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THE LEVERAGE EFFECT AND THE ASYMMETRY OF THE ERROR DISTRIBUTION IN GARCH-BASED MODELS: THE CASE OF BRAZILIAN MARKET RELATED SERIES

Daniel de Almeida and Luiz K. Hotta*

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ABSTRACT. Traditional GARCH models fail to explain at least two of the stylized facts found in financial series: the asymmetry of the distribution of errors and the leverage effect. The leverage effect stems from the fact that losses have a greater influence on future volatilities than do gains. Asymmetry means that the distribution of losses has a heavier tail than the distribution of gains. We test whether these features are present in some series related to the Brazilian market. To test for the presence of these features, the series were fitted by GARCH(1,1), TGARCH(1,1), EGARCH(1,1), and GJR-GARCH(1,1) models with standardized Student t distribution errors with and without asymmetry. Information criteria and statistical tests of the significance of the symmetry and leverage parameters are used to compare the models. The estimates of the VaR (value-at-risk) are also used in the comparison. The conclusion is that both stylized facts are present in some series, mostly simultaneously.

Keywords: asymmetry in volatility models, asymmetric Garch family models, VaR (Value-at-Risk).

1 INTRODUCTION

Two important features usually found in time series of asset returns are the presence of volatility clustering and the high kurtosis. Here volatility is considered as the conditional variance, although some authors define it as the conditional standard deviation. The most popular model used in the literature to explain these two stylized facts is the generalized autoregressive conditional heteroskedastic (GARCH) model of Bollerslev (1986) [3] with symmetric errors (normal or Student t distributions). However, these traditional GARCH models cannot explain some stylized facts found in financial time series. Two important unexplained facts are the skewness, or asymmetry, in the distribution of the errors and the leverage effect. The former consists of losses having a distribution with a heavier tail than gains. Simkowitz & Beedles (1980) [18], Kon (1984) [10], among others drew the attention to the skewness in such distribution. French et al. (1987)

*Corresponding author
University of Campinas, Rua Sérgio Buarque de Holanda, 651, Distr. Barão Geraldo, 13083-859 Campinas, SP, Brazil.
Phone: +55(19) 3521-6081/93101120. E-mails: dani.d.almeida89@gmail.com; hotta@ime.unicamp.br.

[8] found that the conditional asymmetry coefficient significantly differs from zero in the standardized residuals when ARCH family models were fitted to the daily returns of the Standard & Poor 500 (S&P) series. The leverage effect, originally introduced by Black (1976) [2], takes into account that losses have a greater influence on future volatility than do the gains. However, no study has tested yet for the simultaneous presence of these two effects, especially for Brazilian related series.

The aim of the present paper is to verify if these stylized facts are present in some market indices related to the Brazilian market and five of the most important stocks traded in the São Paulo Stock Exchange (BOVESPA). The indices considered are the Ibovespa (IBV, Brazil), MERVAL (Argentina), and S&P (USA), and the five stocks are Itaú-Unibanco (Itaú), Vale PNA (Vale), Petrobrás PN (Petro), Banco do Brasil ON (BB), and Bradesco PN (Brad), in the period from February 1st, 2000 to February 1st, 2011. After filtering the return series with AR(1) models, we fitted the GARCH(1,1), TGARCH(1,1), EGARCH(1,1), and GJR-GARCH(1,1) models with standardized Student t and standardized asymmetric Student t innovations, for a total of eight models.

Three methods are used to compare the models. The first one uses the Akaike information criterion (AIC) (Akaike, 1974 [1]), the Bayesian information criterion (BIC) (Schwarz, 1978 [17]), and the Hannan and Quin information criterion (HQ) (Hannan & Quinn, 1979 [11]), to select the best model. The second method tests the significance of the symmetry and leverage parameters. The third method compares the value at risk (VaR) estimated by the eight models treated. A model is considered adequate if the VaR estimates have the desired properties. Section 2 presents three GARCH family models which have leverage effect and the asymmetric distribution used to model the error term. Section 3 presents the methods used to compare these models and Section 4 presents some applications. Our concluding remarks are in Section 5.

2 ARMA-GARCH MODELS

Denoting the returns by r_t, this series is first filtered by an ARMA model (1), yielding residuals ε_t, serially uncorrelated, but not necessarily independent. In (2)-(4), the series ε_t is fitted by a conditional volatility model. We can write this class of models as

$$\begin{aligned} r_t &= \mu_t + \varepsilon_t & (1)\\ \varepsilon_t &= \sigma_t z_t & (2)\\ \mu_t &= c(\mu|\Omega_{t-1}) & (3)\\ \sigma_t^2 &= h(\mu, \eta|\Omega_{t-1}), & (4)\end{aligned}$$

where $c(.|\Omega_{t-1})$ and $h(.|\Omega_{t-1})$ are functions of $\Omega_{t-1} = \{r_j, j \le t-1\}$, and z_t is an independent and identically distributed (i.i.d.) process, independent of Ω_{t-1}, with $\mathrm{E}(z_t) = 0$ and $\mathrm{Var}(z_t) = 1$. In the ARMA-GARCH model, the residuals are modeled by the generalized autoregressive conditional heteroskedasticity (GARCH) model and the shapes of the functions $c(.|\Omega_{t-1})$ and $h(.|\Omega_{t-1})$ are defined by the orders of the ARMA and GARCH models, respectively. Assuming their existence, μ_t and σ_t^2 are the conditional mean and variance of r_t, respectively.

For example, in the AR(1)-GARCH(1,1) model, the mean and the volatility given by (3) and (4), respectively, are

$$\mu_t = \mu + \phi r_{t-1} \tag{5}$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2, \tag{6}$$

with $|\phi| < 1$ and $\omega > 0$. The conditions $\alpha, \beta \geq 0$, $\alpha + \beta < 1$, which are sufficient conditions for the process to be stationary and have finite variance, therefore, are usually adopted.

2.1 The leverage effect

The leverage effect is caused by the fact that negative returns have a greater influence on future volatility than do positive returns. For a good comparison among several GARCH models with leverage effect, see Rodríguez & Ruiz (2012) [16]. In this paper, we consider three of the most popular models to represent it: the EGARCH, TGARCH, and GJR models.

In the EGARCH model (Nelson, 1991 [15]), the conditional volatility is given by

$$\ln(\sigma_t^2) = \omega + \gamma_E z_{t-1} + \alpha\{|z_{t-1}| - E(|z_{t-1}|)\} + \beta\ln(\sigma_{t-1}^2). \tag{7}$$

Since z_t is an i.i.d. sequence, $|\varepsilon_t| - E(|\varepsilon_t|)$ is also a sequence of i.i.d. random variables with zero mean. γ_E is a real parameter, such that $\gamma_E < 0$ when negative returns have a greater impact on future volatility than positive returns. Due to the volatility specification in terms of the logarithmic transformation, there are no restrictions on the parameters to ensure positive variance. A sufficient condition for stationarity and finite kurtosis is $|\beta| < 1$.

The Threshold GARCH (TGARCH)(see Zakoïan, 1994 [19]) is a particular case of a nonlinear ARCH model and it models the conditional standard deviation instead of the conditional variance. The TGARCH(1,1) is written as

$$\sigma_t = \omega + \alpha|\varepsilon_{t-1}| + \beta\sigma_{t-1} + \gamma_T\varepsilon_{t-1}. \tag{8}$$

Ding et al. (1993) [5] proved that, in order to guarantee the positivity of σ_t, it is sufficient that $\omega > 0$, $\alpha \geq 0$ and $\gamma_T < \alpha$. Furthermore, the model is stationarity if $\gamma_T^2 < 1 - \alpha^2 - \beta^2 - 2\alpha\beta E(|z_t|)$. For example, if z_t is Gaussian, then $E(|z_t|) = \sqrt{\frac{2}{\pi}}$.

The GJR model of Glosten et al. (1993) [9] specifies the conditional variance by

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma_G\mathbb{I}(z_{t-1} < 0)\varepsilon_{t-1}^2, \tag{9}$$

where $\mathbb{I}(.)$ is equal to 1 when the inequality is satisfied and 0 otherwise. Hentschel (1995) [13] showed that σ_t^2 is positive if

$$\omega > 0, \alpha, \beta, \gamma_G \geq 0. \tag{10}$$

A sufficient condition for stationarity and finite variance is

$$\gamma_G < 2(1 - \alpha - \beta). \tag{11}$$

2.2 Asymmetry in the errors

In practice, it is generally assumed that $z_t \sim N(0, 1)$ or $z_t \sim t_v$ standardized, or any distribution that describes the heavy tails of financial time series. For normal errors and GARCH(1,1), the kurtosis is equal to

$$K = \frac{\mathbb{E}(r_t^4)}{[\mathbb{E}(r_t^2)]^2} = \frac{3[1-(\alpha+\beta)]}{1-(\alpha+\beta)^2-2\alpha_1^2} > 3, \tag{12}$$

when the fourth moment is defined, i.e., when the denominator is positive. This shows that even when the error z_t has a standard normal distribution and ε_t follows a GARCH process, the tails of ε_t are heavier than normal. However, in empirical series it is often found that the distribution of the error term z_t has heavier tails than the normal distribution, and is often replaced by the standardized Student t distribution (see, for example, Bollerslev, 1986 [3]).

The standardized Student t distribution with ν $(\nu > 2)$ degrees of freedom is given by

$$g(z) = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{(\nu-2)\pi}\,\Gamma(\nu/2)}\left(1+\frac{z^2}{(\nu-2)}\right)^{-(\frac{\nu+1}{2})}, \tag{13}$$

where Γ is the gamma function.

The distribution given in (13) has skewness coefficient equal to zero and the excess of kurtosis equal to $6/(\nu-4)$, for $\nu > 4$.

While the high kurtosis of returns is a well established fact, the situation is much more obscure for the symmetry of the distribution of z_t. In this paper, we consider the asymmetric Student t distribution. There have been several proposals to include asymmetry in the Student t distribution. Hansen (1994) [12] was the first to use an asymmetric Student t distribution in modeling financial data. Fernández & Steel (1998) [7] proposed a way of introducing asymmetry into any symmetric and unimodal continuous distribution $g(.)$, changing its scale on each side of the mode. Applying this procedure to the Student t distribution, one obtains an asymmetric Student t density. In order to preserve the specifications of the GARCH model, Lambert & Laurent (2001) [14] modified this density to standardize it, that is, to have zero mean and unit variance.

Following Lambert & Laurent (2001) [14], the random variable z_t is said to follow the standardized asymmetric Student t, denoted by SKST(0,1,ξ, υ), with parameters $\upsilon > 2$ (the number of degrees of freedom) and $\xi > 0$ (the parameter associated with the skewness), if its density is of the form

$$f(z_t|\xi,\upsilon) = \begin{cases} \dfrac{2}{\xi+\frac{1}{\xi}}\, s\, g[\xi(sz_t+m)|\upsilon] & \text{if} \quad z_t < -m/s \\ \dfrac{2}{\xi+\frac{1}{\xi}}\, s\, g[(sz_t+m)/\xi|\upsilon] & \text{if} \quad z_t \geq -m/s, \end{cases} \tag{14}$$

where $g(.|v)$ is the density of the standardized symmetric Student t given by (13), and the constants $m = m(\xi, v)$ and $s = \sqrt{s^2(\xi, v)}$ are, respectively, the mean and standard deviation of the SKST(m, s^2, ξ, v) distribution and can be expressed by

$$m(\xi, v) = \frac{\Gamma(\frac{v+1}{2})\sqrt{v-2}}{\sqrt{\pi}\,\Gamma(\frac{v}{2})}\left(\xi - \frac{1}{\xi}\right) \tag{15}$$

and

$$s^2(\xi, v) = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2, \tag{16}$$

respectively (Fernández & Steel, 1998 [7]). The main advantages of this density are its easy implementation and the clear interpretation of its parameters. Ehlers (2012) [6] modeled GARCH model with the error term errors with this distribution and proposed a fully Bayesian approach to estimate the model.

3 CRITERIA FOR COMPARISON OF MODELS

Consider T observations of a volatility process and suppose that we want to verify the presence of the leverage effect and of asymmetry in the perturbations. In order to do this, we use the following eight models: GARCH, TGARCH, EGARCH, and GJR-GARCH with standardized symmetric and asymmetric Student t distributions. In this section, we present the three criteria used to select the most appropriate model.

Information criteria. There are several information criteria suggested in the literature to select a model. In this paper, we consider the AIC, BIC, and HQ criteria. These criteria are the likelihood penalized by different functions of the number of parameters of the model.

Testing hypotheses. By fitting the GJR-GARCH model with asymmetric Student t distribution, for example, we have as special cases a model without leverage when $\gamma_G = 0$ and a model with symmetric innovations when the skewness parameter (ξ) is equal to 1. Thus we can use hypothesis testing to verify the presence or absence of these two stylized facts. We can follow the same procedure with GARCH, TGARCH, and EGARCH models.

The third criterion uses the VaR at the 95% and 99% levels to test the accuracy of the models in making predictions. We use the conditional prediction interval evaluation procedure of Christoffersen (1998) [4]. He proposed a likelihood ratio (LR) test to test the null hypothesis that a statistical method (the model) is good for prediction purpose. This test is defined as follows.

3.1 The likelihood ratio test for the conditional coverage

The VaR can be viewed as a prediction interval. One of the methods to evaluate prediction interval is the LR test of Christoffersen (1998) [4]. In the VaR case, it tests whether the sequence of losses smaller than the VaR comes from a random sample of the Bernoulli distribution with probability equal to the nominal value.

Let $(r_t)_{1\leq t\leq T}$ be the realization of a series of returns of any financial asset and let $[L(p)_{t|t-1}, U(p))_{t|t-1}]$ be the corresponding sequence of interval forecast outside the sample, where $L(p)_{t|t-1}$ and $U(p))_{t|t-1}$ are the lower and upper limits of the forecast intervals at time t, given the information until time $t-1$, at the confidence level p. Set the indicator variable I_t at time t, given information until time $t-1$, as

$$I_t = \begin{cases} 1, & \text{if} \quad r_t \in [L(p)_{t|t-1}, U(p))_{t|t-1}] \\ 0, & \text{if} \quad r_t \notin [L(p)_{t|t-1}, U(p)_{t|t-1}]. \end{cases} \tag{17}$$

We say that the sequence of prediction interval, $[L(p)_{t|t-1}, U(p))_{t|t-1}]$, is efficient with respect to the information set at time $t-1$ (Ψ_{t-1}), if $E(I_t|\Psi_{t-1}) = p$, $\forall t$ if it passes the LR test. Christoffersen (1998) [4] showed that testing $E(I_t|\Psi_{t-1}) = p$, for all t, is equivalent to testing if the sequence $(I_t)_{1\leq t\leq T}$ is i.i.d. with a Bernoulli distribution with parameter p, i.e., $I_t \sim$ i.i.d. Ber(p). Therefore, a sequence of prediction intervals, $[L(p)_{t|t-1}, U(p))_{t|t-1}]$, has a correct conditional coverage if $I_t \sim$ Ber(p) i.i.d., $\forall t$.

In the conditional coverage test, the null hypothesis is that $(I_t)_{1\leq t\leq T}$ is independent and $E(I_t|\Phi_{t-1}) = p$. The test statistics is

$$LR_{cc} = -l(p; I_1, \ldots, I_T) - l(\hat{\pi}_1; I_1, \ldots, I_T)], \tag{18}$$

where $l(\theta; ; I_1, \ldots, I_T)$ is the log likelihood function, i.e., $l(p; I_1, \ldots, I_T) = (n_T)\ \log(\theta) + (T - n_T)\ \log(1-\theta)$ with $n_T = \sum_{i=1}^{T} I_i$, and $\hat{\pi}_1 = n_T/T$. The statistics LR_{cc} has a χ_2^2 distribution under the null hypothesis.

Equation (18) can be written as the sum of the LR test statistics for the correct unconditional coverage and the LR test statistics for independence (Christoffersen, 1998 [4]). Rejecting the null hypothesis implies that the model is not good for prediction purpose.

4 APPLICATIONS

In this section, we analyze the series of returns of IBV, Merval, S&P, Itaú, Vale, Petro, BB, and Brad, from February 1st, 2000 to February 1st, 2011, with a total of 12 years. Each series was previously filtered by an ARMA (p, q) model with appropriate orders.

For each dataset we adopted the following procedure.

1. Consider the observations of the returns of the first eight years.
2. Fit all eight models.
3. Verify which model is selected by the AIC, BIC and HQ criteria.
4. For each estimated model, evaluate the one-step-ahead 95%-VaR and 99%-VaR for the next five days. Test whether the returns are below the estimated VaR values. Note that we are always doing one-step-ahead estimation of the VaR, but the model is not re-estimated every time we include one observation.

5. Include five more observations and exclude the first five observations.

6. Repeat steps (2) to (5) until the end of the period.

For each series and each model, we fitted around 200 models and estimated around 1,000 VaR values. The number of models and VaR estimates depend upon each series, because we ignored non-trading days.

Tables 1 and 2 indicate how many times each of the eight models were selected by the AIC, BIC, and HQ criteria. The main conclusions are:

- The GARCH model was never selected by any criterion for the IBV, S&P, Itaú, Petro, or Brad series. For the Merval and Vale series, the GARCH model was only selected by the BIC (60% of the time for the Merval series and 21% for the Vale series); for the BB series the GARCH model was only selected 31% of the time. This means that there is a clear preference of the information criteria for models with the leverage effect.

- For all of the stocks, the GJR was the most selected model by all the criteria. For the Petro and Brad series, it was always the model selected. For the Merval series, the GJR model was always selected by the AIC, in 91% of the cases by the HQ criterion, and 40% by the BIC. The TGARCH was selected most of the time for the IBV series by all criteria, and the EGARCH model was selected most of the time for the S&P series by all criteria.

- For the IBV and S&P series, the criteria selected models with leverage and asymmetric distributions almost all the time. For the Merval, Itaú, and Brad series, the criteria selected models with leverage and asymmetric distributions most of the time. For the Vale, Petro, and BB series, the criteria selected models with leverage and symmetric distributions most of the time.

Tables 3 and 4 present, respectively, the percentage of cases where the asymmetry and leverage parameters were significant at the 5% level. Figure 1 presents the estimated asymmetry and leverage parameters in the GJR-GARCH asymmetric model for the IBV, Merval, Vale, and BB series, while Figure 2 presents the results for the Itaú, S&P, Petro, and Brad series. We do not present the equivalent graphs for the other models, since their behavior is very similar to that of the GJR model. Under the null hypothesis of no asymmetry, one has $\xi = 1$; and under the null hypothesis of no leverage effect, $\gamma_G = 0$. We use full symbols to indicate rejection of the null hypotheses at the 5% level. The main conclusions are:

- For the GJR model, the leverage effect was detected in the models with symmetric and asymmetric errors in all cases for the IBV, S&P, Itaú, Petro, and Brad series, and in practically all cases for the Vale series. For the Merval and BB series, the leverage effect was detected in approximately 80% and 67% of the cases, respectively. The results were similar in the EGARCH and TGARCH models.

Table 1 – Number of times the model was selected by the AIC, BIC, and HQ criteria. Panels 1–8 correspond to the IBV, Merval, S&P, Itaú, Vale, Petro, BB, and Brad series, respectively. sym., asym. = standardized symmetric and asymmetric Student t innovations, respectively.

Criterion	GARCH		GJR		EGARCH		TGARCH	
	sym.	asym.	sym.	asym.	sym.	asym.	sym.	asym.
AIC	0	0	0	30	0	26	0	141
BIC	0	0	0	30	8	18	3	138
HQ	0	0	0	30	0	26	0	141
AIC	0	0	20	176	0	0	0	0
BIC	66	52	37	41	0	0	0	0
HQ	0	17	46	133	0	0	0	0
AIC	0	0	0	13	0	142	0	46
BIC	0	0	0	13	22	120	0	46
HQ	0	0	0	13	6	136	0	46
AIC	0	0	0	92	0	40	0	65
BIC	0	0	25	53	49	0	38	32
HQ	0	0	9	81	10	30	6	61
AIC	0	0	183	0	0	0	14	0
BIC	42	0	141	0	0	0	14	0
HQ	0	0	183	0	0	0	14	0
AIC	0	0	38	159	0	0	0	0
BIC	0	0	197	0	0	0	0	0
HQ	0	0	152	45	0	0	0	0
AIC	0	42	69	86	0	0	0	0
BIC	81	0	116	0	0	0	0	0
HQ	1	58	102	36	0	0	0	0
AIC	0	0	12	185	0	0	0	0
BIC	0	0	111	86	0	0	0	0
HQ	0	0	62	135	0	0	0	0

- The asymmetry in the errors was detected in all the cases for all models for the IBV and S&P series, and in approximately 75%, 70%, and 50% of the cases for the Merval, Brad, and BB series, respectively. For the Vale series, the null hypothesis was never rejected. For the Itaú and Petro series, the percentage depended on the model. For the Itaú series, the detection of asymmetry varied from 99.5% for the GARCH model to 74.6% for the TGARCH model, while for the Petro series the percentages varied from 34.5% for the GJR model to 9.1% for the EGARCH model.
- From the figures we can observe that there is a certain stability in time and that in most of the cases the leverage effect and the asymmetry are simultaneously significant most of the time.

Table 2 – Percentage of selection of a model with leverage (GJR, EGARCH, TGARCH) and without leverage (GARCH), and with and without asymmetric innovations. The left side panels 1–4 correspond to the IBV, S&P, Vale, and BB series, respectively. The right side panels 1–4 correspond to the Merval, Itaú, Petro, and Brad series, respectively. sym., asym. = standardized symmetric and asymmetric Student t innovations, respectively.

Criterion	Left panel				Right panel			
	Leverage		Innovation		Leverage		Innovation	
	without	with	sym.	asym.	without	with	sym.	asym.
AIC	0.00	100.0	0.00	100.0	0.00	100.0	10.20	89.80
BIC	0.00	100.0	5.58	94.42	59.90	40.10	52.55	47.45
HQ	0.00	100.0	0.00	100.0	23.35	76.65	23.47	76.53
AIC	0.00	100.0	0.00	100.0	0.00	100.0	0.00	100.0
BIC	0.00	100.0	10.95	89.05	0.00	100.0	56.85	43.15
HQ	0.00	100.0	2.99	97.01	0.00	100.0	12.69	87.31
AIC	0.00	100.0	100.0	0.00	0.00	100.0	19.29	80.71
BIC	21.32	78.68	100.0	0.00	0.00	100.0	100.0	0.00
HQ	0.00	100.0	100.0	0.00	0.00	100.0	77.16	22.84
AIC	21.32	78.68	35.03	64.97	0.00	100.0	6.09	93.91
BIC	41.12	58.88	100.0	0.00	0.00	100.0	56.35	43.65
HQ	29.95	70.05	52.28	47.72	0.00	100.0	31.47	68.53

Table 3 – Percentage of times the skewness parameter of the asymmetric Student t distribution were significant at the 5% level.

	IBV	Merval	S&P	Itaú	Vale	Petro	BB	Brad.
GARCH	99.49	77.04	92.54	99.49	0.00	11.17	57.87	72.08
GJR	100.0	79.08	99.50	89.34	0.00	34.52	44.16	68.53
EGARCH	100.0	72.45	99.00	77.66	0.00	9.14	51.78	68.53
TGARCH	100.0	72.96	99.00	74.62	0.00	14.21	49.24	68.53

Table 4 – Percentage of times the leverage parameter of the GJR model was significant at the 5% level.

Distr.	IBV	Merval	S&P	Itaú	Vale	Petro	BB	Brad.
sym.	100.0	79.70	100.0	100.0	98.98	100.0	68.02	100.0
asym.	100.0	82.74	100.0	100.0	95.94	100.0	66.50	100.0

Table 5 presents the percentage of cases with loss larger than the one-step-ahead 95%-VaR and 99%-VaR. A good model should give a percentage close to the nominal value. It is preferred to have percentage smaller than larger that the nominal values. A good model should also have a large p-value for the LR test. The main conclusions are:

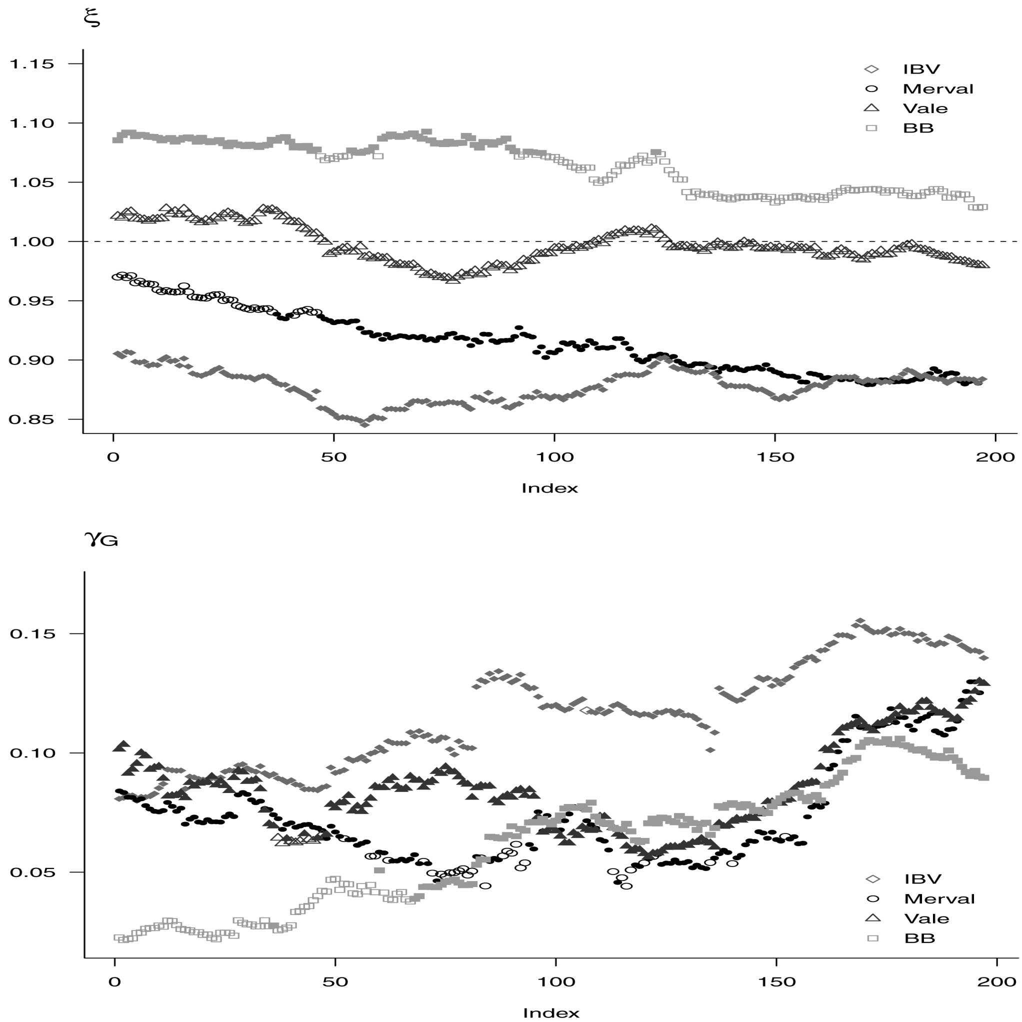

Figure 1 – Estimates of the asymmetry parameter of the error distributions (ξ) and of the leverage parameter (γ_G) of the GJR-GARCH model for the IBV, Merval, Vale, and BB series. Full symbols mean rejection of the null hypothesis at 5%. Under the null hypothesis of no asymmetry, one has ($\xi = 1$); and under the null hypothesis of no leverage effect, ($\gamma_G = 0$).

- There is no meaningful difference in terms of percentage, although the models with asymmetric distributions are generally slightly better.
- For the 99%-VaR, the models with asymmetric error distribution, except for S&P series (for EGARCH and TGARCH), pass the LR test, with the smallest p-value equal to 0.15. When we consider the symmetric error distribution, all the models fail for the IBV and S&P series.

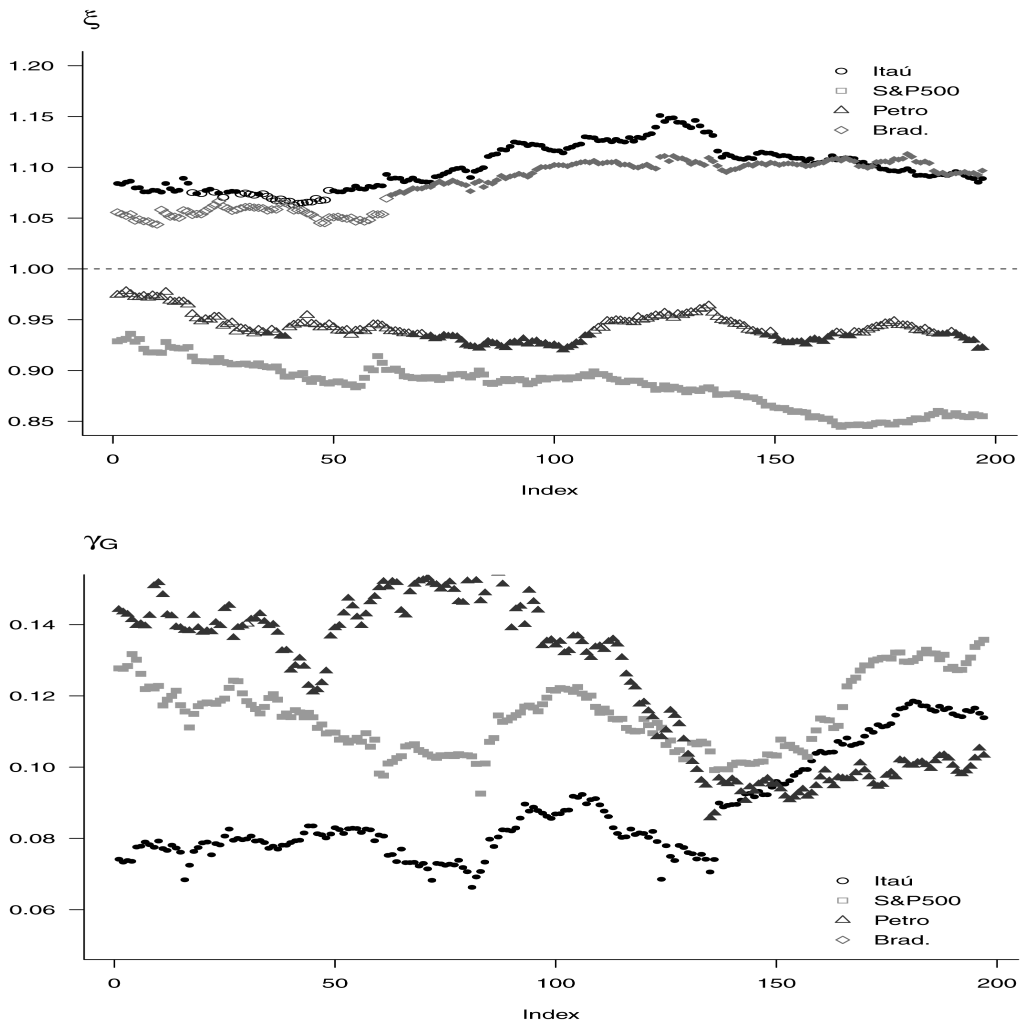

Figure 2 – Estimates of the asymmetry parameter of the error distributions (ξ) and of the leverage parameter (γ_G) of the GJR-GARCH model for the Itaú, S&P, Petro and Brad series. Full symbols mean rejection of the null hypothesis at 5%. Under the null hypothesis of no asymmetry, one has ($\xi = 1$); and under the null hypothesis of no leverage effect, ($\gamma_G = 0$).

- For the 95%-VaR, the models with asymmetric error distribution, except for the Vale, Petro (for GARCH model), and S&P series, pass the LR test at the 5% level. When we consider the symmetric error distribution, all the models fail for the IBV, Merval, S&P, Vale (except for GJR), and Petro series.

Considering the three methods of comparison we can say that the two stylized facts are present in most of the series analyzed, and that models taking into account these two stylized facts improve the estimation of the VaR.

Table 5 – Percentage of cases with loss larger than the VaR and the p-value of the LR test for the conditional 95%-VaR and 99%-VaR. Panels 1–8 correspond to the IBV, Merval, S&P, Itaú, Vale, Petro, BB, and Brad series, respectively.

Model	VaR 95%				VaR 99%			
	Percentage		p-value		Percentage		p-value	
	sym.	asym.	sym.	asym.	sym.	asym.	sym.	asym.
GARCH	93.10	94.11	0.032	0.447	98.27	98.98	0.086	0.901
GJR	93.10	93.91	0.023	0.293	98.27	98.88	0.086	0.827
EGARCH	92.79	93.81	0.006	0.144	98.07	98.68	0.024	0.529
TGARCH	92.99	94.11	0.015	0.447	98.07	98.98	0.0235	0.901
GARCH	92.96	93.47	0.022	0.118	98.47	98.61	0.245	0.545
GJR	93.06	93.88	0.023	0.292	98.37	98.89	0.191	0.629
EGARCH	92.76	93.67	0.010	0.200	98.16	98.77	0.1463	0.474
TGARCH	92.96	93.67	0.022	0.088	98.27	98.77	0.186	0.474
GARCH	92.84	93.23	0.002	0.025	97.61	98.61	0.001	0.287
GJR	92.94	93.53	0.001	0.032	97.81	98.47	0.002	0.247
EGARCH	91.74	92.74	<0.001	0.012	96.82	97.91	<0.001	0.002
TGARCH	91.84	92.94	<0.001	0.020	96.92	98.31	<0.001	0.022
GARCH	94.42	93.50	0.561	0.097	99.39	98.68	0.399	0.529
GJR	94.92	93.81	0.928	0.229	99.19	98.98	0.776	0.901
EGARCH	94.31	93.50	0.473	0.097	99.29	99.09	0.599	0.886
TGARCH	94.62	93.60	0.737	0.132	99.29	99.19	0.599	0.776
GARCH	93.20	93.20	0.046	0.046	98.48	98.48	0.246	0.246
GJR	93.50	93.50	0.118	0.118	98.38	98.38	0.150	0.150
EGARCH	93.10	93.20	0.023	0.034	98.38	98.38	0.150	0.150
TGARCH	92.89	92.99	0.010	0.015	98.48	98.58	0.246	0.374
GARCH	92.99	93.10	0.022	0.032	98.78	98.78	0.691	0.691
GJR	93.30	93.91	0.026	0.071	98.48	98.68	0.246	0.529
EGARCH	93.10	93.50	0.023	0.097	98.58	98.68	0.374	0.529
TGARCH	93.30	93.50	0.026	0.055	98.68	98.68	0.529	0.529
GARCH	95.63	95.23	0.235	0.080	98.88	98.58	0.827	0.374
GJR	95.74	95.43	0.062	0.149	98.88	98.68	0.827	0.529
EGARCH	95.74	95.33	0.381	0.436	98.88	98.48	0.827	0.246
TGARCH	95.74	95.33	0.381	0.436	98.98	98.58	0.901	0.374
GARCH	95.23	94.42	0.591	0.561	99.29	99.19	0.599	0.776
GJR	95.33	94.72	0.588	0.415	99.09	98.98	0.886	0.901
EGARCH	95.13	94.62	0.084	0.360	99.09	98.88	0.886	0.827
TGARCH	95.13	94.62	0.084	0.360	99.19	98.88	0.776	0.827

5 CONCLUDING REMARKS

In this paper we analyzed eight series in order to test whether two stylized facts are present: asymmetry in the error distributions and the leverage effect. We first compared the models using the AIC, BIC, and HQ information criteria, and by using hypothesis testing. In both methods, we found evidence that the two stylized facts are present in most of the series analyzed. In the third method, we compared the VaR estimates and found that in VaR estimation, the models with asymmetric errors performed much better than those with symmetric distributions, in terms of the LR test.

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