u(c_{2,t}) = \frac{(c_{2,t})^{1-\gamma}-1}{1-\gamma}$						**External Habit:** $u(C_{2t}, C_{2,t-1}) = \frac{1}{1-\gamma}\left(\frac{C_{2,t}}{C_{2,t-1}^{k}}\right)^{1-\gamma}$					
	$m_{t+1} = \left[\beta\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + -\gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$			$m_{t+1} = \left[\beta\left(\frac{C_t - \lambda Y_t}{C_{t-1} - \lambda Y_{t-1}}\right)^{k(\gamma-1)}\left(\frac{C_{t+1} - \lambda Y_{t+1}}{C_t - \lambda Y_t}\right)^{-\gamma}\right]$			$\ln m_{t+1} = \ln\beta + k(\gamma-1)\,\Delta[\ln(C_t - \lambda Y_t)]$ $-\gamma\Delta\ln(C_{t+1} - \lambda Y_{t+1})$		
	(I)			(II)			(I)			(II)		
	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$	$\lambda = 0$	$\lambda = 0.3$	$\lambda = 0.6$
β	0.9834*** (0.0167)	0.9805*** (0.0164)	0.9788*** (0.0161)	0.9728*** (0.0172)	0.9697*** (0.0168)	0.9679*** (0.0165)	0.9848*** (0.0176)	0.9850*** (0.0166)	0.9788*** (0.0162)	0.9731*** (0.0182)	0.9746*** (0.0172)	0.9679*** (0.0166)
γ	0.8952 (0.9255)	0.2806 (0.5226)	-0.2895 (0.0536)	0.9498 (0.9594)	0.3337 (0.5499)	-0.0256 (0.0550)	0.9075 (0.9322)	0.5062 (0.5518)	-0.0273 (0.0550)	0.9518 (0.9660)	0.6012 (0.5934)	-0.0237 (0.0565)
k	-	-	-	-	-	-	2.5839 (28.2087)	0.5834 (1.4885)	0.0093 (0.0529)	1.3379 (33.8693)	0.7629 (2.1348)	0.009 (0.0553)
ARCH Test¹	3.6638	3.2959	4.4861	8.3804*	7.9194*	9.3291*	3.7254	2.9212	4.3436	8.3589*	7.2487	9.2253*
LM Test²	3.8568	4.6171	3.3007	4.0793	4.9417	3.3409	4.0308	3.697	3.1803	4.1253	3.9259	3.232
Jarque-Bera³	0.0364	0.0628	0.2428	5.1114	5.1779*	6.5792**	0.0798	0.1178	0.2497	5.2661*	5.8812*	6.5713**

Notes: *, ** and *** refer to rejection of the null hypothesis at the levels 10%, 5% and 1%. Steps of the Gauss-Newton/Marquardt method. (1) ARCH test, least squares method; (2) Breusch-Godfrey serial correlation LM Test; (3) Jarque-Bera test of normality of the residuals. The shaded columns represent the valid models as defined by the diagnostic tests at 5% significance. i) m_{t+1} is the estimator of the stochastic discount factor (SDF), according to the hypothesis proposed by Hansen and Jagannathan (1991) for asset pricing. ii) Specification test: (4 lags)-ARCH Test: with null hypothesis of constant variance. Test of the autoregressive effect of the variance of the errors; iii) Specification test: (4 lags)- LM Test: with null hypothesis of absence of autocorrelation of the errors. Test of the presence of serial correlation in the model; iv) Specification test: (4 lags)- Jarque-Bera, with the null hypothesis of normality of the errors. Tests whether the errors are constant, as in a normal distribution. v) C_{t+1} represents the aggregate consumption; Y_{t+1} represents the aggregate income; and $C_{2,t}$ represents the consumption of individuals that optimize (rational expectations), where: $C_{2,t+1} = C_{t+1} - \lambda Y_{t+1}$. vi) The data of aggregate consumption and aggregate income of the individuals utilized in the model are per capita; vii) Considering for the term (λ) the values 0; 0.3; and 0.6, with minimum value (zero) and maximum value (0.6) set so that consumption cannot be negative. viii) Model I (CRRA) estimates results in level; as well as model I (External Habit). ix) Model II (CRRA) estimated results in logarithms, as well as model II (External Habit).

The results in columns (I) correspond to equations in level and in columns (II) in natural logarithm of the econometric models. The correct specification of the estimated models was based on the diagnosis of the errors of the models, e.g., tests of heteroscedasticity, autocorrelation and normal distribution. In all of them, the null hypothesis is the condition favorable to the correct specification of the model. Thus, the non-rejection of all the null hypotheses at 5 % significance means the model was correctly specified.

Table 3 presents the estimates of the parameters of the consumption models for the SDF-BM. It can be seen that both for the external habits preferences and the CRRA version, the estimates presented according to specification (I) indicate non-rejection of the null hypothesis in all the diagnostic tests, i.e., the premises associated with the errors of the model were satisfied. In particular, the diagnostic tests for the model with consumption habits did not reject the null hypothesis at 5 % significance. Therefore, considering that the models estimated with the external habits function were classified as being well specified, the results for $\lambda = 0.3$ provided statistically significant estimates for the parameters β and γ at 5 % significance, with estimated values of $\hat{\beta} = 0.79$ and $\hat{\gamma} = 2.37$, which are in line with the theory. For the CRRA, only the coefficient β was statistically significant, at the 5 % level. table 4 reports the estimates of the parameters of the consumption model for the SDF-HJ. When considering the estimates for the correctly specified models, it can be seen that in both the CRRA and the consumption habits models, only the coefficient β was statistically significant at 5 %. The best situation was for the external habits model in specification (I) with $\lambda = 0.6$, in which besides the significant beta ($\beta = 0.93$), the parameter γ was statistically significant at 10 %, with estimated value of 0.12. Finally, Table 5 presents the estimates of the parameters of the consumption models for the SDF-CAPM. The majority of the estimated models were correctly specified, but in all of them only the coefficient of the intertemporal discount factor was statistically significant, at the 5 % level. In this case, there was no evidence or risk aversion.

5. Conclusion

This article investigates the problem of optimal intertemporal consumption of the CCAPM type from a new empirical perspective. We present estimators for the SDF to aggregate the available information on the returns on assets in the economy, regardless of the specifications of the utility functions that were used. The econometric analysis is based on use of the equality between the SDF and the marginal rate of intertemporal substitution of consumption, which in the CCAPM is equivalent to the Euler equation resulting from the intertemporal optimization problem of the representative individual. In our empirical example using Brazilian data, we found that the consumption habit preferences and the SDF-BM best fit the hypotheses related to the implications of aggregate consumption behavior. In particular, the choice of $\lambda = 0.3$ led to statistically significant estimates of the parameters corresponding to the intertemporal impatience and relative risk aversion, with values near $\hat{\beta} = 0.8$ and $\hat{\gamma} = 2.4$ respectively. These empirical results show that the implications of the consumption models are in line with the macroeconomic theory.

For possible future research, we suggest some extensions of the empirical approach, among them: the inclusion of consumption preferences and the recursive utility model presented by Kreps and Porteus (1978), as well as other estimates of the stochastic discount factor model, such as those derived from the arbitrage pricing theory (APT) model of Ross (1976). Finally, we suggest using other measures of accuracy, such as the Hansen–Jagannathan distance or even the construction of the HJ frontier to validate and rank the stochastic discount factors.

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