

The Professional Identity in educators who teach mathematics: elements and actions to build a proposal for future research^{1 2 3}

Identidade profissional de professores que ensinam matemática: Elementos e ações para a construção de uma proposta para futuras investigações

Enio Freire de Paula ⁽ⁱ⁾

Márcia Cristina de Costa Trindade Cyrino ⁽ⁱⁱ⁾

⁽ⁱ⁾ Instituto Federal de Educação, Ciência e Tecnologia de São Paulo, São Paulo, SP, Brasil.
<https://orcid.org/0000-0003-0395-4689>, eniodepaula@ifsp.edu.br

⁽ⁱⁱ⁾ Universidade Estadual de Londrina; Centro de Ciências Exatas, Londrina, PR, Brasil.
<https://orcid.org/0000-0003-4276-8395>, marciacyrino@uel.br

Abstract

This article presents a theoretical essay motivated by the desire to learn which elements and actions can be considered in a research work that intends to discuss the professional identity (PI) in educators who teach mathematics. The results are the product of studies performed in recent years, which aimed at helping with the making of a non-prescriptive proposal based on the quadripolar model of investigation and on a characterization of the PI, which can be considered in future investigations on the PI in educators who teach mathematics. Thus, we hope that this study can offer subsidies to those interested in the PI in educators who teach mathematics and who are willing to expand discussions on this research field.

Keywords: professional identity, formation of teacher who teaches Mathematics, research on the professional identity of mathematics teachers, mathematical education

¹ Responsible Editor: Carlos Miguel da Silva Ribeiro – cmribas78@gmail.com, <https://orcid.org/0000-0003-3505-4431>

² Normalization, preparation, and Portuguese review: Luan Maitan – revisao@tikinet.com.br

³ English version: Viviane Ramos - vivianeramos@gmail.com

Resumo

O presente artigo apresenta um ensaio teórico, motivado pelo desejo de conhecer que elementos e ações podem ser considerados em um trabalho que se propõe a discutir a identidade profissional (IP) de professores que ensinam matemática (PEM). Os resultados apresentados são fruto de estudos realizados nos últimos anos com o objetivo de contribuir para a construção de uma proposta, não prescritiva, pautada no modelo quadripolar de investigação e em uma caracterização de IP, que podem ser considerados em futuras investigações a respeito da IP de PEM. Espera-se, assim, que este trabalho possa oferecer subsídios para reflexão aos interessados na temática da IP de PEM, dispostos a ampliar o debate a respeito desse campo investigativo.

Palavras-chave: *identidade profissional, formação de professores que ensinam matemática, investigação sobre identidade profissional de professores que ensinam matemática, educação matemática*

Introduction

To join efforts to reflect on the understanding of the elements present on the constitution of the Professional Identity (PI) in educators who teach mathematics teachers (MTs) is a constant challenge to those interested in this emerging research field.

We understand that the training of MTs is a field of ideological and political fight (Cyrino, 2006; 2017; Souza, Teixeira, Baldino, & Cabral, 1995) and that the movement to establish the PI of MTs should consider the aspects of *complexity, dynamism, temporality, and experience* –CDTE⁴ (De Paula & Cyrino, 2018d, in submission process) –, reaffirming the need to discuss the movement to constitute PI through the lenses of education/training. This discussion is relevant because it can foment the acknowledgement of professional practices of (future) MT and show tension/fragility moments that can trigger the process of formation needed to establish the PI of MTs.

⁴ To facilitate reading, in parts of the text, we will refer to these four characterizing aspects using the acronym CDTE.

In the international and national contexts, the studies on teachers' PI have gained prominence and are used as theoretical lenses to analyze projects and develop teacher training actions (Bennison, 2015; Goos, & Bennison, 2008; Hossain, Mendick, & Adler, 2013; Lloyd, 2006; Oliveira, 2004; Pamplona & Carvalho, 2009; Ponte & Chapman, 2008; Teixeira & Cyrino, 2015; Ticknor, 2012; Van Putten, Stols, & Howie, 2014).

In this theoretical essay, we discuss elements and actions to build a proposal, based on a quadripolar model of investigation (Lessard-Hebert, Goyette, & Boutin, 1994) and on our understanding of the movement to establish the PI of MTs (Cyrino, 2016, 2017).

The reflections presented here, motivated by the question “What elements and actions can be considered in a research that intends to discuss the PI of MTs?” were prompted to find answers to four problems that composed our investigative trajectory:

- i. What elements related to PI were assumed in dissertations and theses produced in Brazil between 2006 and 2016? What are the focuses of these studies? What results do they show? (De Paula & Cyrino, 2018a).
- ii. Which theoretical and epistemological elements are considered in the dissertations and theses produced in Brazil between 2006 and 2016? (De Paula & Cyrino, 2018c).
- iii. What perspectives of MTs' PI are present in scientific articles, published in internationally known journals in Mathematics education, between 2006 and 2016? (De Paula & Cyrino, 2018b).
- iv. What aspects could be considered in investigations on the movement to establish the PI of MTs? (De Paula & Cyrino, 2018d, under review).

Each of those questions guided our actions in our reflection-investigation path to write this theoretical essay. The first question led us to search and establish a *corpus* of 24 Brazilian dissertations and thesis from *stricto sensu* graduate programs in the area of Education and Teaching (*corpus* 1). First, we focused on the research contexts of the studies in this *corpus*, grouping them by ‘focus points’: MTs work condition; public policies, programs, or foment projects; different contexts of teacher training; practice community or study groups; pre-service MT training and pedagogical practices; online MT training; and MT as a secondary approach (De Paula & Cyrino, 2018a).

To answer the second question, also guided by this *corpus*, we analyzed the elements of the epistemological and theoretical poles presented in these studies. We reflected on authors' theoretical choices and identified four epistemological perspectives of the PI of MTs, which we have named *sociological*, *cultural*, *psychological/psychoanalytical*, and *general*. (De Paula & Cyrino, 2018c).

Regarding the third question, we have established a different *corpus* with 23 articles published in internationally renowned journals⁵, which are representative in the research field of Mathematics Education (*Corpus 2*). Then, by discussing, mapping, describing, and analyzing the epistemological perspectives of the concept of PI of MTs – and the important notions related to it –, we have identified the following perspective: *holistic*, *Wengerian*, *political-reformist*, and *internship pedagogy* (De Paula & Cyrino, 2018b).

The following path (question iv) was to confront the *corpora*, aiming to know which aspects could be considered in studies on the movement to establish PI of MTs. To do so, we have analyzed and problematized aspects associated to 9i) the processes of establishing the PI of MT, (ii) the investigation contexts, and (iii) the epistemological perspectives of the PI of MTs identified in these investigations during the *corpora* analysis. We then debate on the aspects considered important in the movement to establish the PI of MTs, among them *complexity*, *dynamism*, *temporality*, and *experience* (De Paula & Cyrino, 2018d, under review).

The time delimitation to build the *corpora* was drawn based on a project that nationally mapped Brazilian dissertations and theses⁶ focusing on MT published between 2001 and 2012, in *strictu sensu* graduate programs in the Education area. In the analysis of these studies, we have identified that the theme of PI was restricted⁷ to the period 2006-2012. Considering that PI is an emerging research field (Beijaard, Meijer, & Verloop, 2004; André, 2011; Cyrino, 2016a; Darragh,

⁵ *Educational Studies in Mathematics, International Journal of Science and Mathematics Education, Journal for Research in Mathematics Education, Journal of Mathematics Teacher Education, Mathematics Education Research Journal, Mathematical Thinking and Learning, Mathematics Teacher Education and Development, The Journal of mathematical behavior and ZDM Mathematics Education, Boletim de Educação Matemática (Bolema), and Revista Latinoamericana de Investigación en Matemática Educativa (Relime)*.

⁶ *Mapeamento e estado da arte da pesquisa brasileira sobre o professor que ensina matemática*. Call Chamada Universal; MCTI/CNPQ n° 014/2014, coordinated by Prof. Dr. Dario Fiorentini (FE/Unicamp). The regional results, as well as the analysis and more details on the project are published on the E-book *Mapeamento da pesquisa acadêmica brasileira sobre o professor que ensina matemática: período 2001-2012*. The complete reference is presented in the end of the article.

⁷ The fact that we do not find studies about PI of MT during the period 2001-2005 can be associated to the idea that the studies on teachers' PI (in general), in the Brazilian context, are relatively new and emerging, as pointed out by André (2009; 2011).

2016), we have enlarged the mapping of *Corpus 1* until 2016, keeping the same criteria used for the time frame 2001-2012, i.e., studies focusing on MT's approaching PI. We have used these criteria to establish *Corpus 2*, consolidating the *corpora* period of 2006-2016, resulting in 47 studies⁸ (15 dissertations, 9 theses, and 23 articles). We choose to present here our pathway to support this theoretical essay, as we understand that this is important and necessary, pointing out the imperative difficulties (and diligence) connected to this theme.

Therefore, we focus on elements and actions that can be considered when building a proposal, based on a quadripolar model of investigation pointed out by Lessard-Hebert et al. (1994) articulated with the characterization of the movement to constitute the PI of MT defended by Cyrino (2016; 2017), from what we consider to be relevant in our investigations and reflections faced by the analysis of the aforementioned *corpora* (De Paula & Cyrino, 2018a; 2018; 2018c; 2018d, under review).

In the next sections, we present the characterization of PI created by Cyrino (2016; 2017), with which we agree, problematizing it with the fact that we assume *complexity*, *dynamism*, *temporality*, and *experience* as significant aspects to be considered in studies about the movement to establish the PI of MTs (De Paula & Cyrino, 2018d, under review). Later, we discuss the quadripolar model of investigation, proposed by Lessard-Hebert et al. (1994), as a possibility to outline a methodological framework of investigation in this direction. We continue by intertwining these components and discussing elements and actions that can be considered in a proposal that supports studies in this direction and, finally, we draw some (in)conclusion remarks.

However, before that, we highlight that we are aware that the problems regarding the PI of MTs involve questions that hardly accept definitive answers, yes/no, right/wrong, or pro/con, as these dichotomies do not help us to further the discussion. We hope that this article can be a possibility to gather those willing to talk and reflect in the movement to establish the PI of MT; we do not have the illusion to find any definitive answers, but wish that new thoughts can emerge and enable new pathways.

⁸ The complete list of these studies (dissertations, theses, and articles) and their references can be found at De Paula and Cyrino (2018b; 2018c).

The ship of Theseus and the studies of PI of MT: singularities

The PI of MTs has similarities with an interesting paradox: the ship of Theseus. The legend says that among the many exploits of the Greek hero Theseus, is his victory to combat the Minotaur, a monster that lived imprisoned in a labyrinth in the Island of Crete⁹. From this story emerges an interesting paradox known as the *paradox of Theseus's ship*. In one version, after winning his combat, Theseus and the young Athenians who escaped from the Minotaur returned to Athens on a ship. To remind this conquest, the ship is docked for many years. New pieces would then replace her old ones, worn by time. The question is: *after a long time and after all the pieces exchanged, would the docked ship still be Theseus' ship?*

Now, we see the case of the PI of MTs. We assume that *complexity, dynamism, temporality, and experience* are important aspects in its constitutive processes (De Paula & Cyrino, 2018d, under review). *Complexity* comes from the understanding that the movement to constitute the PI of MT is a process influenced by an array of factors, for example, personal, social, cultural, psychological, contextual, and political ones. *Dynamism* is shown in the inconclusive and dynamic nature of the formative processes and in the repercussions of continuity arose from the (personal, professional, and inter-relational) contexts. The inter-relations of/on the contexts implement personal and professional transformations (metamorphosis). Thus, each of us, individually and as MT, is now different from what we were in the beginning of our professional life and, certainly, will be different in the future. That is why, we consider *temporality, and experience*

Articulated with this, we share Cyrino's characterization (2016; 2017), in which PI is understood as a *movement* that "takes place considering a set of beliefs and concepts interconnected to self-knowledge and knowledge about the profession, associated with the autonomy (vulnerability and sense of agency), and political commitment (Cyrino, 2017, p. 704)".

⁹ According to the Greek mythology, the famous Cretean labyrinth was built by the skillful Daedalus with the help of his son Icarus, by the demand of King Minos, to imprison the Minotaur, as it was half-man, half-bull. King Minos (from Crete) as a compensation for the death of his son Androgeus (who died on the attempt to capture the Marathonian Bull) demanded to King Aegeus (of Athens) that all young Athenians should be offered as tributes. These youngsters were sent a certain death in the labyrinth, as food for the Minotaur) (Stephanides, 2004).

Based on these ideas, it is possible to see that the PI of (future) MT is transformed during their professional and personal trajectories. Their beliefs and concepts are modified, as are their considerations/interpretations on their practices, the senses, and expectations towards the professional. Therefore, we need to draw another question: *in the case of MT, once substituted those “epistemological pieces”, would his/her PI as MT still be the same?* We believe not.

Theseus’ ship, as an inanimate object, after the replacement of all pieces could be seen as a representation of the essence of the original ship, though the pieces could have been built by different material and techniques. The MT, on the other hand, is a social-cultural-political being, a result (among other factors) of his/her multiple relations with others. After many epistemological transformations, his/her essence is altered. The PI of MT and alterity go hand in hand. Thus, during training actions/practices, the experience of *being, seeing oneself and being recognized as a mathematics teacher* involve the relation identity/alterity, and this particularity does not guarantee a convergence/agreement during these actions. His/her experiences (Larrosa, 2002) changed him/her. As an individual, the MT is complex and time-defined. His/her personal and professional relations are constantly changing. They are dynamic. The characterizing aspects of his/her PI (CDTE) show, therefore, that PI is produced in *movement*.

The quadripolar research model: singularity of *movement*

Lessard-Hebert et al. (1994) defend a research model articulated on four poles – *epistemological, theoretical, technical, and morphological* – as an interactive system establishing dynamic research processes. This model was our theoretical lenses, when analyzing *Corpus 1* (De Paula & Cyrino, 2018c) in which we specifically focused the epistemological and theoretical elements of the studies researched. Table 1 shows a synthesis of the main dimensions of each of the four poles in the research process.

Table 1 – Informative synthesis of the four poles of the research process

Poles	Dimensions
<i>Epistemological</i>	Guarantees the construction of the object, directly involving the discursive dimension and the paradigm related to the research problem.
<i>Theoretical</i>	Corresponds to the methodological instance in which the hypotheses are organized and the concepts defined. The theoretical pole encompasses the theoretical context of the research and the methodological procedures. During the research or after data collection, the theoretical pole also has an analytical role through data interpretation.
<i>Technical</i>	Establish the relation between the construction of the scientific object and the world of the events. In this dimension the researcher collects information on the real world and the investigations are converted into pertinent data to the research problem. It corresponds to the technical operations to collect data (unit and systems of observation, method of investigation).
<i>Morphological</i>	Related to the structuring of the scientific object, involving the display of the knowledge object; the style through which the researchers presents the results, referring to a causation space (articulating scientific facts in an operational configuration) to its explanation and understanding.

Source: Created by authors, based on Lessard-Hebert et al. (1994)

The connection among poles is evident during the research actions. The construction of the scientific object and the delimitation of the research problem integrate the *discursive processes*, as they are languages through which the scientific object takes shape. Such processes involve the construction of the investigative object (epistemological pole), the theoretical and investigative discussion on the problem (theoretical and technical pole), which, together, allow us to structure the object researched and draw the intersubjective criticisms of the researcher faced by these processes (morphological pole). Therefore, it is not only a question of configuring the researched object, but includes the display of the investigative process elements that allowed its construction (Lessard-Hebert et al., 1994). Visualizing the research action through discursive processes evidences a key characteristic of the quadripolar model: *its constitution is topologic (not chronologic nor sequential)*. Thus, the epistemological, theoretical, technical, and morphological are present in different moments of the research action, as there is an interaction among its characteristic elements. To highlight the analytical dimensions, related to the model articulated in four poles, we present a synthesis on Figure 1.

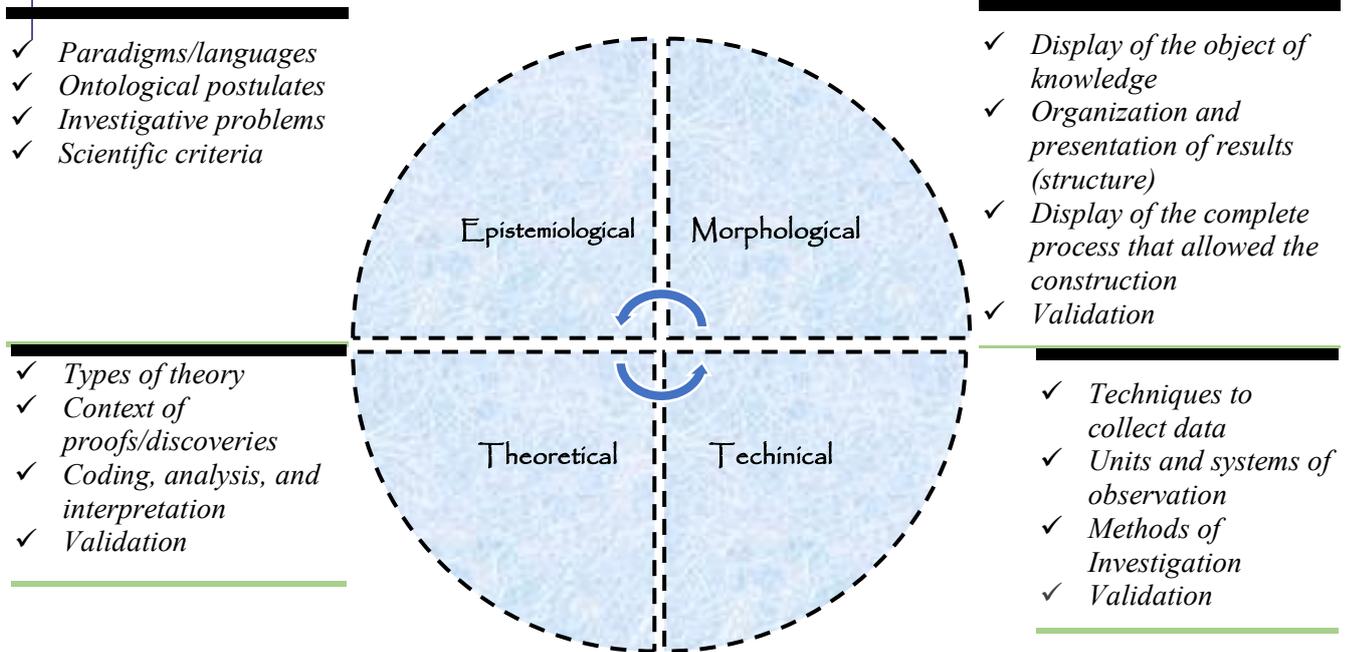


Figure 1: Connection of the analytical dimension in each investigative pole

Source: Created by authors, based on Lessard-Hebert et al. (1994)

The specificities of the quadripolar research model, associated to the characteristic aspects of the PI of MT (CDTE), help to understand the characterization proposed by Cyrino (2016; 2017), in which the idea of movement is present. Therefore, we believe that relating some guiding elements for future studies on the PI of MTs, interconnecting them, is an urgent need emerged from the current research demand, as expressed in the introduction of this essay.

In the following section, we discuss the interconnection of the elements we consider relevant to build a proposal for those who wish to investigate/reflect the PI of MT.

The emergence of elements to build a proposal that supports investigations on the PI of MTs: a possible Ariadne's thread

Returning to our initial character, another mythical protagonist is associated to Theseus' victory. The legend says that Ariadne, King Minos's daughter, with a ball of wool, helped Theseus to find the entrance of the labyrinth in the Island of Crete, beat the Minotaur, and leave the place¹⁰. The task was accomplished with the help of *Ariadne's thread*. From this story, the term '*Ariadne's thread*' became symbolically related to a logical effort to solve a complex problem.

In our case, the allegory to discuss elements to create a proposal that supports studies in the PI of MT as a *possible Ariadne's thread* is connected to perusing a *labyrinth-like pathway* more than the solution to the problem. The discussion of elements and actions that can be considered in creating an investigative proposal on the PI of MT, listed in this essay, offers a *possible pathway* for researchers who wish to *walk down the labyrinth-like pathway* of the studies on the PI of MTs. To walk (or not) on this path is a personal choice of the researchers. Our *Ariadne's thread* is only *one* (more) hypothesis to be considered. We defend the idea that the researchers should privilege some elements we consider key to the argumentative/analytical process based on consistence, a result of careful choices, on the idea of the PI of MTs.

On table 2, we list the actions associated to the elements we consider to be relevant to studies on the PI of MT and relate them to the poles of the quadripolar model. As we write on paper and thus are stuck to a bi-dimensional universe, we opted to delimitate the frontiers of the poles and the elements listed using dotted lines (as in figure 1). This is intentional and reaffirms the characteristics discussed in previous section on the quadripolar model (*its constitution is topological, non-chronological, non-sequential*). Thus, the elements present on table 2 though (more closely) guided towards a certain pole, are related to the others, as all of them (*epistemological, theoretical, technical, and morphological*) are present in several moments of the research. We use the dotted line to point out the relationship among the elements and the poles. After, we discuss about the actions associated to these elements and their possibilities, interconnecting

¹⁰ Exterminating the Minotaur was an extremely difficult task. However, to accomplish Theseus still need to leave the labyrinth Legend says that princess Ariadne, in love with Theseus, offered him her help to accomplish the task as long as he married her. Her solution was to give him a ball of wool, which Theseus would tie in the entrance of the labyrinth and would enroll as he entered it. This way, if he could beat the Minotaur, he could find his way out by following the thread. Theseus could accomplished the feat and left the labyrinth. However, the promised wedding did not take place (Stephanides, 2004). But this is another story...

them. We start from the research model to which it is guided, followed by its organizational structure, to discuss the elements we have listed and their singularities related to the study of the PI of MTs.

Table 2 – Actions associated with the elements that can be considered in an investigation on PI of MT

Poles	Actions associated to the elements
<i>Epistemological</i>	Assume a characterization of the PI of MT's Explain the understanding of MT's Adopt a theoretical perspective of the PI of MT's Be careful to the moments of tension/fragility during the research Recognize the dialogic, diachronic, and dialectic character of the PI of MT's
<i>Theoretical</i>	Choose theoretical support in the field of Mathematics education Be careful not to use theoretical generalization when dealing with PI Be careful not to confront different situations/contexts Harmonize the theoretical choices and the analytical perspectives
<i>Technical</i>	Consider the number of participants Recognize the importance of the context of data/information collection Provide (and foment) a group environment for the study Be careful when constructing/selecting the instruments to collect data/information
<i>Morphological</i>	Structure writing, in the case of dissertations and theses, in the <i>multipaper</i> format To explicit and disseminate the main results found to the research community Point out the limitations of the study and future pathways

Source: Created by authors

When delimitating these actions, *we guide them towards a long-term research*. One of the justifications for this is its temporal character. For instance, in a PhD study, excluding the time to do the subjects demanded by the program, there is a considerable interval between the actions intrinsically related to the research activities (reading, theoretical reflection and discussion; planning and organizing the methodological pathway; select the most adequate methodological tools to collect data/information; collect data; establish the analytical criteria to analyze the data; write the work). Another plausible justification to draw a long-term research proposal is the recognition of the dialogic, diachronic, and dialectic nature of the studies on the PI of MTs (action *v*, Table 2). These natures are evident in the discussion of the PI of MTs, connected to the relations of MTs with other(s), in an *attempt to understand* the constitutive dynamic of their PI

as MTs, in which, without a doubt, there are contrapositions/contraindications of ideas, allowing (other) actions/practices that empower this *constant* movement.

However, this is not a restrictive factor for researchers that, for example, are guided to the studies on the PI of MT since their master's degrees (academic or professional). The time to do the master's degree can difficult the research actions (in the way previously described), especially due to the elements we assume (CDTE) to characterize the PI of MT. This means that master's studies on the PI of MT need to have a more acute look on *which aspects of the PI of MT will be focused* on the research process.

Paired with this factor, we recommend the use of the *multipaper* format¹¹ to master's and PhD students (action *xiv*, Table 2), and in the case of researchers who have finished their dissertations/theses, we recommend them to disseminate their results via articles/essays. Using the *multipaper* writing style in master's and PhD studies have been broadly recommended (Barbosa, 2015; Boote & Beile, 2005; Duke & Beck, 1999; Thomas, West, & Rich, 2016), as it is a possibility to disseminate these studies and potentialize its reach.

Together with this, in the last years, there has been an increase on the use of *multipaper* format in the programs of Science Teaching and Mathematics Education in Brazil. In this direction, Barbosa (2015) defends that the presentation of dissertations and theses could be interpreted as a movement of creative insubordination (D'Ambrósio & Lopes, 2015) against the strict academic criteria, as this paradigm favors different ways in which researchers can organize and write their studies.

In a recent publication (De Paula & Cyrino, 2018d under review), when confronting *Corpus 1*, which encompasses the so-called *grey literature*¹², with *Corpus 2*, we have identified that a great part of the authors on *Corpus 1* were not present on *Corpus 2*. This shows that in many cases the results from dissertations and theses were not published (at least not in the period

¹¹ The term *multipaper* in this case refers to a dissertation/thesis built and organized as a collection of articles articulated and interconnected by a research question.

¹² During the 12th International Conference on Grey Literature, held in Prague in 2010, a new definition of the term was proposed: "Grey literature stands for manifold document types produced on all levels of government, academics, business and industry in print and electronic formats that are protected by intellectual property rights, of sufficient quality to be collected and preserved by library holdings or institutional repositories, but not controlled by commercial publishers i.e., where publishing is not the primary activity of the producing body." (Schöpel, 2010, p. 24).

analyzed). We understand that the studies on the PI of MTs could be stimulated to use the *multipaper* format as a way to *abandon* the status of *grey literature*¹³.

To elucidate and disseminate the main results to the research community (action *xv*, Table 2) are converging purposes on the recommendation for master's and PhD students to use *multipaper* format, and the writing of articles from the researchers who have finished their thesis and/or theses.

It is essential that researchers assume a position on what they understand as the PI of MTs, to know whom they are talking about when referring to Mathematics teachers and establish a theoretical perspective of the PI of MT (actions *i*, *ii*, *iii*, Table 2). During the analysis of the *corpora*, especially *Corpus 2* (De Paula & Cyrino, 2018b), we have seen that when the researchers assume a clear characterization of the PI and a theoretical perspective of the PI of MT (coherent to the characterization assumed) they can establish deeper discussions. In these studies, the researchers recognize the idiosyncrasies and the (inevitable) consequences of their choices. However, among the main concerns of this endeavor is the care of not falling into mistakes due to the confrontation between situations/contexts that cannot be compared (action *viii*, Table 2).

This fact influences and is influenced by the choice of theoretical references on PI, as the scope of the studies in the theme of identity (general) and PI (in particular) is broad and involves multiple research themes (for instance, PI, student identity, identity towards mathematics, issues of power, gender, and mastery of learning) (Darragh, 2016) (action *vii*, Table 2).

Considering this, researchers need to have a clear theoretical base, if the PI of MTs is the specific theme to discuss, then their reflections and analysis are part of the Mathematics education field (action *vi*, Table 2). Finding theoretical support written by researchers in the field of Mathematics Education who discuss the paradigm of the PI of MT is (still) a complex task, though the theme is an emerging field of research. In the *corpora* analyzed, we have identified the difficulties of several researchers to find authors in the field of Mathematics Education who did this theoretical problematization. The researchers interested on discussing the PI of pre-

¹³ The works of Oliveira (2010), of *Programa de Pós-Graduação em Ensino, Filosofia e História das Ciências da UFBA/UEFS*; Estevam (2015), of *Programa de Pós-Graduação em Ensino de Ciências e Educação Matemática da UEL*; Dantas (2016), of *Programa de Pós-Graduação em Educação Matemática da Unesp/RC*; Lima (2017) and Januário (2017), of *Programa de Pós-Graduação em Educação Matemática da PUC/SP*; and Lautenschlager (2017), of *Programa de Pós-graduação em Ensino e História das Ciências e da Matemática da UFABC*, are examples of the multiple/diverse ways of the *multipaper* perspective.

service/early teacher training, for instance, assume the lack of studies as a difficulty during the bibliographic survey.

Still on the methodological pathway to build a proposal, the contextual elements of the research process are extremely important for the studies of PI of MT and still lack researchers' attention (action *xi*, Table 2). To start, delimiting the number of participants is crucial (action *x*, Table 2). Studies of the PI of MTs demand detailed descriptions/characterization of the participants instead of generic appointments on a great number of individuals (Darragh, 2016). From this, among other facts, comes our understanding on the *complexity*, the *dynamism*, *temporality*, and *experience* (De Paula & Cyrino, 2018d, under review) as aspects that characterize the processes to constitute a PI of MTs. Studies on the PI of MTs centered in a reduced number of participants are better to identify the analytical elements and, consequently, explore them in detail. (De Paula & Cyrino, 2018b).

However, the small number of participants is not restricted to an individualist relation of the researcher nor among between him/her and the participants. A recurrent characteristic seen in the process to establish and analyze the *corpora* is the *locus* in which participants are immersed during the collection of data/information. Studies conducted in contexts of group learning (such as study groups, collaborative groups, Communities of Practice- CoP) or actions related to the demands of Supervised Internships during the undergraduate degree, for example, involve in-service teachers (at Basic and Higher education) and pre-service future MTs, allowing a discussion of robust elements triggered by the problematization of the movement to establish the PI of the MTs involved (action *xii*, Table 2) (De Paula & Cyrino, 2018a; 2018b; 2018c, under review). On our opinion, this resource leads to qualitative improvements to the research.

In this context of group learning, involving pre- and in-service MTs, debates involving self-image, self-esteem, motivation to work, perception of duties, and future work perspectives (Kelchtermans, 2009), as well as problematize sociocultural situations can be interesting to discuss *vulnerability* and search for a *sense of agency* (Beijaard, Meijer, & Verloop, 2004; Cyrino, 2017; Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013; Lasky, 2005; Oliveira & Cyrino, 2011) (action *in*, Table 2).

Vulnerability can be understood as a multidimensional and multifaceted experience that encompasses the emotions triggered by influenced and influencing situations in/of professional teacher leaning. When involving contextual problems where MTs are inserted (Lasky, 2005),

vulnerability can be related to questions/situations of frustration/destitution (anxiety, impotence) or to moments/episodes in which certain actions/certainties are discussable/questioned (or put to test), without weaken the teacher¹⁴.

Agency arises as an important element in the process of establishing PI and can be practiced/enacted in different ways (Beijaard, Meijer, & Verloop 2004). The exercise of agency emerges in occasions in which (future) teachers

interacting with the social structures in a mutually constitutive way, exert influence, make choices, take decisions that affect their work and reveal their professional and ethic commitment, through their ideas, motivations, interests, and objectives (Cyrino, 2017, p. 706).

In this universe, the emotions are established as one of the influent aspects that imply a movement of constituting the PI of MTs and lack the necessary attention (Hobbs, 2012; Hodgen & Askew, 2007; Kelchtermans, 2009; Kelchtermans & Hamilton, 2004). Many subtleties can be taken into account when researching the movement to establish the PI of MTs. Therefore, besides highlighting the importance of the researchers' theoretical perspective (actions *i*, *iii*, *v*, *vi*, *vii*, *viii*, *ix*, Table 2), it is also important to be careful when constructing/selecting the tools to collect data/information (action *xiii*, Table 2).

Considering the aspects we believe characterize this movement (CDTE), their influencing elements are peculiar and closely associated to the contexts of MT (action *xi*, table 2). Therefore, the process to collect data/information should prioritize the ways through which it is possible to observe *evidences* of this movement. We refer to *evidences* in the perspective of Ginzburg (1989), who defends the evidential paradigm as a methodological support to shed a light on the object/phenomenon researched. Through clues, evidences, marks, or signs, that is, "from apparently negligible data", it is possible to "reassemble a complex reality not directly experienced" (p. 152). Hence, through inferences and relations between the clues, researchers can find evidences or signs not immediately perceived, leading to the identification of possible

¹⁴ "It is not the vulnerability that weakens, sensitizes, and paralyzes...but that which allows us to suspend for some time, longer or shorter, more or less frequent, our certainties and convictions. The one that makes us question ourselves. Also vulnerability in the sense of exposing ourselves to others and, as such, become "targets for criticism, contestation" [emphasis added]. From our point of view, sometimes, what we consider to be an inability of the future teacher to reflect on his/her learning and practice, in the context of supervised internship, can be related to the fact that this reflection depends on personal conditions, such as confidence and the wish to take risks" (Oliveira & Cyrino, 2011, p. 112).

convergence (or divergence) points and incorporate new hypotheses and conjectures about their research focus.

Regarding the use of instruments to collect data/information, the issues of constitutive nature – how they are built- and propositional – what are the intentions to create certain instruments and why use them- are aspects that need attention and should be taken into account.

The use of multiple instruments that allow researchers to gather material to reflect on the ideas, thoughts, and opinions of (future) MTs related to the research topic, through written or audio/visual records, are indispensable to the studies of the PI of MTs. Some examples are interviews, memorial, narratives, narrative interviews, metanarratives, diaries, (auto)biographies or other ways that can be used in different moments of the research to gather data on the peculiarities of MT in their dialogic and diachronic dimensions (actions *xi*, *xii*, *xiii*, Table 2).

When thinking about the elements that can compose an investigative proposal for future researchers on the PI of MTs, we highlight two points. The first is that researchers' choices related to the epistemological and theoretical poles influence the choices of the technical and morphological poles, which, besides guiding the paradigms of investigation are also present during the methodological development of the research. *It needs to be clear that the epistemological and theoretical research choices influence the analytical process.* The second point relates to the cohesion in the process of methodological development: the harmony and articulation between epistemological perspectives, the theoretical choices, the research apparatus to collect data/information, and the processes of analysis are key for the research of PI of MTs (action *ix*, Table 2).

Resuming the elements most intimately associated to the epistemological and theoretical tools (actions from *i* to *ix*, Table 2), it is equally important to deepen the epistemological discussion of the PI of MT. Even in the case of a PhD study, a proposal that is concerned with multiple perspectives (for instance considering those we identified as *cultural, generalist, holistic, pedagogy of internships, political-reformist, psychological/psychoanalyst, sociological, and Wengerian*) can be a Herculean task ¹⁵ (De Paula & Cyrino, 2018d, under review), if attending to the epistemological, theoretical,

¹⁵ As we have referred to mythology during the essay, the adjective Herculean could not be ignored. Legend says that Hercules, when attempting to purify himself from a crime fomented by Hera, goes to the Oracle of Delphi and offers himself to Eurystheus, king of the Mycenaean city, who gave him 12 labors that were supposed to purify Hercules: (1) kill the Nemean lion and; (2) the Lernaean Hydra; (3) capture the Erymanthian Boar; and the (4) Ceryneian Hind; (5) kill the Slay the Stymphalian birds; (6) clean King Augean stables; (7) Capture the Cretan Bull,

and analytical care we recommend in this essay. Adopting a theoretical perspective of PI of MTs is key for the analysis of the research. Recognizing the investigative limitation, when embarking in the studies of PI of MTs, is not a flaw but a sign of maturity. To point out (new) questions should be a guiding principle for those interested in the research field (action *xvi*, Table 2).

(In)Conclusions, implications, and possible future investigations

It is a great challenge to present elements and actions to build a research proposal on the PI of MTs. We believe that the establishment of group contexts which allow connecting basic education in-service MTs with university professors and future teachers, questioning social-cultural-political aspects, and the methodological care to build/use instruments to collect data/information can be a way to find evidences on the movements to constitute the PI of MTs. Together with this, the writing of dissertations and theses in the *multipaper* format, demanding a careful synthesis, and of articles is an undertake that requires thinking about the aspects that characterize the PI of MTs (CDTE) in articulation with the actions and the elements we highlighted.

Based on the ideas presented in this essay, instead of drawing conclusions, we point out new directions:

- (1) *Build a theoretical reference that problematizes the relations of MTs and their peers in social-cultural-political contexts is a political commitment that all of us, Brazilian mathematics educations, must undertake, as we defend an education of MTs that takes place in the field of ideological and political fights* (Cyrino, 2006; 2017; Souza et al., 1995) *and recognize the specificities of the Brazilian context.* This position results from assuming the characterizing aspects (CDTE) of the PI of MTs (De Paula & Cyrino, 2018d, under review) and the understanding that the social, cultural, and political contexts are extremely relevant to carry on defending the problematization of PI as an educational perspective for (future) MT.
- (2) *Discuss the PI of MTs in the context of in- and pre-service teacher training, problematize the (in)efficiencies of certain public policies that directly or indirectly involve the roles of MTs is key.*

and; (8) the mares of Diomedes; (9) obtain the girdle of Hippolyta, queen of the Amazons; (10) capture the cattle of Geryon; (11) get the Golden apples of Hesperides; and (12) capture the three-headed dog Cerberus, gate-keeper of the Underworld (Stephanides, 2000). Research, in only one study the broad array of epistemological perspectives of PI of MTs would, undoubtedly, be the 13th labor of Hercules!

Defending the problematization of the PI of MTs in the context of pre-service training does not mean, in any way, to simply include a curricular component to reflect on these themes. On the contrary, it means reflecting on its characterizing principles (CDTE) in an integrating and questioning fashion, permeating the structure of the course itself. Mainly, it is to understand and identify that these aspects are present in the pre-service training (and before it). The vulnerability and the sense of agency can be potentially productive in the reflection on the actions of the internships; likewise, they are intertwined with the other curricular and non-curricular components that present/reflect public policies, normative aspects, and the specificities of the knowledge interrelated to the profession. In the context of teacher training, the issues related to vulnerability and the sense of agency are not restricted to the subjects on the field of Mathematics Education.

- (3) *Guarantee that, during the actions involving the morphological pole, the questions/discussion of the researcher, as an epistemological character, are also evident. After all, the PI of the researcher is in process, in movement.* The researchers' position, especially when presenting their research problem, different limitations, results, and indications of future studies, are part of the political commitment of the PI proposed by Cyrino (2016, 2017). *Positioning oneself* is a necessary, (im)pertinent, and insubordinate political act (D'Ambrósio & Lopes, 2015) and should be expected in a study on the PI of MT. Discussing the processes involved in the movement to constitute the PI of MT contributes and promotes ethical, philosophical, and moral reflections. These reflection help (future) MTs to broaden their view on the possibilities and scopes of their actions. They are inviting (and contribute) to raise the awareness of (future) MTs towards their responsibilities and convince them on the importance of assuming the political, social, and cultural commitment necessary to perform their roles aiming an equitable society, guaranteeing freedom, solidarity, and equal opportunities. In this context, we assume that the political commitment encompasses the notions of ethics, social justice, and equity.

The ideas presented here, as well as the three appointments listed, are a reflex on the idea of incompleteness towards the PI of MT: they *point out possibilities*, they do not prescribe them.

To foment these reflections/problematisations seem to be a promising path for future studies. For those (*Thesens(s)*) interested to brave the many research possibilities (*labyrinths*) on the studies of the PI of MT's, the elements discussed in this essay (*a possible Ariadne's thread*) can help in this endeavor.

References

- André, M. E. D. (2011) A Pesquisas sobre formação de professores: tensões e perspectivas. In H. A. Fontoura; M. Silva (Orgs.). *Formação de professores, culturas: desafios à pós-graduação em Educação em suas múltiplas dimensões*. Rio de Janeiro: ANPEd Nacional, 24-36.
- André, M. E. D. (2009) A produção acadêmica sobre formação de professores: um estudo comparativo das dissertações e teses defendidas nos anos 1990 e 2000. *Formação Docente*, 1(1), 41-56.
- Barbosa, J. C. (2015). Formatos insubordinados de dissertações e teses na Educação Matemática. In B. S. D'Ambrósio, & C. E. Lopes (Orgs.). *Vertentes da subversão na produção científica em educação matemática* (1a ed., vol. 1, pp. 347-367). Campinas: Mercado de Letras.
- Beijaard, D., Meijer, P. C., & Verloop, N. (2004). Reconsidering research on teachers' professional identity. *Teaching and Teacher Education*, 20, 107-128. doi:10.1016/j.tate.2003.07.001
- Bennison, A. (2015). Developing an analytic lens for investigating identity as an embedder-of-numeracy. *Mathematics Education Research Journal*, 27(1), 1-19.
- Boote, D. N., & Beile, P. (2005). Scholars before researchers: on the centrality of the dissertation literature review in research preparation. *Educational Researcher*, 34(6), 3-15.
- Cyrino, M. C. C. T. (2006). Preparação e emancipação profissional na formação inicial do professor de matemática. In A. M. Nacarato, & M. A. V. Paiva (Orgs.). *A formação do professor que ensina matemática: perspectivas e pesquisas* (pp.77-86). Belo Horizonte: Autêntica.
- Cyrino, M. C. C. T. (2016). Mathematics teachers' professional identity development in communities of practice: reifications of proportional reasoning teaching. *Bolema*, 30(54), 165-187. doi: 10.1590/1980-4415v30n54a08.

- Cyrino, M. C. C. T. (2017). Identidade Profissional de (futuros) Professores que Ensinam Matemática. *Perspectivas da Educação Matemática*, 10, 699-712.
- D'Ambrósio, B., & Lopes, C. E. (2015, abril). Insubordinação criativa: um convite à reinvenção do educador matemático. *Bolema*, 29(51), 1-17.
- Dantas, S. C. (2016). *Design, implementação e estudo de uma rede sócio profissional de professores de matemática*. Tese de Doutorado em Educação Matemática, Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista.
- Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, 93(1), 19-33.
- De Paula, E. F., & Cyrino, M.C.C.T. (2017a). Identidade Profissional de professores que ensinam matemática: panorama de pesquisas brasileiras entre 2001-2012. *Zetetiké*, 25(1), 27-45. doi: 10.20396/zet.v25i1.8647553
- De Paula, E. F., & Cyrino, M. C. C. T. (2018a). Perspectivas de identidade profissional de professores que ensinam matemática presentes em dissertações e teses brasileiras. In M. C. C. T. Cyrino (Ed.). *Temáticas emergentes de pesquisas sobre a formação de professores que ensinam matemática: desafios e perspectivas* (pp. 126-154). Brasília: SBEM, GT7.
- De Paula, E. F., & Cyrino, M. C. C. T. (2018b). Perspectivas de Identidade Profissional de Professores que ensinam Matemática presentes em artigos científicos publicados entre 2006-2016. *Acta Scientiae*, 20(5), 778-799.
- De Paula, E. F., & Cyrino, M. C. C. T. (2018c). Polos teórico e epistemológico presentes em pesquisas brasileiras sobre Identidade Profissional de Professores que Ensinam Matemática. *Imagens da Educação*, 8(2), 1-23.
- De Paula, E. F., & Cyrino, M. C. C. T. (2020). Aspectos a serem considerados em investigações a respeito do movimento de constituição da Identidade Profissional de professores que ensinam matemática. *Educação (UFSM)*, 45, e28/ 1-29. doi: 10.5902/1984644434406.
- Duke, N. K., & Beck, S. W. (1999). Education should consider alternative forms for the dissertation. *Educational Researcher*, 28(3), 31-36.
- Estevam, E. J. G. (2015). *Práticas de uma comunidade de professores que ensinam matemática e o desenvolvimento profissional em educação estatística* (192 pp.). Tese de Doutorado, Programa de

Pós-Graduação em Ensino de Ciências e Educação Matemática, Universidade Estadual de Londrina.

- Eteläpelto, A., Vähäsantanen, K., Hökkä, P., & Paloniemi, S. (2013). What is agency? Conceptualizing professional agency at work. *Educational Research Review*, 10, 45-65.
- Ginzburg, C. (1989). *Mitos, emblemas, sinais*. São Paulo: Companhia das Letras.
- Goos, M., & Bennison, A. (2008). Developing a communal identity as beginning teachers of mathematics: Emergence of an online community of practice. *Journal of Mathematics Teacher Education*, 11, 41-60.
- Hobbs, L. (2012). Examining the aesthetic dimensions of teaching: Relationships between teacher knowledge, identity and passion. *Teaching and Teacher Education*, 28(5), 718-727.
- Hodgen, J., & Askew, M. (2007). Emotion, identity and teacher learning: Becoming a primary mathematics teacher. *Oxford Review of Education*, 33(4), 469-487.
- Hossin, S., Mendrick, H., & Adler, J. (2013). Troubling “understanding mathematics in-depth”: Its role in the identity work of student-teachers in England. *Educational Studies in Mathematics*, 84, 32-48.
- Januário, G. (2017). *Marco conceitual para estudar a relação entre materiais curriculares e professores de matemática* (194 pp.). Tese de Doutorado em Educação Matemática, Pontifícia Universidade Católica de São Paulo, São Paulo.
- Kelchtermans, G. (2009). Who I am in how I teach is the message: self-understanding, vulnerability and reflection. *Teachers and Teaching: Theory and Practice*, 15(2), 257-272.
- Kelchtermans, G., & Hamilton, M. L. (2004) The dialectics of passion and theory: Exploring the relation between self-study and emotion. In *International handbook of self-study of teaching and teacher education practices* (pp. 785-810). Springer, Dordrecht.
- Larrosa,, J. L. (2002). Notas sobre a experiência e o saber de experiência. *Revista Brasileira de Educação*, 19, 20-28.
- Lasky, S. (2005). A sociocultural approach to understanding teacher identity, agency and professional vulnerability in a context of secondary school reform. *Teaching and Teacher Education*, 21(8), 899-916.

- Lautenschlager, E. (2017). *Conhecimento matemático para o ensino de polinômios na educação básica*. Tese de Doutorado em Neurociência e Cognição, Universidade Federal do ABC.
- Lessard-Hebert, M., Goyette, G., & Boutin, G. (1994). *Investigação qualitativa: fundamentos e práticas*. Lisboa: Instituto Piaget.
- Lima, K. (2017). *Relação professor-materiais curriculares em Educação Matemática: uma análise a partir de elementos dos recursos do currículo e dos recursos dos professores* (163 pp.). Tese de Doutorado em Educação Matemática, Pontifícia Universidade Católica de São Paulo, São Paulo.
- Lloyd, G. M. (2006). Preservice teachers' stories of mathematics classrooms: Explorations of practice through fictional accounts. *Educational Studies in Mathematics*, 63(1), 57-87.
- Oliveira, A. M. P. (2010). *Modelagem matemática e as tensões nos discursos dos professores*. Tese de Doutorado em Ensino, Filosofia e História das Ciências, Universidade Federal da Bahia, Universidade Estadual de Feira de Santana.
- Oliveira, H. (2004). Percursos de identidade do professor de Matemática em início de carreira: O contributo da formação inicial. *Quadrante*, 13(1), 115-145.
- Oliveira, H. M., & Cyrino, M. C. C. T. (2011). A formação inicial de professores de Matemática em Portugal e no Brasil: narrativas de vulnerabilidade e agência. *Interações*, 7, 104-130.
- Pamplona, A. S., & Carvalho, D. L. de. (2009). Comunidades de prática e conflitos de identidade na formação do professor de matemática que ensina estatística. In D. Fiorentini, R. C. Grandó, & R. G. S. Miskulin (Eds.). *Práticas de formação e de pesquisa de professores que ensinam Matemática* (pp. 211–232). Campinas: Mercado de Letras.
- Ponte, J. P.; Chapman, O. (2008). Preservice mathematics teachers' knowledge and development. In L. D. English (Ed.). *Handbook of international research in mathematics education: Directions for the 21st century* (2nd, pp. 225-263). New York: Routledge.
- Schöpfel, J. (2010). Towards a prague definition of grey literature. In Twelfth International Conference on Grey Literature: Transparency in Grey Literature. Grey Tech Approaches to High Tech Issues (pp. 11-26). Prague, Czech Republic.
- Souza, A. C., Teixeira, M. V., Baldino, R. R., & Cabral, T. C., (1995). Novas diretrizes para a licenciatura em matemática. *Temas e Debates*, 8(7), 41-65.
- Stephanides, M. (2000). *Hércules* (M. P. Michael, Trad.). São Paulo: Odysseus.

- Stephanides, M. (2004). *Teseu, Perseu e outros mitos* (J. R. M. Potzamann, Trad.) São Paulo: Odysseus
- Teixeira, B. R., & Cyrino, M. C. C. T. (2015). Desenvolvimento da identidade profissional de futuros professores de Matemática no âmbito da orientação de estágio. *Bolema*, 29(52), 658-680.
- Thomas, R. A., West, R. E., & Rich, P. (2016). Benefits, challenges, and perceptions of the multiple article dissertation format in instructional technology. *Australasian Journal of Educational Technology*, 32(2), 82-98.
- Ticknor, C. (2012). Situated learning in an abstract algebra classroom. *Educational Studies in Mathematics*, 81, 307-323.
- Van Putten, S., Stols, G., & Howie, S. (2014). Do prospective mathematics teachers teach who they say they are? *Journal of Mathematics Teacher Education*, 17, 1-24.
- Wenger, E. (1998). *Communities of practice: learning, meaning and identity*. Cambridge: Cambridge University Press.

Submission:

Submitted for evaluation in September 24, 2018; revised in June 23, 2019; accepted for publication in May 24, 2020.

Corresponding author:

Instituto Federal de Educação, Ciência e Tecnologia de São Paulo, São Paulo, SP, Brasil. Rua José Ramos Junior, 27-50 - Jardim Tropical - Presidente Epitácio - SP - Brasil - Cep: 19470-000