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ARTICLES

AN OUTRANKING ELECTRE-BASED MODELING FOR SETTING THE NAVAL CORE COMPONENTS OF AN AMPHIBIOUS TASK FORCE

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ABSTRACT. Selecting the naval components to be part of a task force is a typical setting decision problem wherein each component of the task force should complement the others. This decision situation calls for the use of outranking modeling tools instead of the additive ones, which are usually used when selecting group components. This study applies Electre I to develop an original approach to this hypothetical problem: the composition of the Amphibian Readiness Group, used in the United States. Despite being a hypothetical decision situation, the work uses real data from American media. We use Electre I because it is a native outranking multicriteria decision method, designed to address choice decisions. Moreover, we take the support of The Visual Outdeck web app (Outranking Decision and Knowledge app) to implement the model and the Electre I multicriteria algorithm and a sensitivity analysis of the results regarding the weights and the agreement and disagreement cut-off levels. Further we apply the same input data to an additive decision algorithm, allowing comparison of the results. Consequently, we established the setting of the task force by utilizing elements that optimally complement the others in the task force configuration. Furthermore, the study finds that this task force differs from the one that should be constituted using traditional additive modeling. The main contribution of this study is that it provides a robust discussion and original modeling that supports military decision-makers while selecting a better set of military taskforces.

Keywords: multicriteria, decision, outranking, visual outdeck, warship selection, Electre I, operations research.

1 INTRODUCTION

This study presents modeling for setting the core components of an amphibious task force for a hypothetical decision. This kind of problem is a typical non-additive decision problem, and according to Costa (2016), in such problems, it is recommended to adopt an outranking multicriteria method to support the choice modeling.

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Multicriteria methodologies applied to military problems are not a novelty. According to Pessôa & Costa (2020), and Costa et al. (2022), the examples are diverse, encompassing mainly logistical and administrative aspects of the military context, commonly using the additive Analytic Hierarchy Process (AHP) (Saaty, 1980), most of which originate from researchers linked to institutes in China and the United States. Despite the advances brought forth by previous works, a gap still exists as most of the previous models used additive multicriteria methods.

Amphibious operations are complex military activities, due to the diversity of military means employed. Normally, they involve simultaneous operations of Navy, Marines, Army, and Air Force units.

The synchronicity of actions is a fundamental pillar of an operation's success. Those units must work together, and that demands complex coordination. However, the main characteristic of an Amphibious Operation is the sea-land transition of troops and equipment. Hence, the ships transporting such troops and equipment are the capital of amphibious operations. Therefore, such vessels are vulnerable to the enemy during land-sea engagements, as they lose mobility to perform landing operations.

Due to the importance of the transport and land ships, their neutralization or destruction makes a littoral terrain conquest, known as a beachhead, infeasible. To achieve an amphibious mission's desired effect, it is necessary to analyze the composition of the core nucleus of its task force.

A natural question arises from such analysis: What is the most efficient composition: Large ships with greater load capacity, or faster and more maneuverable small ships?

The task force definition may be classified as a configuration problem, typically a non-additive one. However, we did not find an outranking modeling for this specific purpose in the literature.

In this study, we cover a part of this gap by developing an original model, based on non-compensatory and non-additive (Electre I), addressing the problem of ship selection to replace an amphibious expeditionary force parcel. Such problem is close to reality and is presented in a case study, demonstrating its importance and political context.

The remainder of this paper is structured as follows. Section 2 provides a background on multicriteria decision methods. Section 3 presents the methodology. Section 4 presents the case study and provides discussion of the results. Finally, Section 5 concludes the paper.

2 BACKGROUND ON MULTICRITERIA DECISION METHODS

Roy (1968) classifies the decision problems as choice, ranking, sorting, and description. This classification is extended by Sant'Anna et al. (2015) by including clustering and sharing (or portfolio) problems. Under this categorization, the setting or configuration problem is a kind of choice problem in which a subset of alternatives is selected from a whole set of options.

Beyond this classification, a different kind is addressed to explore the differences between situations in which the alternatives interact because of their synergy and situations in which synergistic relationships among alternatives do not occur. In the Multiple Criteria Decision

Aid/Making(MCDA/M) subject, the situations where synergy occurs are usually classified as additive situations, while the situations where this is not the case are usually classified as non-additive ones.

Based on the discussion shown in Costa (2016), there is a need to pay attention to this classification to avoid misinterpretations of results, mainly when modeling the problem of alternative selection in a setting problem. Thus, we introduce a briefing about this classification in the following subsections.

2.1 Additive decision situations

As it appears in Equation 1, in additive decision situations, the multi-attribute utility function that is usually applied is additive. Therefore, the total utility of a subset X of options chosen from the whole set A is a scalar, whose value equals to the weighted sum of the utilities of each alternative $(x_i)\ \varepsilon\ X$, considering an entire set of criterion composed by n criteria. According to Fishburn (1970) and Keeney & Raiffa (1993), the models that use an additive approach are based on the Multi-Attribute Utility Theory (MAUT).

$$U(X) = U(x_1, x_2, \dots x_{ns}) = \sum_{j=1}^{n} k_j \sum_{i=1}^{mx} u(x_i) \tag{1}$$

where:

- X is composed by the options chosen from A
- mx is the number of alternatives that compose X
- n is the number of criteria used in the modeling
- k_j is the constant of scale (sometimes called as weight) of the $k-esim$ criterion

By closely examining Equation 1, we conclude that the additive effects occur in two paths: inner, and inter-criteria. Because the additive occurs in inter-criteria, the MAUT modeling can also show a compensatory behavior, which means that a high performance or utility $u_j(a)$ of a generic alternative a under criterion j compensates or balances a low performance of this alternative under another criterion.

Hence, as stated by Diaby & Dias (2017), the use of value function methods entails accepting that an inferior performance on one criterion can always be compensated for by an excellent performance on some other criterion. Therefore, these methods may not be the most appropriate when such compensatory effects are not considered adequate in the decision-making process.

There are several situations in which compensatory and additive effects are not undesirable and others in which they are not welcome. Thus, one should carefully understand whether a MAUT based modeling is suitable for the specific decision situation.

The multicriteria methods that implement additive approaches are usually classified as based on MAUT, which is presented in the seminal works of Fishburn (1970) and Keeney & Raiffa (1993). Nowadays, the most used MAUT-based methods are: TOPSIS (Hwang & Yoon, 1981), AHP (Saaty, 1980), and FiTradeoff (De Almeida et al., 2016).

2.2 Non-additive decision situations

In non-additive decision situations, the multiattribute utility function usually applied for modeling the performance, or final utility, of a subset of alternatives considers the synergies or interactions among them. It is the basis of the outranking methods as described in Roy (1968).

For example, consider a decision problem in which a soldier carries two weapons with him: - weapon x having a range of 500 yards and weapon y with a range of 700 yards.

In terms of range, the usefulness of this set of weapons that the soldier carries is not given by an additive function.

If one models this situation as an additive one, the result may be represented by Equation 2, which is clearly not suitable for the real decision problem.

$$U(x,y) = u(x) + u(y) = 500 + 700 = 1200\ yards \tag{2}$$

In contrast, one can say that $U(x,y)$ should be modeled as it appears in 3. Applying this modeling to the example provides a result closer to reality concerning this specific problem, as shown in 4. This is because the problem described in the model is a typical non-additive decision situation.

$$U(x,y) = max\{u(x), u(y)\} \tag{3}$$

$$U(x,y) = max\{u(x), u(y)\} = max\{500, 700\} = 700\ yards \tag{4}$$

One perspective to deal with such problems is the Outranking perspective. The Outranking methods were developed to deal with the difficulties derived from a value function approach, and associated with Bernard Roy and the Electre methods family Bouyssou (2009).

The outranking method Electre looks to discover a subset of X alternatives from a whole set A that hits the best performance in each one of the criteria. The seminal work of Roy (1968) has been deployed into the Electre family of outranking methods, as it appears in Table 1, adapted from Nepomuceno & Costa (2015). The Electre I used in this study is detailed in the methodology section.

2.3 Some remarks on modelling either additive or non-additive decision situations

As explained in the previous two subsections, there are situations that are naturally additive and others that are non-additive. Hence, an in-depth knowledge of the nature of the decision is helpful in deciding the method to approach the problem.

Table 1 – Outranking multicriteria methods: summary of seminal references.

Decision Situation	Non-additive Method
Choice	Electre I Roy (Roy (1968)) and Electre IS (Roy & Skalka (1984))
Ranking	Electre II Roy & Bertier (1971)), Electre III (Roy (1978)), Electre IV (Roy & Hugonnard (1982)) and PROMETHEE (Brans et al. (1986))
Sorting	PROMETHEE TRI (Figueira (2004)), PROMSORT (Araz & Ozkarahan (2005)), aara, Electre TRI (Mousseau & Slowinski (1998)), Electre TRI-C (Almeida-Dias et al. (2010)), Electre TRI-nC (Almeida-Dias et al. (2012)) and CPP TRI (Sant'Anna et al. (2015)) and (Gomes Costa et al. (2019))

In contrast, another point that should not be forgotten in this discussion is the nature of the Decision Maker's (DM) rationality. In other words, the success of an MCDA/M also depends on the preference modeling of the decision maker, which type of preference structure the DM feels most comfortable using.

Therefore, as a previous step in the modeling, the decision maker needs to understand first the nature of the decision situations:

- If the DM understands the problem as having a non-additive behavior and non-compensatory rationality and feels comfortable participating in this kind of modeling, he/she should apply an outranking method and adopt the same thinking path.
- If the DM classifies the decision situation as having additive behavior and compensatory rationality and is comfortable with this, he/she should apply a MAUT-based method.

2.4 Multicriteria modelling in military decision making

From a military perspective, the traditional planning method is based on the choice of different strategies available for a decision where a commander has a staff to assess the necessary information about the feasibility and main characteristics of each different course of action (COA).

Regarding the decision strategies from a military perspective, they "have been broken down into three categories: managing the number of options, using compensatory techniques, and using noncompensatory techniques" (Pounds & Fallesen, 1994).

Multicriteria decisions applied to military problems are not a novelty. According to Pessôa & Costa (2020), the examples are diverse, encompassing mainly logistical and administrative aspects of the military context, commonly using the AHP methodology (that compensatory behavior), and most of them are produced by researchers affiliated with institutes in China and the United States.

Pessôa & Costa (2020) presented a taxonomy for the uses of multicriteria in Defense. That taxonomy comprises both "Emergency Management" and "Military and Defense Department capabilities." This study highlights some results shown in Pessôa & Costa (2020), regarding the branch "Military and Defense Department capability."

If one wants to access a complete discussion about the use of multicriteria in the military or even in the defense context, we suggest reading the reviews provided by Pessôa & Costa (2020) and Costa et al. (2022).

Crary et al. (2002) asserted that, for the U.S. Navy to be successful, it must make suitable investments in combatant ships and uses AHP combined with the mixed-integer programming model. Nikou & Moschuris (2016) and Nikou et al. (2017) used combined AHP hybrid models to deal with supplier selection and assessment.

Cheng et al. (1999) proposed a method for evaluating weapon systems based on AHP and fuzzy linguistic variable weight. Fan et al. (2015) described the construction of the evaluation index system method and constructed an index system for missile penetration effectiveness evaluation, using a combination of AHP and firefly algorithm.

Linkov et al. (2010) used AHP to prioritize capability gaps for a small-arms-specific Department of Defense (DOD) Joint Capabilities Integration, and Alomair et al. (2016) also used AHP to evaluate defense simulation tools.

As exceptions to the use of AHP, Austin & Mitchell (2008) described the application of Value Focused Thinking (VFT) within MoD to support decisions involving conflicting objectives, complex alternatives, and significant uncertainties. Gazibey et al. (2015) employed DEMATEL to determine the criteria and their sub-criteria affecting Main battle tanks (MBTs) selection. Dewispelare & Sage (1980) integrated MOOT and MAUT when developing a process for specific military equipment acquisition involving aircraft retrofit. In contrast, Dou et al. (2015) used linear programming optimization and multi-objective integer programming, selecting a suitable portfolio from several candidate multi-function weapon systems in the defense acquisition and manufacturing process.

One can deduce that most related works have used AHP at least as a component. Moreover, despite these advances brought forth by previous works, a gap still exists because the previous model used mainly additive compensatory methods: AHP, DEMATEL, or even optimization (linear program).

Perhaps, the reason for the preponderance of compensatory methods, especially on additive methods, comes from their proximity to the traditional military decision-making process on the comparison of COAs, where an additive matrix or even AHP is used to compare the alternatives.

As utilities "rely on a sophisticated and cognitively demanding way of representing decision alternatives" (Pounds & Fallesen, 1994) and "the expected utility and other analytic models have produced accurate predictions of policy decisions and outcomes in numerous instances."

Simultaneously, ”this type of strategy is considered high in processing requirements because all cues considered relevant to the decision are examined for all alternatives” (Pounds & Fallesen, 1994). “Such models seldom capture the underlying cognitive processes involved in decision making” (Mintz, 1993).

In contrast, when the military decision process reaches higher levels (strategic or political), and if an alternative is unacceptable in one criterion, it cannot be compensated by a high score in another dimension, so a compensatory model may not be viable to execute the solution (Brummer & Oppermann, 2021).

Moreover, the DM may be led to a bounded rational model, searching for an acceptable outcome, where many aspects must be considered. Thus, the cognitive effort demanded will be constrained by the processing capabilities (Mintz, 1993). Therefore a non-compensatory method will address these requirements. In addition it turns handy a computational aid to help the facilitation process and the communications with the DM.

In this work, we covered this gap by applying a non-compensatory and non-additive modeling for selecting a configuration of a naval unit meant for an amphibious expeditionary force. Nevertheless, such procedure only enables an MCDM/A problem to be solved if the DM has non-compensatory rationality.

3 METHODOLOGY

This section enumerates the steps applied in the modeling. The details about the modeling processing are provided in the next section (Case study), where we apply the stages and steps described here to a hypothetical realistic military situation decision. As shown in Figure 1, the methodology is based on the following four main stages:

A. To map the decision situation, highlighting its main aspects and constraints.

B. To choose the multicriteria decision method to model the decision situation

C. To apply the MCDA method chosen in the previous stage

D. To analyze the sensitivity of the results to the modeling parameters.

Figure 1 – Stages.

As described in the next section (case study), in the stage (c) the MCDA method applied in the modeling is the Electre I, proposed in (Roy, 1968). Thus, we insert a deployment of stage (c) into the steps performed when applying such a decision method.

The ELimination Et Choice Traduisant la REalité (Electre I) was designed to approach the outranking choice of a subset $\underline{\mathbf{N}}$ from a whole set of options or alternatives $\underline{\underline{\mathbf{A}}} = (a_1, a_2, a_3, \ldots, a_m)$. Figure 2 illustrates the steps on which the Electre I method is structured. The description of these steps follows just after this figure.

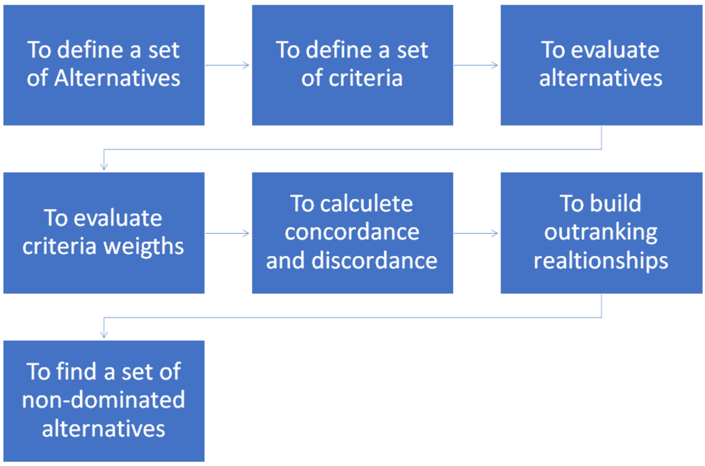

Figure 2 – Steps to apply Electre I.

i. To identify a whole set of feasible options or alternatives: $\underline{\underline{\mathbf{A}}} = (a_1, a_2, a_3, \ldots, a_m)$

ii. To define a set or family of criteria or decision drivers: $\underline{\mathbf{F}} = (k_1, k_2, \ldots, k_n)$

iii. to elicitate the weight or relevance of each criterion in $\underline{\mathbf{F}}$, which results in a set of weights: $\underline{\mathbf{W}} = (w_1, w_2, \ldots, w_n)$

iv. To evaluate the performance of each option in $\underline{\underline{\mathbf{A}}}$ under the viewpoint of each criterion in $\underline{\mathbf{F}}$, which results in the matrix of grades or performance:

$\underline{\underline{\mathbf{G}}} = g(\underline{\underline{\mathbf{A}}}) = ((g_1(a_1), \ldots, g_n(a_1)), (g_1(a_2), .., g_n(a_2)), \ldots, (g_1(a_m), .., g_n(a_m)))$.

v. To calculate the concordance matrix $\underline{\underline{\mathbf{C}}}$ and the discordance matrix $\underline{\underline{\mathbf{D}}}$, respectively, through Equations 5 and 6.

$$C(a,b) = \frac{1}{\sum_{i=1}^{n} w_i} * \sum_{j=1}^{n} w_j c_j(a,b) \tag{5}$$

$$D(a,b) = \frac{max[g_j(b) - g_j(a); 0]}{\gamma_j} \tag{6}$$

Where

- a and b are generic alternatives belonging to the set $\underline{\underline{\mathbf{A}}}$

- j is the j^{th} criterion in $\underline{\mathbf{F}}$
- $g_j(a)$ and $g_j(b)$ e grade or performance of a and b the criterion i
- w_j is the importance or relevance of the j^{th} criterion
- $C(a,b)$ and $D(a,b)$ respectively, imply how one agrees or disagrees with the assertive "*a* is at least as good as *b*"

vi. To build outranking relationships among the alternatives, so that aSb if, and only if $c(a,b) \geq c^*$ and $d(a,b) \leq d^*$

vii. To construct a partition that splits A into two subsets:

- The kernel or non-dominated subset N, composed of options with no outranking relationship.
- The dominated or outranked subset D, composed of alternatives that are outranked by at least one in N

We also highlight that, although it was proposed in 1968, the Electre I is singular once it provides a multicriteria outranking-based selection of the members of a set (so it is addressed to setting problems) without having to build a rank, as highlighted in Costa (2016). Building a rank is a need in other outranking methods, such as Electre II/III and Promethee and even in non-additive compensatory methods such as AHP and TOPSIS. It is worthy in setting problems, once it avoids rank reversal problems that could happen when using some methods mentioned above, and mainly because it seeks the best set of alternatives that "outranks" the others (which may not be the set composed of the best alternatives).

4 THE CASE STUDY

This section details the application of each of the stages and steps described in the previous section to a hypothetical, realistic military situation decision.

i. To map the decision situation, highlighting its main aspects and constraints

   The strategic scenario used an American decision-maker evaluation of replacing amphibious units for potential use in the South China Sea, in which both the U.S. and China are military powers and permanent members of the U.N. Security Council.

   Recently, the South China Sea environment has encompassed the development of the Chinese strategy to use the "nine-dash line", which concerns a representation of Chinese expansion claims in the South China Sea, represented inaccurately by dashed lines, as shown in Figure 3.

   The Chinese strategy achieves the desired effects at the political level, which, from an international point of view, is linked to the dominance of international trade with a policy

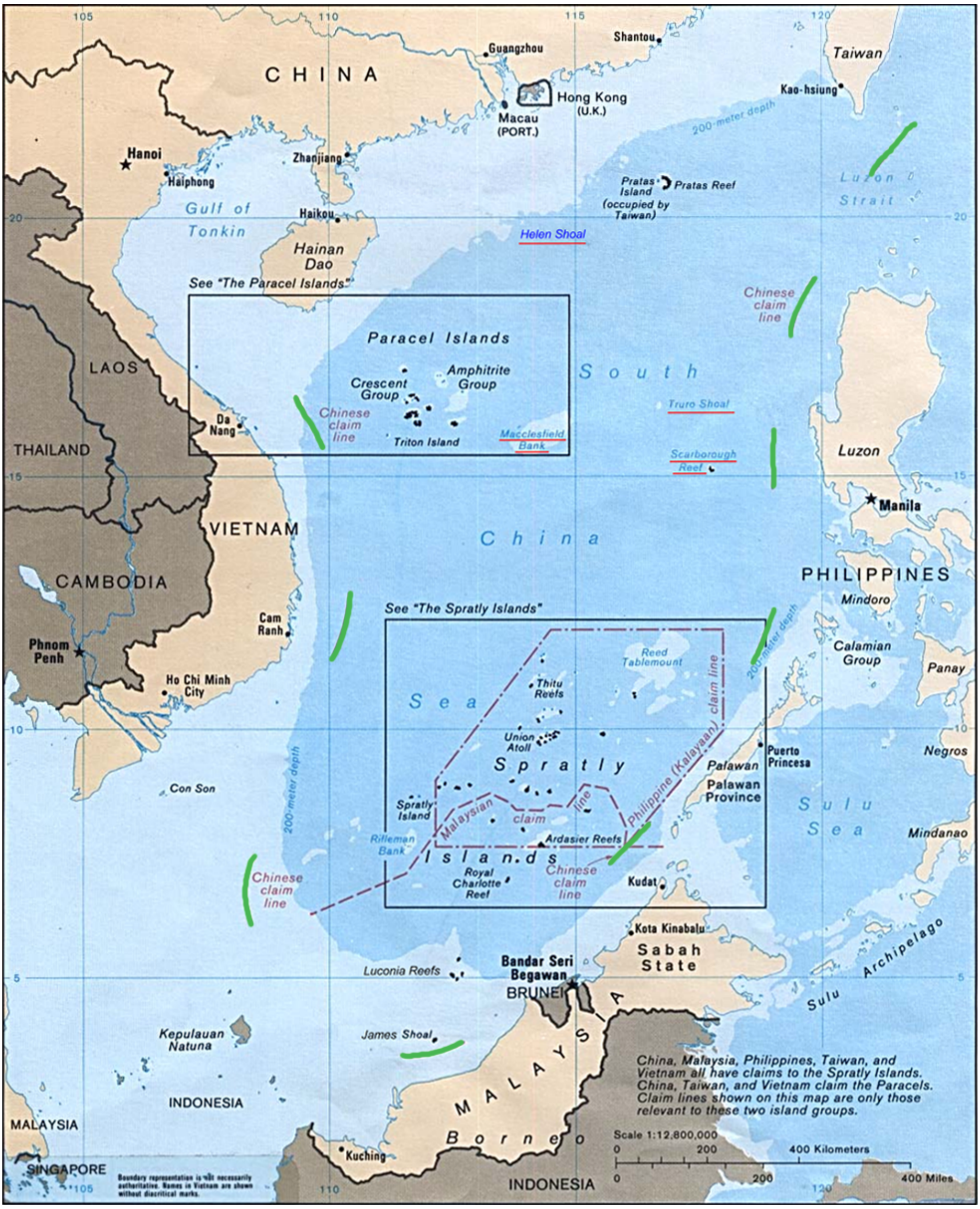

Figure 3 – Nine Dotted line. Source: Agency (1988).

named "One Belt, One Road" (Cai, 2017), which is an application, in the modern era, of the "silk route".

It is possible to deduce that the leading Chinese interest is in maintaining trade with the world. The best way to put this into practice is through sea routes. China has nine container ports. They represent the hubs for China's production.

China aims to secure the output of its merchant ships to fulfill the goal of "One Belt, One Road." However, that naturally happens by securing, through force or deterrence, the exit of its merchant ships. There is a perception that the entry of any ships into the South China Sea is denied, using existing Chinese means: ships, bases, islands, aircraft missiles, submarines, and so on.

These means create the perception that the entry of military means from other states in the event of a crisis or conflict in the China Sea region will engender losses in the event of fighting.

The United States has termed this strategy A2/AD Anti Access and Area Denial (Krepinevich & Watts, 2003) due to the difficulty in entering and navigating the region. It is worth noting that this Chinese strategy does not use submarines to deny the use of the sea.

In contrast, over the past decades, the U.S. has invested in an operational concept called ARG (Amphibious Ready Group), which provides the U.S. president with credible deterrence power capable of conducting amphibious operations in response to crises, conducting contingency operations, and supporting special operations. This group is present in various regions of the world, ready to act as an armed arm of diplomacy. The ship composition of an ARG consists primarily of three elements (US Marine Corps, 2003):

- a Landing helicopter assault (LHA) - or a Landing helicopter dock (LHD);
- a LPD; and
- a LSD

Table 2 present their characteristics.

The ARG composition in large assets does not provide an excellent counter to the A2/AD strategy, because if only one of the assets is hit, the material and personal losses will be so great that they will compromise the Combat Power advantage and, consequently, mission accomplishment.

Therefore, this study will use the replacement of the LSD by one, or a set of two new types of a ship called LAW and LAWalt (Light Amphibious Warship), both smaller than the LSD, but with advantages by allowing the reduction of technical risks and costs by being adapted from commercial ship designs, aspects that motivated the initiation of Request for Information, by the U.S. Navy (Rourke, 2021).

It is possible to assume that the economic factor hugely influences the decision process and should be used as criteria, combined with tactical and technical aspects, to make this decision.

Table 2 – Comparing table ARG. Source: US Marine Corps (2003).

LHD 1 Wasp Class	LPD 17 San Antonio Class	LSD 41 Whidbey Island Class
22 knots	22 knots	22 knots
Crew: 1,123	Crew: 360	Crew: 413
Troops: 1,687 (+184 surge)	Troops: 720 (+80 surge)	Troops: 402 (+102 surge)
20,000 sq ft vehicle storage	24,000 sq ft vehicle storage	12,500 sq ft vehicle storage
125,000 cubic ft cargo storage	34,000 cubic ft cargo storage	5,000 cubic ft cargo storage
9 land.spots & aircraft hangar	*4 land. spots & aircraft hangar	2 land. spots
3 LCACs or 2 LCUs	2 LCACs or 1 LCU	4 LCACs or 3 LCUs
536,343 gal JP-5	318,308 gal JP-5	52,160 gal JP-5
64 hospital beds	24 hospital beds	8 hospital beds
6 operating rooms		

ii. Choosing the multicriteria decision method

If one focuses on the problem, he/she can note that it is an outranking one because we select a subset providing the best performance in most of the criteria, instead of a subset allowing higher performance in one criterion to compensate for low performance in another.

As reported in Nepomuceno & Costa (2015), among the outranking methods, the Electre I was designed to support the choice of a set of alternatives in a setting problem. So, we decided to apply it in our solution.

iii. Applying the MCDA method chosen in the previous stage

The application of Electre I follows according to the steps listed in the previous section. We note that the Visual OutDecK APP is a web app that supports modeling that uses outranking methods, enables graphical analysis and calculus routines, as in (Costa, 2021).

(a) Identifying a whole set of feasible options or alternatives

In this hypothetical case study, the alternatives are described in Table 3.

We observe that

- alternatives LAW and LAWalt are hypothetical, although close to reality.
- The LSD alternative is a real one, which considers the "LSD-41-Whidbey Island" class.
- There are two new LAW classes: LAW and LAWalt.
- Alternatives are either composed by sole unit (LAW and LAWalt alternatives) or by compositions of units of the same class (2LAW,4LAW, and 8LAW; or 2LAWalt and 3LAWalt).
- Arrangements of ships of different classes were not considered, preserving the speed and operational range of the alternative in a uniform manner.

$\underline{\mathbf{A}} = (LSD, 2LAW, 4LAW, 8LAW, LAWalt, 2LAWalt, 3LAWalt,)$

(b) Defining a set or family of criteria or decision drivers

The criteria set was obtained from the opinion of two experts in the study of marine war operations, that considers the following set as drivers to select a subset from the alternatives that appear in $\underline{\underline{\mathbf{A}}}$:

$\underline{\mathbf{F}} = (StorageCapacity, TroopCapacity, Speed, OperatingRange, Resilience, Cost)$

The resilience criterion considers the mathematical expectancy of troop maintenance higher than one-third, given a survival probability of 50 percent. Therefore, the binomial distribution was changed accordingly to more vessels in each alternative. It is noteworthy that the comparison depends on the characteristics of proposed mission used as an evaluation context.

(c) Defining the weight or relevance of each criterion in $\underline{\mathbf{F}}$

As we used the Visual OutDeck app, which facilitates exploring the influence of weight variation on the results, we began applying a weight equal to 10 for all the criteria in $\underline{\mathbf{F}}$. Later, a sensitivity analysis was performed, exploring the sensitivity of the result to variation in the weights. Thus we started with

$\underline{\mathbf{W}} = (10, 10, 10, 10, 10, 10)$

(d) Evaluating the performance of each option in $\underline{\mathbf{A}}$ under the viewpoint of each criterion in $\underline{\mathbf{F}}$

These evaluations were made by consensus between the two experts that estimated the performance of each alternative under each criterion. Table 3 presents their characteristics

Table 3 – Estimate of alternatives performances.

Alternative	Cost	Troop	Storage	Velocity	Operating Range	Resilience
LSD	100	402	17500	22	3000	50
LAW	20	100	10000	20	3500	75
2LAW	38	200	20000	20	3500	69
4LAW	56	400	40000	20	3500	86
LAWalt	35	75	8000	25	2900	50
2LAWalt	68	150	16000	25	2900	75
3LAWalt	102	225	24000	25	2900	88

The values in Table 3 were organized into the matrix of grades or performances:

$$\underline{\underline{\mathbf{G}}} = \begin{bmatrix} 100 & 402 & 17500 & 22 & 3000 & 50 \\ 20 & 100 & 10000 & 20 & 3500 & 75 \\ 38 & 200 & 20000 & 20 & 3500 & 69 \\ 56 & 400 & 40000 & 20 & 3500 & 86 \\ 35 & 75 & 8000 & 25 & 2900 & 50 \\ 68 & 150 & 16000 & 25 & 2900 & 75 \\ 102 & 225 & 24000 & 25 & 2900 & 88 \end{bmatrix}$$

(e) To calculate the concordance matrix $\underline{\underline{\mathbf{C}}}$

Inserting the data that appears in $\underline{\underline{\mathbf{G}}}$ into 6, it results in

$$\underline{\underline{\mathbf{C}}} = \begin{bmatrix} 1 & 0.6667 & 0.5000 & 0.5000 & 0.8333 & 0.6667 & 0.3333 \\ 0.3333 & 1 & 0.5000 & 0.3333 & 0.6667 & 0.3333 & 0.1667 \\ 0.5000 & 0.8333 & 1 & 0.3333 & 0.8333 & 0.5000 & 0.1667 \\ 0.5000 & 1 & 1 & 1 & 0.8333 & 0.6667 & 0.5000 \\ 0.3333 & 0.3333 & 0.1667 & 0.1667 & 1 & 0.3333 & 0.3333 \\ 0.3333 & 0.8333 & 0.5000 & 0.3333 & 1 & 1 & 0.3333 \\ 0.6667 & 0.8333 & 0.8333 & 0.5000 & 1 & 1 & 1 \end{bmatrix}$$

We have previously evaluated all the alternatives in A as feasible, or in other words, we have vetted candidates that are not feasible. We considered it unnecessary to calculate the discordance matrix. As reported in Roy (1968), it provides a veto to select an alternative.

(f) Building outranking relationships among the alternatives

In this step, we constructed an outranking graph. In this graph, an arrow from a generic alternative a addressed to another alternative b, means that a outranks b, as it follows: aSb if, and only if $c(a,b) \geq c^*$

In the more exigent situation, we have $c^* = 1$. Applying this cut level to the data of the case study results in the graph in Figure 4.

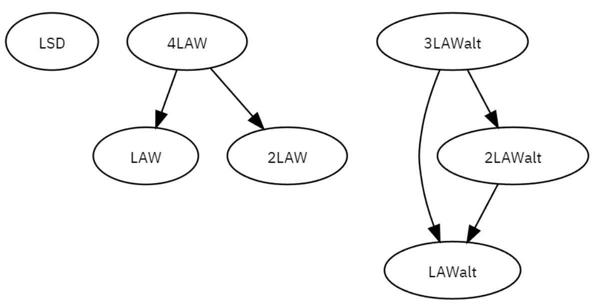

Figure 4 – Initial outranking relationships ($c^* = 1$) .

(g) Constructing a partition of $\underline{\mathbf{A}}$ composed by the kernel $\underline{\mathbf{N}}$ and outranked subset D. One must note that

- there are no outranking relationships among the members of $\underline{\mathbf{N}}$, or in other words, there is not an arrow between any pair of alternatives in $\underline{\mathbf{N}}$.
- each alternative in the subset $\underline{\mathbf{D}}$ is outranked by at least one in $\underline{\mathbf{N}}$. In other words, each alternative in the subset $\underline{\mathbf{D}}$ is targeted by an arrow from at least one in $\underline{\mathbf{N}}$.

As a result
$\underline{\mathbf{N}} = (LSD, 4LAW, 3LAWalt)$
$\underline{\mathbf{D}} = (LAW, 2LAW, LAWalt, 2LAWalt)$
Thus, the best solution indication by Electre I is to select $\underline{\mathbf{N}}$, which provides the best performance reachable from a subset of options selected from $\underline{\underline{\mathbf{A}}}$: $g(\underline{\mathbf{N}}) = (102, 402, 40000, 24, 3500, 88)$.
However, it must be highlighted that the model aims to evaluate a substitution of LSD for smaller assets, enhancing the probability of achieving Combat Power advantage because it would be more challenging to sink small substitutes than one huge vessel.

4.1 Discussion and analysis

This section performs a sensitivity analysis to determine further impacts on the selection. As shown in Figure 5, the sliders on the left side of the screen of the Visual OutDecK App (Costa, 2023) allow to easily change the values of the cut-level of concordance (c^*) and also to vary the values in the vector of criteria weights $\underline{\mathbf{w}} = (w_1, w_2, ..., W_n)$, which are, as default, assigned to ($c^* = 1.0$) and $\underline{\mathbf{w}} = (10, 10, 10, 10, 10, 10)$.

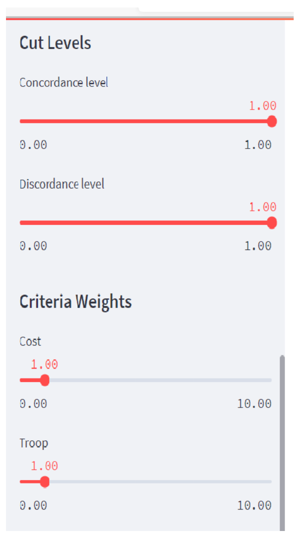

Figure 5 – App dynamic control of weights and cut level.

- Sensitivity to the weights while the value of $c* = 1.0$

  We observe that, while $c^* = 1.0$, the results are not sensitive to variation in the weights. It is because the variations in the weights can change the results of the concordance matrix. Such changes do not alter the result if $c^* = 1.0$. Thus, as c^* remains equal to 1, there is no change in the graph, and the remain the same even if we change the values in $\underline{\mathbf{w}}$ to select $\underline{\mathbf{N}} = (LSD, 4LAW, 3LAWalt)$, which provides the best performance reachable from a subset of options selected from $\underline{\underline{\mathbf{A}}}$: $g(\underline{\mathbf{N}}) = (102, 402, 40000, 25, 3500, 88)$.

- Sensitivity to the value of $c*$, while the weights equal to 10

  We observe that, while fixing $\underline{\mathbf{w}} = (10, 10, 10, 10, 10, 10)$ and relaxing the concordance exigence to lower values of c^*, the results remain the same for $0.67 \leq c^* = 1.0$. However, for values in the range $0.51 \leq c^* \leq 0.66$, the solution changes to: select $\underline{\mathbf{N}} = (4LAW, 3LAWalt)$. This solution provides the performance $g(\underline{\mathbf{N}}) = (102, 225, 40000, 24, 3500, 88)$. Table 4 compares the performance resulting from these solutions against the one brought forth by the initial solution. As one can see, the solution reaches the best reachable results for five criteria among the six as taken into account; it reaches 225 in the criterion "Troop" instead of 402, as available in the original solution.

Table 4 – Comparing performances.

Criteria	$\underline{\mathbf{N}} = (LAW, 3LAWalt)$	$\underline{\mathbf{N}} = (LSD, LAW, 3LAWalt)$
Cost	102	102
Troop	402	225
Storage	4000	4000
Velocity	25	25
Operating range	3500	3500
Resilience	88	88

- Sensitivity to variations in $c*$ and $\underline{\mathbf{w}}*$

  If we continue relaxing the concordance exigence degree to values $c* \leq 0.5$, the kernel will be empty, or in other words, $\underline{\mathbf{N}} = \{\}$, or $\underline{\mathbf{N}} = Null$. That is because the criteria have the same weight and the number of criteria in which LAW is at least as good as 3LAWalt (in which $g(4LAW) \geq= 3LAWalt)$ is equal to the number of criteria in which 3LAWalt is at least as good as LAW (in which $g(3LAWalt) \geq= 4LAW)$. It implies in $c(4LAW, 3LAWalt) = c(3LAWalt, 4LAW) = 0.5$. However, changes in the criteria weight cause a change in the concordance degree and solve the situation described above. For example, changing the weight of the criterion "velocity" to 2.0 instead of 1.0 causes the outranking relationship $3LAWaltS4LAW$ to appear, which means: 3LAWalt outranks 4LAW. In this case $\underline{\mathbf{N}} = 3LAWalt$.

5 CONCLUSION

The study pioneers the use of Electre I for a distinct configuration/setting military problem. The modeling was designed to select a subset to compose a task force composed of non-dominated or non-outranked options $\underline{\mathbf{N}} = (LSD, 4LAW, 3LAWalt)$ from an entire subset of alternatives $\underline{\underline{\mathbf{N}}}$. This is a typical setting problem, a particular case of choice problem. Additionally, this setting problem, has criteria with non-additive behaviors.

This subset provides the best performance solution one can select on a non-additive decision base - the context of selecting the task force components.

Therefore, the use of Electre I is suitable once addressed to solve multicriteria selection of members of a set (i.e., addressed setting problems) in non-additive decision situations. Also, as underlined in Costa (2016)), it does require building a rank, once it seeks the best set of alternatives that "outrank" the other ones (which may not be the set composed of the best alternatives, which is useful to avoid rank reversal that could happen when using some of the multicriteria ranking methods).

The analysis of the provided context reveals that the 4 LAW or 3LAWalt alternatives are the most suitable substitutes for LSD. Those options have a better performance on resilience, representing more robustness in opposition as less material and personnel losses will arise if only one of the assets is hit, with a lower probability of compromising the Combat Power advantage. It must be emphasized that such affirmation is adequate for a specific operations context and decision-maker. Interestingly, it demonstrates the dominance of alternative 4LAWalt regarding 8LAWalt, which is not naturally expected.

As a literature contribution, the study explored the original use of Visual Outdeck, a novel app that implements Electre I in a practical military example. Thus it serves as a possible approach for other military decisions, especially for situations of higher levels, due to the bounded rationality and politically related aspects (Brummer & Oppermann, 2021; Mintz, 1993). Moreover, the study supports an uncommon decision: not a single purchase of a vessel, but the parts of a task force, a team. Hence, the decision considers not only the individual potentiality, but also how they will act in a complete set for covering the defined purpose.

The sensitivity analysis revealed that the criteria weights do not matter if one is looking for the subset that provides the best performance from the alternatives available and the criteria decision set.

The sensitivity analysis also highlighted that, if for any reason, there is a need to reduce the number of components of a task force, the solution will consider the difference among the criteria weights, looking to discard from N the options that provide better performance in the criteria with a lower weight.

The "Visual Outdeck" representation enables a complete exploration by the DM to evaluate the application of the outranking Electre method because the app is easy to operate. It allows sensitivity analysis on the weight of criteria and presents a graphical response of its results. These graphs directly represent the dominance relations between the proposed alternatives and how changes in weights and cut levels of concordance impact the results.

Therefore, in this hypothetical case, the suggestion for the DM is to replace LSD with 3LAWalt, using the dominance relationship as justification.

Despite being a hypothetical case, it is realistic and worth for DMs once it provides a reliable instrument to test different cut levels and present it to the DM in a timely manner, test the robustness of the solution provided, and adapt the analysis for different contexts.

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References

AGENCY USCI. 1988. English: Map of the South China Sea, with 9-dotted line highlighted in green. Available at: https://commons.wikimedia.org/w/index.php?curid=17066897.

ALMEIDA-DIAS J, FIGUEIRA JR & ROY B. 2010. Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions. *European Journal of Operational Research*, **204**(3): 565–580.

ALMEIDA-DIAS J, FIGUEIRA JR & ROY B. 2012. A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-nC method. *European Journal of Operational Research*, **217**(3): 567–579.

ALOMAIR Y, AHMAD I, ALGHAMDI A, E AMIN F & ALHAZNAWI SS. 2016. Evaluating defense simulation packages using analytic hierarchy process. *Journal of Internet Technology*.

ARAZ C & OZKARAHAN I. 2005. A multicriteria sorting procedure for financial classification problems: The case of business failure risk assessment. *Lecture Notes in Computer Science*, **3578**: 563–570. Available at: https://link.springer.com/chapter/10.1007/11508069_73.

AUSTIN J & MITCHELL IM. 2008. Bringing Value Focused Thinking to bear on equipment procurement. *Military Operations Research*.

BOUYSSOU D. 2009. *Outranking methods*. pp. 2887–2893. Boston, MA: Springer US. Available at: https://doi.org/10.1007/978-0-387-74759-0_495.

BRANS JP, VINCKE P & MARESCHAL B. 1986. How to select and how to rank projects: The Promethee method. *European Journal of Operational Research*, **24**(2): 228–238.

BRUMMER K & OPPERMANN K. 2021. Poliheuristic Theory and Germany's (Non-) Participation in Multinational Military Interventions. The Non-compensatory Principle, Coalition Politics and Political Survival. *German Politics*, **30**(1): 106–121.

CAI P. 2017. Understanding China's Belt and Road Initiative.

CHENG CH, YANG KL & HWANG CL. 1999. Evaluating attack helicopters by AHP based on linguistic variable weight. *European Journal of Operational Research*, **116**(2): 423–435. Available at: http://linkinghub.elsevier.com/retrieve/pii/S0377221798001568.

COSTA HG. 2016. Graphical interpretation of outranking principles: Avoiding misinterpretation results from ELECTRE I. *Journal of Modelling in Management*, **11**(1): 26–42.

COSTA HG. 2021. Visual Outranking Decision Knowledge. Available at: https://py-testes.herokuapp.com/.

COSTA HG. 2023. Visual OutDecK: A Web APP for Supporting Multicriteria Decision Modelling of Outranking Choice Problems. In: Abraham A, Hong TP, Kotecha K, Ma K, Manghirmalani Mishra P & Gandhi N. (eds) *Hybrid Intelligent Systems*. HIS 2022. Lecture Notes in Networks and Systems, vol 647. Springer, Cham. Available at: https://doi.org/10.1007/978-3-031-27409-1_83.

COSTA IPDA, COSTA APDA, SANSEVERINO AM, GOMES CFS & SANTOS MD. 2022. Bibliometric studies on Multi-Criteria Decision Analysis (MCDA) methods applied in military problems. *Pesquisa Operacional*, **42**: e249414. Available at: http://www.scielo.br/scielo.php?script=sci_arttext&pid=S0101-74382022000100209&tlng=en.

CRARY M, NOZICK L & WHITAKER L. 2002. Sizing the US destroyer fleet. *European Journal of Operational Research*, **136**(3): 680–695. Available at: http://linkinghub.elsevier.com/retrieve/pii/S0377221701000315.

DE ALMEIDA AT, DE ALMEIDA JA, COSTA APCS & DE ALMEIDA-FILHO AT. 2016. A new method for elicitation of criteria weights in additive models: Flexible and interactive tradeoff. *European Journal of Operational Research*, **250**(1): 179–191. Available at: http://dx.doi.org/10.1016/j.ejor.2015.08.058.

DEWISPELARE AR & SAGE AP. 1980. On the application of multiple criteria decision making to a problem in defence systems acquisition. *International Journal of Systems Science*.

DIABY V & DIAS L. 2017. *Beyond Value Function Methods in MCDA for Health Care*. pp. 299–310. Cham: Springer International Publishing. Available at: https://doi.org/10.1007/978-3-319-47540-0_15.

DOU Y, ZHANG P, GE B, JIANG J & CHEN Y. 2015. An integrated technology pushing and requirement pulling model for weapon system portfolio selection in defence acquisition and manufacturing. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, **229**(6): 1046–1067. Available at: http://journals.sagepub.com/doi/10.1177/0954405414534640.

FAN YT, WANG ML, WEN MM & CHEN DL. 2015. Analysis of ballistic missile penetration effectiveness based on FA-AHP. *Systems Engineering and Electronics*.

FIGUEIRA J. 2004. MCDA methods for sorting and clustering problems: Promethee TRI and Promethee. Tech. rep.. Universit´e Libre de Bruxelles. Belgium. Available at: https://www.semanticscholar.org/paper/MCDA-methods-for-sorting-and-clustering-problems-%3A-Figueira-Smet/b4b3c5dbfc9285b61b3b01f98807fe79d046b04f.

FISHBURN PC. 1970. *Utility Theory For Decision Making*. 234 pp.

GAZIBEY Y, KANTEMIR O & DEMIREL A. 2015. Interaction among the Criteria Affecting Main Battle Tank Selection: An Analysis with DEMATEL Method. *Defence Science Journal*, **65**(5): 345. Available at: http://202.159.220.131/ojs/index.php/dsj/article/view/8924.

GOMES COSTA H, BERNARDA M & TEIXEIRA D. 2019. Applying ELECTRE TRI ME for Evaluating the Quality of Services Provided by a Library. In: *ICETC 2019: Proceedings of the 2019 11th International Conference on Education Technology and Computers*. pp. 278–281. Amsterdam: Association for Computing Machinery (ACM). Available at: https://doi.org/10.1145/3369255.3369313.

HWANG CL & YOON K. 1981. *Multiple Attribute Decision Making*. vol. 186 of *Lecture Notes in Economics and Mathematical Systems*. Berlin, Heidelberg: Springer Berlin Heidelberg. Available at: http://link.springer.com/10.1007/978-3-642-48318-9.

KEENEY RL & RAIFFA H. 1993. *Decisions with multiple objectives : preferences and value tradeoffs*. Cambridge University Press. 569 pp.

KREPINEVICH AF & WATTS B. 2003. Meeting the Anti-Access and Area-Denial Challenge. Available at: https://csbaonline.org/research/publications/a2ad-anti-access-area-denial.

LINKOV I, SATTERSTROM FK & FENTON GP. 2010. Prioritization of capability gaps for joint small arms program using multi-criteria decision analysis. *Journal of Multi-Criteria Decision Analysis*, **16**(5-6): 179–185. Available at: http://doi.wiley.com/10.1002/mcda.446.

MINTZ A. 1993. The Decision to Attack Iraq: A Noncompensatory Theory of Decision Making. *The Journal of Conflict Resolution*, **37**(4): 595–618. Available at: https://www.jstor.org/stable/174541.

MOUSSEAU V & SLOWINSKI R. 1998. Inferring an ELECTRE TRI Model from Assignment Examples. *Journal of Global Optimization*, **12**(2): 157–174.

NEPOMUCENO LDO & COSTA HG. 2015. Analyzing perceptions about the influence of a master course over the professional skills of its alumni: A multicriteria approach. *Pesquisa Operacional*, **35**(1): 187–211.

NIKOU C & MOSCHURIS S. 2016. An integrated approach for supplier selection in military critical application items. *Journal of Public Procurement*.

NIKOU C, MOSCHURIS SJ & FILIOPOULOS I. 2017. An integrated model for supplier selection in the public procurement sector of defence. *International Review of Administrative Sciences*, **83**(1_suppl): 78–98. Available at: http://journals.sagepub.com/doi/10.1177/0020852316634446.

PESSÔA LAM & COSTA HG. 2020. Multicriteria applied to Defence: a panorama of the scientific literature. Available at: http://www.abepro.org.br/proceedings/artigo.asp?e=icieom&a=2020&c=37390.

POUNDS JF & FALLESEN JJ. 1994. Understanding Problem Solving Strategies. Tech. Rep. Technical Report 1020. Army research Institute. Available at: https://apps.dtic.mil/sti/pdfs/ADA290350.pdf.

ROURKE R. 2021. *Navy Light Amphibious Warship (LAW) Program: Background and Issues for Congress*. R46374. Available at: https://crsreports.congress.gov/product/pdf/R/R46374/33.

ROY B. 1968. Classement et choix en présence de points de vue multiples. *Revue française d'informatique et de recherche opérationnelle*, **2**(8): 57–75. Available at: http://www.rairo-ro.org/10.1051/ro/196802V100571.

ROY B. 1978. ELECTRE III: Un alghoritme de méthode de classements fonde sur une representation floue des préfé-rences em présence de critéres multiples. *Cahiers du CERO*, **20**(1): 3–24.

ROY B & BERTIER P. 1971. La méthode ELECTRE II: Une méthode de classement en présence de critéres multiples.

ROY B & HUGONNARD J. 1982. Classement des Prolongements de Lignes de Stations en Banlieu Parisienne. *Cahiers du CERO*, **24**(2): 153–171.

ROY BM & SKALKA J. 1984. ELECTRE IS: Aspécts methodologiques et guide d'utilization.

SAATY TL. 1980. *The analytic hierarchy process : planning, priority setting, resource allocation*. McGraw-Hill International Book Co. 287 pp.

SANT'ANNA A, COSTA H & PEREIRA V. 2015. CPP-TRI: A sorting method based on the probabilistic composition of preferences. *International Journal of Information and Decision Sciences*, **7**(3): 193–212.

US MARINE CORPS. 2003. *Amphibious Ready Group And Marine Expeditionary Unit Overview*. Available at: https://www.marines.mil/Portals/1/AmphibiousReadyGroupAndMarineExpeditionaryUnitOverview.pdf.

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