https://doi.org/10.1590/jatm.v15.1315 ORIGINAL PAPER

Investigating the Effect of Applying Uniform Distributed Load on the Deflection of Simply Supported Axial - Functionally Graded Beam

Zainab M Shukur[1,*], Raisan Faris Hamad[1], Yassar Khadhim Ali[1], Luay Sadiq Al-Ansari[1], Mohammedh Helayel Al-Karaishi[1]

1.University of Kufa – Faculty of Engineering – Department of Mechanical Engineering – Najaf – Iraq.

*Corresponding author: Zainab.shukur@uokufa.edu.iq

ABSTRACT

Axially-functionally graded materials are/ types of traditional composite materials in which the mechanical and physical properties are gradually varied from one end to the other. They were used extensively in industries such as defense, automotive and aerospace because of the ability to design its mechanical and physical properties. Two numerical models are built in this work in order to investigate the deflection of a simply supported beam made by axial-functionally graded material. The first model is the new model and it is built by adopting the Rayleigh Method, while the second model used the Finite-Element technique to build an 1D model utilizing the ANSYS APDL. The mechanical and physical of the axial-functionally graded beam were changed in axial direction according to Power-Law Equation. The new model, based on Rayleigh Method ANSYS- 1D model, shows an excellent agreement with the results and available literature. In addition to the validation of the two models, the influences of elastic moduli ratio and material distribution on the maximum static deflection and its position were studied. In ANSYS- 1D model, the position of the maximum deflection was deviated from the middle span of A-FG beam and this deviation in position of maximum deflection reduces, as well as increases the index of power-law equation and the elastic moduli ratio (ME-Ratio) when it diverges from 1.

Keywords: Axially-functionally graded beam; Power-law equation; Uniform distributed load; Simply supported beam.

INTRODUCTION

In mechanical and structural applications, metals (Kassner *et al.* 2015), alloys (Saleh *et al.* 2021; Xu *et al.* 2019), ceramics (Ashby 2000; Majeed *et al.* 2017; Shao 2018), polymers (Kalyan*et al.* 2017; Osswald *et al.* 2010), and conventional composites (i.e., chopped-fiber composites, longitudinal-fiber composites and laminated composites) (Harris 1999; Kaw 2006; Xu *et al.* 2020), were manufactured to get homogenous properties (Saleh *et al.* 2020b). Sometimes these materials cannot meet the requirements of engineering applications because of limitations of their material properties (Saleh *et al.* 2019; Fathi *et al.* 2020). Therefore, it was essential to enhance mechanical and physical properties to achieve the requirements of engineering applications (El-Galy *et al.* 2018) by manufacturing a new material, called "Functionally Graded Materials (FGMs)". The FGM can be defined as "Functionally

Received: June 30, 2022 | **Accepted:** Sept. 09, 2023
Section editor: Melih Kushan
Peer Review History: Single Blind Peer Review.

Grade Materials (FGMs) are kind of composite materials in which the material properties are designed to vary continuously and gradually from one surface to the other in order to eliminate the discontinuity effects in properties" (Bhavar *et al.* 2017; Ebhota and Jen 2018; Helal 2020; Jamian 2012; Walaa 2021); or "Functionally graded materials (FGMs) are a broad research area and attract considerable tremendous attention today in the materials science and engineering society" (Saleh *et al.* 2020a). Due to enhance the material properties of FGM, the static and dynamic characteristics of FG plates and beams are improved. Generally, three models can be adopted to define the variation of material properties along the beam dimensions (thickness, width and length) and these models are exponential, power-law and sigmoid.

Generally, the classical beam theories are used to investigate the static and dynamic problems of beams and the main assumptions of these theories are the uniform area and homogeneous material of the beam. Several applications require enhancing the responses of the beam under various static and dynamic loads, and this enhancement in the static and dynamic responses of the beam is done by improving the material properties. For homogeneous and nonhomogeneous beams, several studies used Rayleigh Method (RM) to determine the frequency and deflection of uniform and non-prismatic beams (Abdulsamad *et al.* 2021; Al-Ansari 2012;2013; Al-Ansari *et al.* 2018;2019; Diwan *et al.* 2019).

When the material properties changed in thickness-direction, several studies estimated the deflection of uniform beam using analytical and numerical methods (Chakraborty *et al.* 2003; Gayen *et al.* 2021; Gayen 2022; Helal 2020; Li 2008; Nie *et al.* 2013; Şimşek *et al.* 2013; Zainy *et al.* 2018; Zhong and Yu 2007; Zhu and Sankar 2004). On the other hand, several researchers studied the effect of crack on static or dynamic behaviors of functionally graded beams or shafts (Gayen *et al.* 2018; 2020a 2020b). For example, Gayen (2022) introduced "an exact solution for thermo-elastic behavior of radially functionally graded hollow shafts" assuming different models to describe the material properties in the radial direction such as linear law, power law and exponential law models. By using a linear strain-displacement relations and steady-state Fourier equation of heat conduction, he solved analytically thermo-elastic equations to obtain displacement and stress fields as functions of radial distances, material gradient indices, and temperature gradients. Gayen *et al.* (2020a) used aluminum oxide (Al_2O_3) and stainless steel to represent the radially Functionally Graded shaft appling power law model. They studied the effects of temperature gradient and gradient index in addition to crack size and crack orientation on the direct and cross-couple local flexibility coefficients in a cracked structure. Gayen *et al.* (2018) represented the local flexibility coefficients as a function of crack depth using finite element method to investigate the effects of crack depth, crack position, slenderness ratio, thermal gradient and gradient parameter on the free vibration of the cracked radially FG shaft. Gayen *et al.* (2020b) studied the effect of transverse surface crack in addition to temperature and material gradients on the vibration of radially functionally graded rotating shafts considering nonlinear material properties applying finite element formulation and basing on Timoshenko beam theory. They used power law model to describe the material properties for two types: FG shafts (FGM I (SS/Al_2O_3) and FGM II (SS/ZrO_2)).

Shahba *et al.* (2011) applied Timoshenko beam theory to investigate the vibration behavior of tapered axial-FG beam. While Nguyen, N-T *et al.* (2014) used Euler–Bernoulli beam theory to study the static transverse deflection of thickness functionally graded beam (T-FGB) and axially-functionally graded beam (A-FGB) with non-uniform area, and they introduced the evaluation between the results calculated by the new model besides that of finite element model. Nguyen *et al.* (2013) calculated a large deflection of tapered cantilever axial-FG beam using the finite elements technique and they investigated the impact of slenderness ratio and non-uniformity (type and ratio) on the transverse deflection. Rajasekaran and Khaniki (2019) based on nonlocal strain gradient theory and used the finite-element method for studying the mechanical behaviors of non-uniform size dependent axial-FG beam with different types of materials. Lin *et al.* (2019) investigated the large deformation behavior of a cantilever axial-FG beam under load at a free end using the "homotopy analysis method". They adopted a power-law equation to represent the mechanical and physical properties and obtained the deformation solution of axial-functionally graded beams by applying the solution of the corresponding homogeneous beam as the initial guess. They observed a good convergence between the analytical results and the finite-element solutions. Soltani and Asgarian (2019) used power series technique and Timoshenko theory to develop a new solution for static and buckling behavior of axially-functionally graded beam (A-FGB). They studied the linear stability of non-prismatic cantilever A-FGB and they compared the new solution results with the results obtained by finite-element method and further solutions available. Mahmoud (2019) presented a general solution of the vibration behavior of a non-uniform axial-FG beam when the A-FG beam was loaded by masses at the free end of the beam. In Daikh *et al.* (2022), using a "new higher order shear deformation theory", researchers investigated the deflection and buckling stability of axially single-walled (SW) functionally graded (FG) carbon nanotubes reinforced composite (CNTRC) plates. Walaa (2021) studied the transverse deflection and free vibration of non-uniform axial-functionally graded beam with different boundary conditions using finite-element technique. The material properties of the axial-functionally graded beam vary using the power-law model.

Finally, in Karamanli and Thuc (2021), by applying the finite-element method and using both a quasi-3D and modified strain gradient theories, the structural problem of 2-D FG porous microbeams was investigated. They applied the "Hermite - cubic beam element" with various supporting types to develop and solve the differential equations of static deflection, buckling, and free vibration behaviors. On the other hand, Karamanli and Thuc (2018) used the finite-element technique to calculate the deformations of conventional functionally graded micro-beams with uniformly distributed load. They applied the power-law equation to represent the change of mechanical and physical properties for conventional functionally graded micro-beams.

In prior work, Wadi *et al.* (2022) applied the Rayleigh method to determine the static deflection of cantilever axial-FG beam under distributed and transverse tip loads. In this work, the Rayleigh method (RM) is modified to study the influence of different material properties of the axial-FG beam along the length of a simply supported beam. The influence of material distribution, modulus ratio (ME-Ratio) and load type on transverse displacement are investigated and the results estimated by Rayleigh method are evaluated to the finite-element results, technique by employing "BEAM" element.

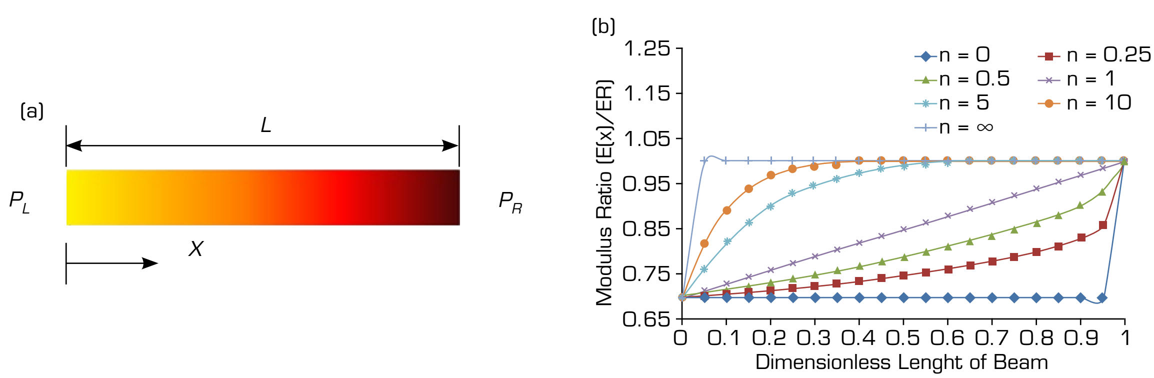
PROBLEM DESCRIPTION AND MATERIALS

In FGM, the material properties are changed along the length of beam using three general equations and these equations are exponential, sigmoid and power-law (Wadi *et al.* 2022; Walaa 2021).

Now, the power-law equation (Eq. 1) is adopted to define the material distribution in axial direction and the mechanical and physical properties can be described in Fig. 1:

$$P(x) = \left(P_L - P_R\right) * \left(1 - \left(\frac{x}{L}\right)\right)^n + P_R \quad (1)$$

Where: P_L and P_R are the properties at the left and right beam end, respectively; *P(x)* is the property at any point along the length of the beam; (x) is the position of the point starting from the left side of the beam; (L) is the length of the axial-functionally graded beam, and (n) is the power-law index. Generally, the deflection of any axial-FGB affects the "stiffness" of this beam, and the "stiffness" of FGB is the multiplication of "modulus of the elasticity (E) and second moment of area (I)". In axial-FGB, the beam modulus changes in axial-direction, therefore, the stiffness of FGB changes too. The deflection of an axial-functionally graded beam is affected by the supporting conditions, type of applied load, modulus ratio of the parents and power-law index.



Source: Elaborated by the authors.

Figure 1. Variation of Material Properties in Axial Direction.

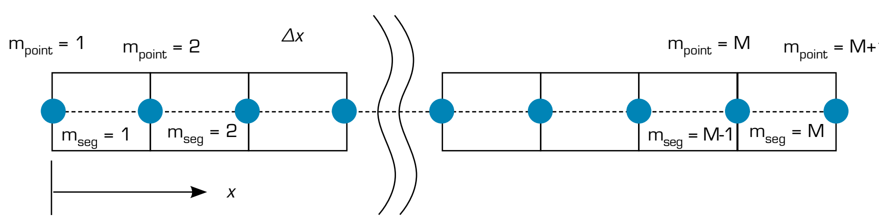
RAYLEIGH METHOD

The non-uniformity area and non-homogenous mechanical and physical properties cause a variation in stiffness of beam, and the calculation of "equivalent stiffness" of uniform simply supported axial-functionally graded beam is the main challenge in Rayleigh method (RM). Firstly, the equivalent stiffness of the axial-FGB is estimated and then the deflection at any point in the axial-functionally graded beam is determined. The following points are used to determine the deflection:

- The axial-FGB is divided into (M) segments and the number of points is (M+1).
- The elastic modulus and Poisson's ratio at each point (m_{point}) using Eqs. 2 and 3 when the location of any point in axial direction is determined by (x = Δx*i) where (Δx=L/M) and i=1,2,3,….M+1 (see Fig. 2).

$$E(x) = \left(E_L - E_R\right) * \left(1 - \left(\frac{x}{L}\right)\right)^n + E_R$$
$$E(i) = \left(E_L - E_R\right) * \left(1 - \left(\frac{\Delta x * i}{L}\right)\right)^n + E_R \,;\, i = 1, 2, 3, \ldots\ldots M + 1 \quad (2)$$

$$\nu(x) = \left(\nu_L - \nu_R\right) * \left(1 - \left(\frac{x}{L}\right)\right)^n + \nu_R$$
$$\nu(i) = \left(\nu_L - \nu_R\right) * \left(1 - \left(\frac{\Delta x * i}{L}\right)\right)^n + \nu_R ; i = 1, 2, 3, \ldots\ldots M + 1 \quad (3)$$



Source: Elaborated by the authors.

Figure 2. The Division and Numbering of A-FG Beam.

- In each segment, the elastic modulus and Poisson's ratio are estimated in Eq. 4 and 5:

$$E\left(x + \left(\frac{\Delta x}{2}\right)\right) = \frac{(E(x)+E(x+\Delta x))}{2}$$
$$E(j) = \frac{(E(i)+E(i+1))}{2} \,;\, j = 1, 2, 3, \ldots\ldots M \quad (4)$$
$$\nu\left(x + \left(\frac{\Delta x}{2}\right)\right) = \frac{(\nu(x)+\nu(x+\Delta x))}{2}$$

$$\nu(j) = \frac{(\nu(i)+\nu(i+1))}{2} \,;\, j = 1, 2, 3, \ldots\ldots M \quad (5)$$

The equivalent stiffness of the simply supported axial-functionally graded beam is determined by:

- The center of the area of the axial-functionally graded beam (X_c) is determined using the Eq. 6:

$$X_c = \frac{\sum_{j=1}^{M} X_j * A_j}{\sum_{j=1}^{M} A_j} \tag{6}$$

Where A_j is the cross section area of (j) segment. The center of axial-functionally graded beam in this work is (0.5*L), because it has a uniform area.

- The simply supported axial-functionally graded beam is divided into two cantilever beams as shown in Fig. 3.
- The "equivalent stiffness" of free-clamped and clamped-free axial-FG beams are estimated depending on the supporting type (Clamped – Free or Free - Clamped A-FG beam) using Eq 7 and 8 (for more details, see Abdulsamad *et al.* (2021), Al-Ansari (2012; 2013), Al-Ansari *et al.* (2018; 2019), Diwan *et al.* (2019), Wadi *et al.* (2022), Zainy *et al.* (2018)):

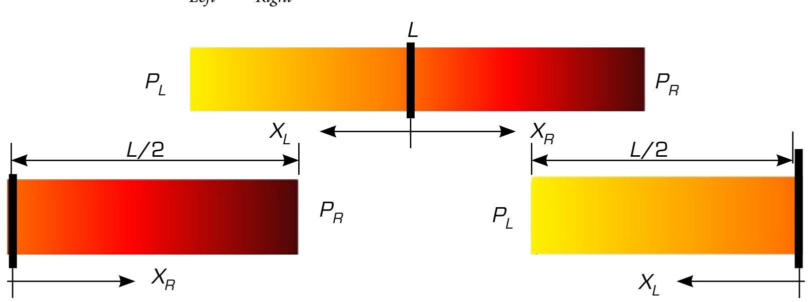
$$\left((EI)_{eq}\right)_L = \frac{(L_{Left})^3}{\sum_{k=1}^{M_{Left}} \frac{(L_k)^3 - (L_{k-1})^3}{(EI)_k}} \; ; K = 1, 2, \ldots . M_{Left} \tag{7}$$

$$\left((EI)_{eq}\right)_R = \frac{(L_{Right})^3}{\sum_{k=1}^{M_{Right}} \frac{(L_k)^3 - (L_{k-1})^3}{(EI)_k}} \; ; K = 1, 2, \ldots . M_{Right} \tag{8}$$

The total equivalent stiffness is (Eq. 9):

$$(EI)_{eq} = \frac{\left(L_{Right} + L_{Left}\right) * \left(L_{Right}\right)^2 * \left(L_{Left}\right)^2}{\left(\left(\sum_{K=1}^{M_{Right}} \frac{l_{K-1}^{\;3} - l_K^{\;3}}{I_K}\right) * L_{Right}^{\;2}\right) + \left(\left(\sum_{K=1}^{M_{Left}} \frac{l_K^{\;3} - l_{K-1}^{\;3}}{I_K}\right) * L_{Left}^{\;2}\right)} \tag{9}$$

Where: $L_{Left} + L_{Right} = L$ and in this work $L_{Left} + L_{Right} = L/2$



Source: Elaborated by the authors.

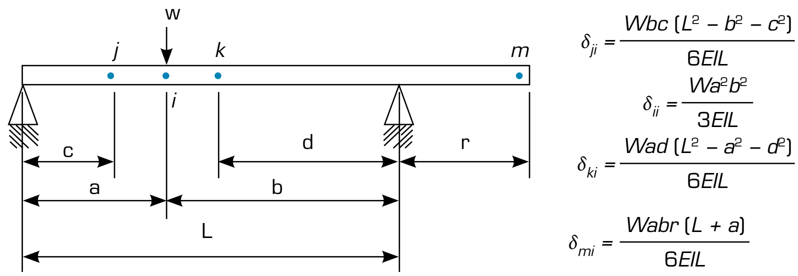
Figure 3. Dividing the Simply Supported A-FG Beam From the Center of Area into Two Cantilever A-FG Beams.

- The deflections of simply supported axial-functionally graded beam are estimated using the Eq. 10:

$$[Y] = [\delta][F] \tag{10}$$

Where: $[Y]$ is the deflection matrix, $[\delta]$ is delta matrix using Fig. 4 to estimate this matrix assuming W=1, and $[Y] = f_i$ *where* $i = 1, 2, 3, \ldots M + 1$ is the force matrix and when the distributed load (ω) (N/m) is applied, the values of f_i are calculated in Eq. 11:

$$f_i = \{\frac{\omega^* \Delta x}{2}\ when\ i\ =\ 1\ and\ i\ =\ M\ +\ 1\ \ \omega\ *\ \Delta x\ when\ i{\neq}1\ and\ i{\neq}M\ +\ 1 \tag{11}$$



Source: Retrieved from Walaa (2021).

Figure 4. Deflections Formula of Simply Supported Beam at Different Points.

The calculating procedure, described in this section, is programmed using FORTRAN Power Station.

FINITE ELEMENT MODEL

In this work, the commercial software ANSYS APDL is used to simulate the simply supported A-FG beam to study the static deflection under the uniform-distributed load. The "BEAM189" is used and the properties and characteristics of this element is "The BEAM189 element is suitable for analyzing slender to moderately stubby/thick beam structures. The element is based on Timoshenko beam theory which includes shear-deformation effects. The element provides options for unrestrained warping and restrained warping of cross-sections. The element is a quadratic three-node beam element in 3-D. With default settings, six degrees of freedom occur at each node; these include translations in the x, y, and z directions and rotations about the x, y, and z directions. An optional seventh degree of freedom (warping magnitude) is available. The element is well-suited for linear, large rotation, and/or large-strain nonlinear applications" (ANSYS, Inc., 2016, ANSYS Version 17.2).

In ANSYS model, the axial-functionally graded beam is divided into 20 parts and the part is also divided into 5 elements to apply the convergence criteria discussed by Wadi *et al.* (2022). The A-FG beam in ANSYS software is drawn using twenty-one key points and the twenty lines which represent twenty segments. The elastic modulus and Poisson ratio of each segment are calculated using Eq. 4 and 5 (i.e. M=20 in ANSYS model).

Accuracy of the Present Models:

In order to exam the accuracy of Rayleigh and FE models, was there a comparison between the non-dimensional deflection estimated by Rayleigh and FE models and that obtained by Walaa (2021). Walaa (2021) applied 2-D FE model using ANSYS Workbench.

In this study, dimensions of axial-FGB and mechanical properties are listed in Table 1 and 2 respectively. The non-dimensional static deflection can be determined by the Eq. 12:

$$Y(x) = \frac{y(x)}{\left(y_{max.}\right)_{at\ n=0}} \tag{12}$$

Where: $Y(x)$ is the non-dimensional static deflection at any point (x) in axial-direction. $y(x)$ is the static deflection at any point (x) in axial-direction. $(y_{max.})_{at\ n=0}$ is the maximum static deflection at zero power index (n=0).

Table 1. The Used Dimensions of the Axial-FGB.

No.	Dimension	Magnitude
1	Length of Beam (l)	1 meter
2	Width of Beam (W)	0.01 meter
3	Thickness of Beam (h)	0.01 meter

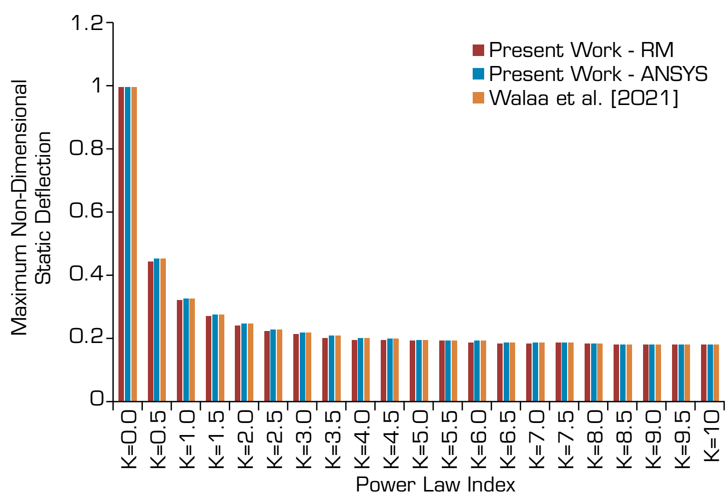
Source: Elaborated by the authors.

Table 2. The Mechanical Properties of Left and Right Materials of the Axial-FGB.

Material	Property	Magnitude
Metal (Right Material)	Elastic Modulus of (E_R)	210 G Pa.
	Poisson Ratio (ν_R)	0.3
	Density (ρ_R)	7800 kg/m³
Ceramic (Left Material)	Elastic Modulus (E_L)	390 G Pa.
	Poisson Ratio (ν_L)	0.23
	Density (ρ_L)	3960 kg/m³

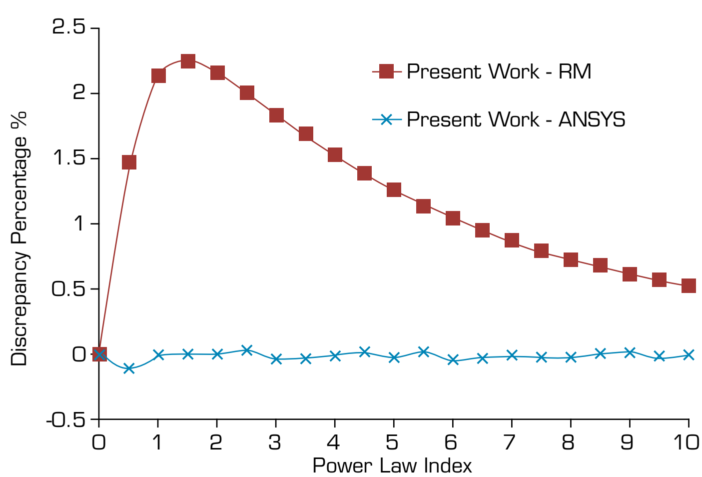
Source: Retrieved from Walaa (2021).

Figure 5 illustrates the comparison between the maximum non-dimensional static deflection estimated by present models and that obtained by Walaa (2021), while Fig. 6 shows the discrepancy percentage of the Rayleigh and 1D ANSYS results with respect to Walaa (2021) results. From Fig. 5 and 6, an excellent agreement is found between the results of Rayleigh method and that calculated by ANSYS software: the maximum discrepancy percentage is smaller than 2.5 % when the power-law index is approximately 1.5 and then the discrepancy percentage decreases with increasing power-law index (i.e. when the power-law index increase, the A-FG beam tend to be pure ceramic).



Source: Elaborated by the authors.

Figure 5. The Comparison Between the Non-Dimensional Static Deflection of the Present Model and That Calculated by Walaa (2021).



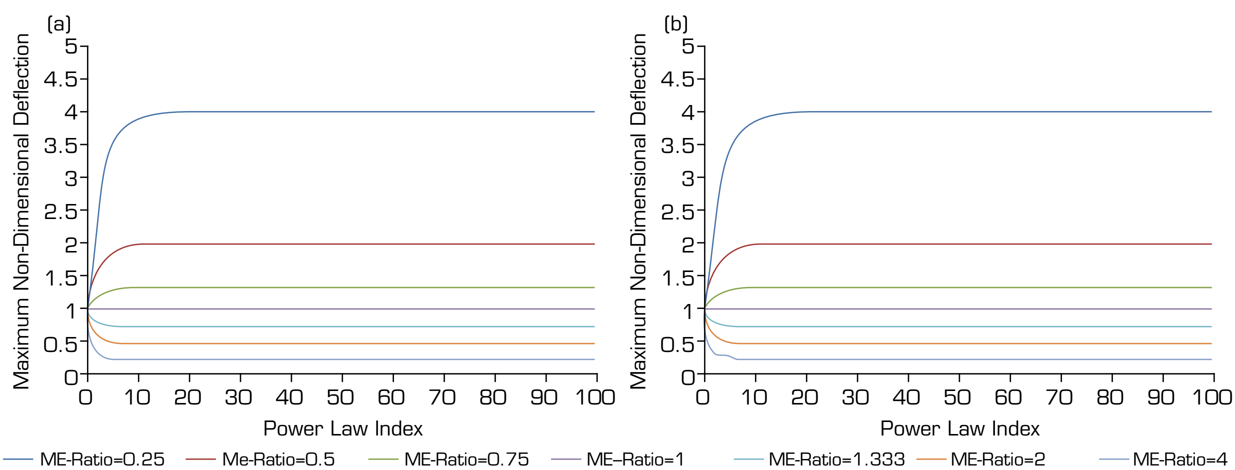
Source: Elaborated by the authors.

Figure 6. The Discrepancy Percentage of the Present Model with Respect to Results of Walaa (2021).

RESULTS AND DISCUSSION

In addition to the accurateness of the present numerical models, the influences of two important parameters (modulus-ratio (ME-Ratio) and power-law index (material distribution index)) on the non-dimensional static deflection of a simply supported A-FG beam are studied in this work. The power-law index (n) refers to the material distribution along the A-FG beam and it changes from (0 to 100). Modulus ratio (ME-Ratio) refers to the ratio of the elastic moduli of the parents of A-FG beam (i.e. E_L/E_R), in other words, it also refers to materials positions at the two ends of the A-FG beam. The values of modulus ratio (ME-Ratio), used in this work, are (0.25, 0.5, 0.75, 1, 1.333, 2 and 4).

Figure 7 illustrates the effect of power-law index (n) on the maximum non-dimensional static deflection of a simply supported A-FG beam at a different elastic moduli ratio (ME-Ratio). Generally, the maximum non-dimensional static deflection changes sharply when the power-law index is smaller than (10). This change in maximum non-dimensional static deflection is affected by the value of elastic moduli ratio (ME-Ratio). When the elastic moduli ratio (ME-Ratio) equals 1, the A-FG beam has a homogenous properties (i.e. pure material), in this case, the maximum non-dimensional static deflection is constant because the effective modulus of FG beam is constant. If the elastic moduli ratio (ME-Ratio) is smaller than 1 (i.e. $E_R>E_L$) , the maximum non-dimensional static deflection increases with increase of the power law index (n), because the effective modulus of FG beam decreases with the increasing of the power law index (n) and equal to E_R at n>10 (i.e. the effective modulus $\rightarrow E_R$). But, the maximum non-dimensional static deflection decreases with the increase of the power law index (n) when the elastic moduli ratio (ME-Ratio) is more than 1 (i.e. $E_R<E_L$).

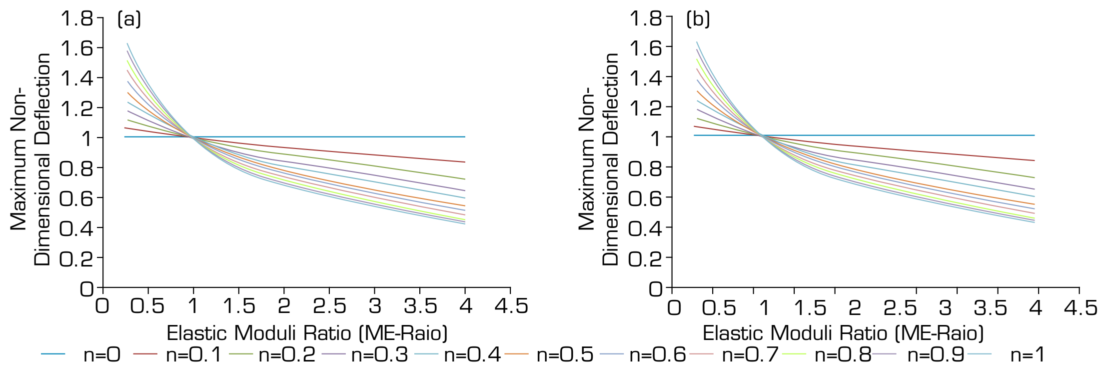


Source: Elaborated by the authors.

Figure 7. The Variation of the Maximum Non-Dimensional Static Deflection Due to an Increase in the Material Distribution Index (n) at Different Elastic Moduli Ratio (ME-Ratio) for the Present Models (a) Rayleigh Method (RM); (b) ANSYS- 1D Model.

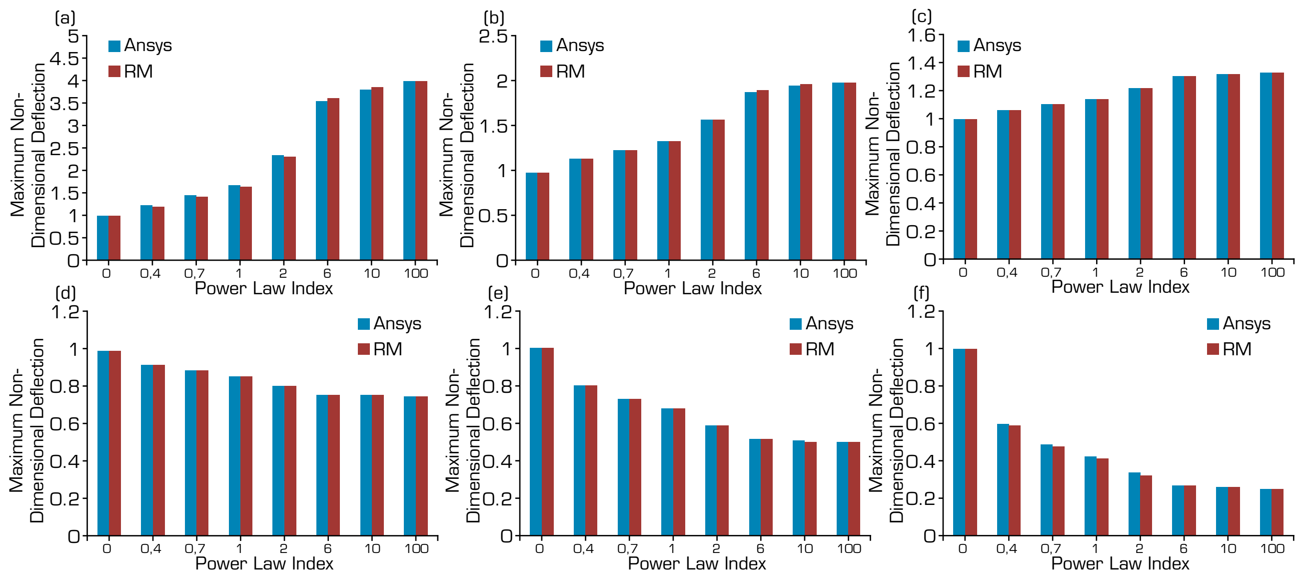
When power-law index equals zero (i.e. the material of axial-FGB is pure right material) (see Eq. 1), the maximum non-dimensional static deflection is not affected by the elastic modulus ratio (ME-Ratio) because the increase of elastic modulus ratio (ME-Ratio) is made by increasing the modulus of left material (E_L). The maximum non-dimensional static deflection decreases with the increase of the elastic moduli ratio (ME-Ratio) for any power-law index (n,) and the rate of decreasing of maximum non-dimensional static deflection reduces with increasing power-law index (n) as displayed in Fig. 8.

For more comparison between the present models (ANSYS- 1D Model and Rayleigh Method (RM)), Fig. 9 illustrates the convergence between the results of non- dimensional static deflection calculated be the two models.



Source: Elaborated by the authors.

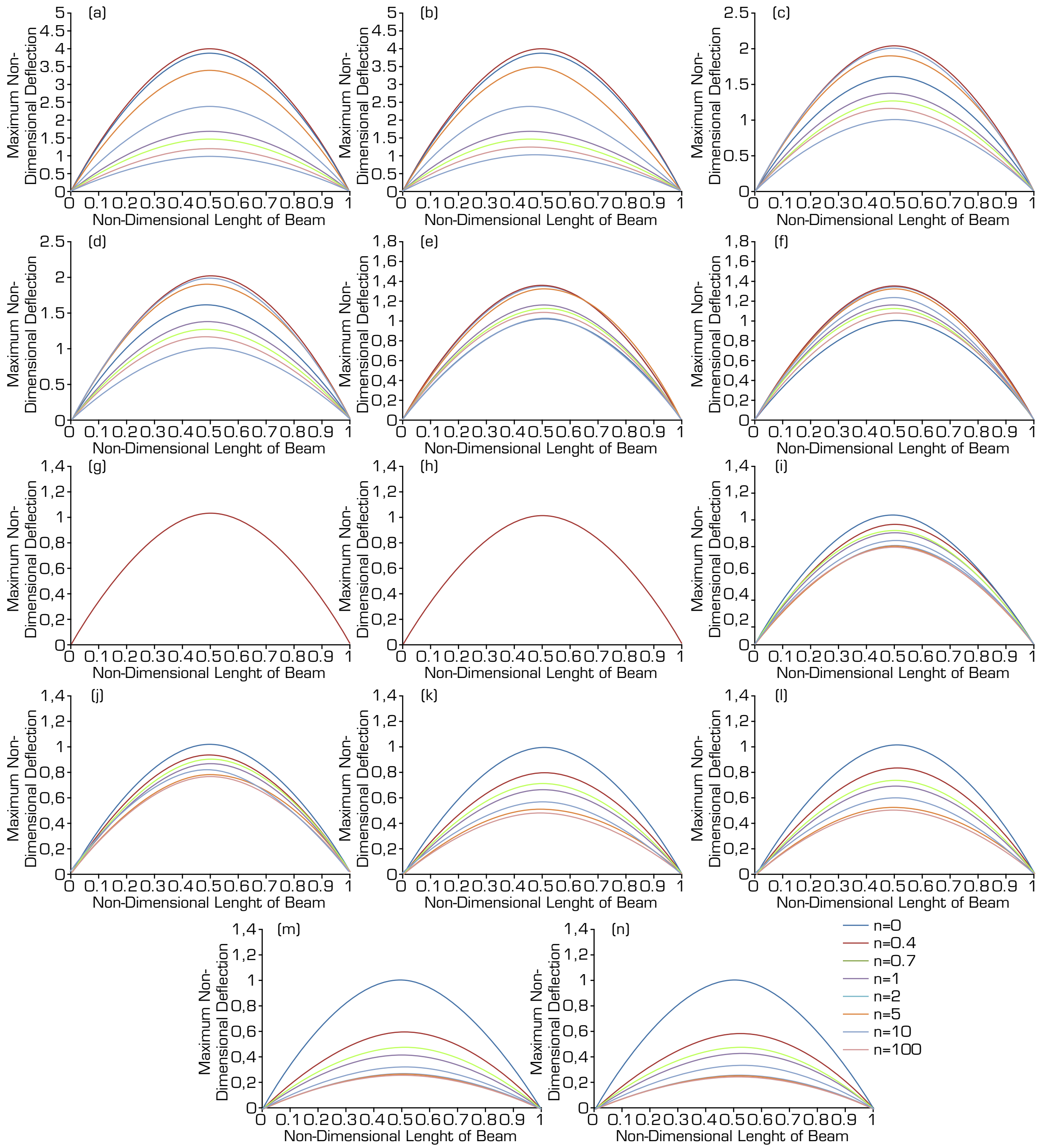
Figure 8. The Variation of the Maximum Non-Dimensional Static Deflection Due to increase Elastic Moduli Ratio (ME-Ratio) at Different Material Distribution Index (n) for the Present Models (a) Rayleigh Method (RM); (b) ANSYS- 1D Model.



Source: Elaborated by the authors.

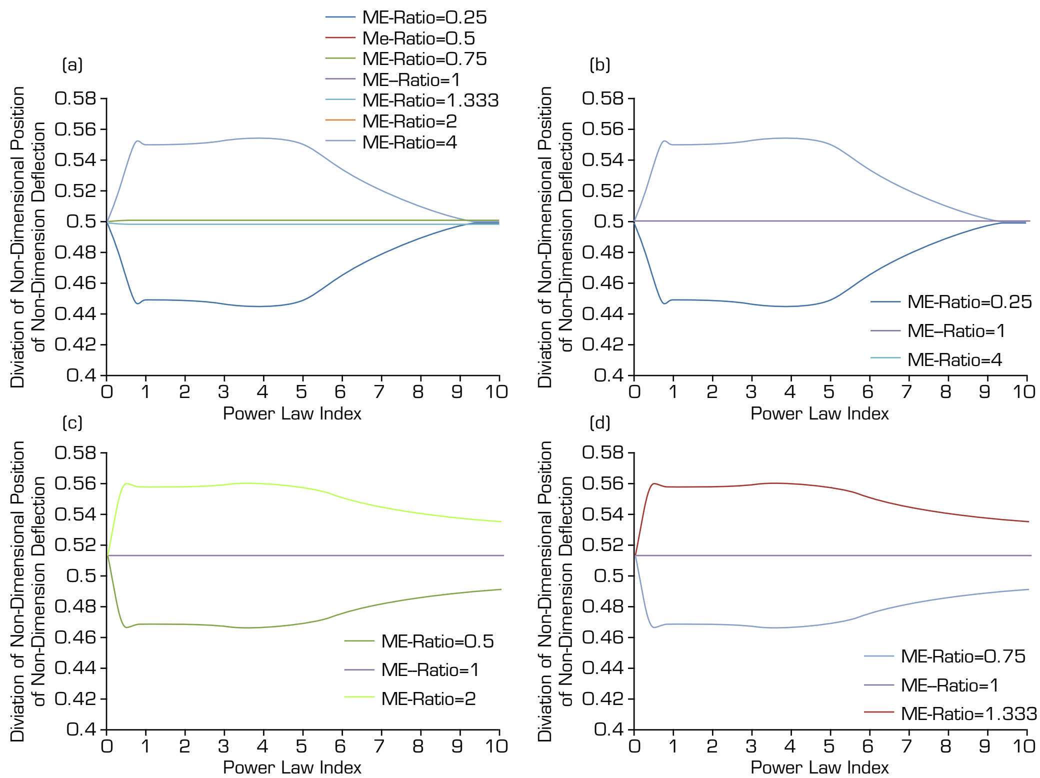
Figure 9. Comparison Between Non-Dimensional Static Deflections Calculated by ANSYS-1D Model and Rayleigh Method (RM) for Different Power-law Index (n) and Elastic Moduli Ratio (ME-Ratio). (a) ME-Ratio = 0.25; (b) ME-Ratio = 0.5; (c) ME-Ratio = 0.75; (d) ME-Ratio = 1.333; (e) ME-Ratio = 2; (f) ME-Ratio = 4.

Figure 10 shows the profile of non-dimensional static deflection at any point on the axial direction for different elastic moduli ratio (ME-Ratio) and power-law index (n). Similar notes can be seen for the non-dimensional static deflection at any point in the axial-direction of axial-FGB comparing with the curves of maximum non-dimensional static deflection (i.e. Fig. 7 and 8). When the elastic moduli ratio (ME-Ratio) equals 1 or the power-law index (n) equals zero, the axial-FGB has homogenous properties (i.e. pure material) and the non-dimensional static deflection is constant. If (E_R>E_L), the non-dimensional static deflection increases due to an increase of the power-law index (n) while the non-dimensional static deflection decreases with an increase of the power-law index (n), if (E_R<E_L). The non-dimensional static deflection increases with the decrease of the elastic moduli ratio (ME-Ratio) for any power-law index (n) and the rate of decreasing of non-dimensional static deflection reduces with increasing power-law index (n). But the position of the maximum non-dimensional static deflection calculated by ANSYS 1D model does not appear at the mid span of A-FG beam and there is a small deviation in position of maximum non-dimensional static deflection. This deviation in position of maximum non-dimensional static deflection is shown in Fig. 11 and it decreases when the power-law index increases. Also, the deviation ME-Ratio =0.25 is similar to the deviation when ME-Ratio =4. This symmetrical deviation profile is also found for each value of elastic moduli ratio and its inverse (i.e. ME-Ratio and (1/ ME-Ratio)). There is no deviation in position of maximum non-dimensional static deflection when ME-Ratio =1, because the beam is homogenous and uniform at the same time. The value of the deviation increases when the elastic moduli ratio (ME-Ratio) diverges from 1. Finally, the deviation in position of maximum non-dimensional static deflection is not found in the results of Rayleigh Method.



Source: Elaborated by the authors.

Figure 10. Comparison Between Non-Dimensional Static Deflections Along the Dimension Length of FG Beam Calculated by ANSYS- 1D Model and Rayleigh Method (RM) for Different Power-law Index (n) and Elastic Moduli Ratio (ME-Ratio). (a) Rayleigh Method (RM), ME-Ratio = 0.25; (b) ANSYS- 1D Model, ME-Ratio = 0.25; (c) Rayleigh Method (RM), ME-Ratio = 0.5; (d) ANSYS- 1D Model, ME-Ratio = 0.5; (e) Rayleigh Method (RM), ME-Ratio = 0.75; (f) ANSYS- 1D Model, ME-Ratio = 0.75; (g) Rayleigh Method (RM), ME-Ratio = 1; (h) ANSYS- 1D Model, ME-Ratio = 1; (i) Rayleigh Method (RM), ME-Ratio = 1.333; (j) ANSYS- 1D Model, ME-Ratio = 1.333; (k) Rayleigh Method (RM), ME-Ratio = 2; (l) ANSYS- 1D Model, ME-Ratio = 2; (m) Rayleigh Method (RM), ME-Ratio = 4; (n) ANSYS- 1D Model, ME-Ratio = 4.



Source: Elaborated by the authors.

Figure 11. Deviation of Non-Dimensional Position of Maximum Non-Dimensional Static Deflection Calculated by ANSYS-1 D Model at Different Power-law Index (n) and Elastic Moduli Ratio (ME-Ratio). (a) Total ME-Ratio; (b) ME-Ratio=0.25,1 and 4; (c) ME-Ratio=0.5,1 and 2; (d) ME-Ratio=0.75,1 and 1.333.

CONCLUSIONS

In this work, two numerical models are built for the studying of the deflection of a simply supported axial-functionally graded beam, the Rayleigh method is adopted to build the first model (new one) while the second model used the finite element technique to build 1 D – model utilizing the ANSYS APDL. In addition to the accuracy of the two models, the effects of elastic moduli ratio (ME-Ratio) and material distribution (n) on the maximum static deflection and its position are studied. From the figures, these points are concluded:

- The new model based on Rayleigh method gives results with an excellent agreement with the results of ANSYS– 1D model and available literatures;
- The A-FG beam has homogenous properties when the elastic moduli ratio (ME-Ratio) equals 1 or the power-law index (n) equal zero and in this case, the non-dimensional static deflection and its maximum are constant;
- The non-dimensional static deflection and its maximum decrease with the increase of the power-law index (n) when ($E_R<E_L$). If ($E_R>E_L$), the non-dimensional static deflection and its maximum increase with the increase of the power-law index (n);
- The non-dimensional static deflection increases with the decrease of the elastic moduli ratio (ME-Ratio) for any power-law index (n) and the rate of the decreasing of non-dimensional static deflection reduces with increasing power-law index (n);
- In ANSYS– 1D model only, the position of the maximum non-dimensional static deflection is deviated from the mid span of A-FG beam and this deviation in position of maximum non-dimensional static deflection decreases when the power-law index increases and increases when the elastic moduli ratio (ME-Ratio) diverges from 1;

- The deviation in position of maximum non-dimensional static deflection is not found in the results of the Rayleigh method.

In future work, the deflection and dynamic behavior of non-prismatic simply supported axial-functionally graded beams will be investigated considering the effect of symmetrical and unsymmetrical boundary conditions.

CONFLICT OF INTEREST

Nothing to declare.

DATA AVAILABILITY STATEMENT

All dataset were generated or analyzed in the current study.

AUTHOR CONTRIBUTIONS

Conceptualization: Shukur ZM; **Formal analysis:** Hamad RF, Ali YK; **Research:** Hamad RF, Al-Karaishi M; **Methodology:** Al-Ansari LS, Shukur ZM; **Software:** Al -Ansari LS; **Supervision:** Al-Ansari LS, Shukur ZM; **Validation:** Al-Ansari LS, Shukur ZM; **Writing - Preparation of original draft:** Al-Ansari LS, Shukur ZM; **Writing - Proofreading and editing:** Al-Ansari LS, Shukur ZM.

FUNDING

Not applicable.

ACKNOWLEDGMENTS

Not applicable.

REFERENCES

Abdulsamad HJ, Wadi KJ, Al-Raheem SK, Al-Ansari LS (2021) Investigation of Static Deflection in Internal Stepped Cantilever Beam. Journal of Mechanical Engineering Research and Developments 44(5):87-125.

Al-Ansari LS (2012) Calculating of Natural Frequency of Stepping Cantilever Beam. Int J Mech Mechatron Eng 12(5):59-68.

Al-Ansari LS (2013) Calculating Static Deflection and Natural Frequency of Stepped Cantilever Beam Using Modified Rayleigh Method. IJMPERD 3(4):107-118.

Al-Ansari LS, Al-Hajjar AMH, Jawad H (2018) Calculating the natural frequency of cantilever tapered beam using classical Rayleigh, modified Rayleigh and finite element methods. Int J Eng Technol 7(4):4866-4872.

Al-Ansari LS, Zainy HZ, Yaseen AA, Aljanabi M (2019) Calculating the natural frequency of hollow stepped cantilever beam. Int J Mech Eng Technol 10(1):898-914.

Ashby MF (2000) Materials Selection in Mechanical Design. Oxford: Butterworth-Heinemann.

Bhavar V, Kattire P, Thakare S, Patil S, Singh RKP (2017) A Review on Functionally Gradient Materials (FGMs) and Their Applications. IOP Conf Ser: Mater Sci Eng 229(1):012021. https://doi.org/10.1088/1757-899X/229/1/012021

Chakraborty A, Gopalakrishnan S, Reddy JN (2003) A new beam finite element for the analysis of functionally graded materials. Int J Mech Sci 45(3):519-539. https://doi.org/10.1016/S0020-7403(03)00058-4

Daikh AA, Houari MSA, Belarbi MO, Chakraverty S, Eltaher MA (2022) Analysis of Axially Temperature-dependent Functionally Graded Carbon Nanotube Reinforced Composite Plates. Engineering with Computers 38(Suppl 3):2533-2554. https://doi.org/10.1007/s00366-021-01413-8

Diwan AA, Al-Ansari LS, Al-Saffar AA, Al-Anssari QS (2022) Experimental and theoretical investigation of static deflection and natural frequency of stepped cantilever beam. Aust J Mech Eng 20(2):303-315. https://doi.org/10.1080/14484846.2019.1704494

Ebhota WS, Jen T-C (2018) Casting and Applications of Functionally Graded Metal Matrix Composites. In: Vijayaram TR, editors. Advanced Casting Technologies. InTech. https://doi.org/10.5772/intechopen.71225

El-Galy IM, Bassiouny BI, Ahmed MH (2018) Empirical model for dry sliding wear behaviour of centrifugally cast functionally graded Al/SiCp composite. Key Eng Mater 786:276-285. https://doi.org/10.4028/www.scientific.net/KEM.786.276

Gayen D, Chakraborty D, Tiwari R (2018) Free Vibration Analysis of Functionally Graded Shaft System with a Surface Crack. J Vib Eng Technol 6:483-494. https://doi.org/10.1007/s42417-018-0065-9

Gayen D, Chakraborty D, Tiwari R (2020a) Determination of Local Flexibility Coefficients of a Functionally Graded Shaft with Breathing Crack. In: Dutta S, Inan E, Dwivedy S editors. Advances in Rotor Dynamics, Control, and Structural Health Monitoring. Lecture Notes in Mechanical Engineering. Singapore: Springer. https://doi.org/10.1007/978-981-15-5693-7_13

Gayen D, Chakraborty D, Tiwari R (2020b) Transverse Vibration and Stability of a Cracked Functionally Graded Rotating Shaft System. In: Li L, Pratihar D, Chakrabarty S, Mishra P, editors. Advances in Materials and Manufacturing Engineering. Lecture Notes in Mechanical Engineering. Singapore: Springer. https://doi.org/10.1007/978-981-15-1307-7_71

Gayen D, Tiwari R, Chakraborty D (2021) Thermo-Mechanical Analysis of a Rotor-Bearing System Having a Functionally Graded Shaft with Transverse Breathing Cracks. In: Rao JS, Arun Kumar V, Jana S editors. Proceedings of the 6th National Symposium on Rotor Dynamics. Lecture Notes in Mechanical Engineering. Singapore: Springer. https://doi.org/10.1007/978-981-15-5701-9_8

Gayen D (2022) Analysis of Temperature, Displacement, and Stress in Shafts Made of Functionally Graded Materials with Various Grading Laws. Adv Eng Mater 24(5):2101328. https://doi.org/10.1002/adem.202101328

Harris B (1999) Engineering Composite Materials. London: Institute of Materials.

Helal SHB (2020) The Static Analysis of a Functionally Graded Euler Beam under Mechanical Loads (Master Thesis). Kufa: University of Kufa.

Jamian S (2012) Application of functionally graded materials for severe Plastic Deformation and Smart Materials: Experimental Study and Finite Element Analysis (Doctoral dissertation). Nagoya: Nagoya Institute of Technology.

Kalyan V, Swamy SB, Snehith N, Karthik UPS, Ommi NS (2017) A review on application of polymers in mechanical engineering. Int J Adv Res Sci Eng 6(11):1972-1979.

Karamanli A, Thuc PV (2018) Size dependent bending analysis of two directional functionally graded microbeams via a quasi-3D theory and finite element method. Compos B Eng 144:171-183. https://doi.org/10.1016/j.compositesb.2018.02.030

Karamanli A, Thuc PV (2021) Bending, vibration, buckling analysis of bidirectional FG porous microbeams with a variable material length scale parameter. Appl Math Model 91:723-748. https://doi.org/10.1016/j.apm.2020.09.058

Kassner ME, Smith KK, Campbell CS (2015) Low-temperature creep in pure metals and alloys. J Mater Sci 50:6539-6551. https://doi.org/10.1007/s10853-015-9219-2

Kaw AK (2006) Mechanics of Composite Material. Boca Raton: Taylor & Francis Group.

Li X-F (2008) A unified approach for analyzing static and dynamic behaviors of functionally graded Timoshenko and Euler–Bernoulli beams. J Sound Vibr 318(4-5):1210-1229. https://doi.org/10.1016/j.jsv.2008.04.056

Lin X, Huang Y, Zhao Y, Wang T (2019) Large deformation analysis of a cantilever beam made of axially functionally graded material by homotopy analysis method. Appl Math Mech 40:1375-1386. https://doi.org/10.1007/s10483-019-2515-9

Mahmoud MA (2019) Natural frequency of axially functionally graded, tapered cantilever beams with tip masses. Eng Struct 187:34-42. https://doi.org/10.1016/j.engstruct.2019.02.043

Majeed T, Wahid MA, Sharma N (2017) Ceramic materials: processing, joining and applications. Paper presented at International Conference On Communication & Computational Technologies by RIET, Jaipur & IJCRT.ORG. Jaipur, India.

Nguyen DK (2013) Large displacement response of tapered cantilever beams made of axially functionally graded material. Composites: Part B 55:298-305. https://doi.org/10.1016/j.compositesb.2013.06.024

Nguyen N-T, Kim N-I, Cho I, Phung QT, Lee J (2014) Static analysis of transversely or axially functionally graded tapered beams. Materials Research Innovations 18(Suppl 2):260-264. https://doi.org/10.1179/1432891714Z.000000000419

Nie GJ, Zhong Z, Chen S (2013) Analytical solution for a functionally graded beam with arbitrary graded material properties. Compos Pt B-Eng 44(1):274-282. https://doi.org/10.1016/j.compositesb.2012.05.029

Osswald TA (2010) Material Science of Polymers for Engineers. Ohio: Hanser Publications.

Rajasekaran S, Khaniki HB (2019) Finite element static and dynamic analysis of axially functionally graded nonuniform small-scale beams based on nonlocal strain gradient theory. Mech Adv Mater Struct 26(14):1245-1259. https://doi.org/10.1080/15376494.2018.1432797

Saleh B, Jiang J, Ma A, Song D, Yang D (2019) Effect of main parameters on the mechanical and wear behaviour of functionally graded materials by centrifugal casting: a review. Met Mater Int 25:1395-1409. https://doi.org/10.1007/s12540-019-00273-8

Saleh B, Fathi JJR, Al-hababi T, Xu Q, Wang L, Song D, Ma A (2020a) 30 Years of functionally graded materials: An overview of manufacturing methods, Applications and Future Challenges. Composites Part B 201:108376. https://doi.org/10.1016/j.compositesb.2020.108376.

Saleh B, Jiang J, Fathi R, Xu Q, Wang L, Ma A (2020b) Study of the microstructure and mechanical characteristics of AZ91–SiCp composites fabricated by stir casting. Arch Civ Mech Eng 20:71. https://doi.org/10.1007/s43452-020-00071-9

Saleh B, Jiang J, Xu Q, Fathi R, Ma A, Li Y, Wang L (2021) Statistical analysis of dry sliding wear process parameters for AZ91 alloy processed by RD-ECAP using response surface methodology. Met Mater Int 27:2879-2897. https://doi.org/10.1007/s12540-020-00624-w

Shahba A, Attarnejad R, Marvi MT, Hajilar S (2011) Free vibration and stability analysis of axially functionally graded tapered Timoshenko beams with classical and non-classical boundary conditions. Compos Part B 42(4):801-808. https://doi.org/10.1016/j.compositesb.2011.01.017

Shao W (2018) Research and Application of Engineering Ceramic Material Processing Technology. IOP Conf Ser: Mater Sci Eng 394(3):032030 https://doi.org/10.1088/1757-899X/394/3/032030

Şimşek M, Kocatürk T, Akbaş ŞD (2013) Static bending of a functionally graded microscale Timoshenko beam based on the modified coupling stress theory. Compos Struct 95:740-747. https://doi.org/10.1016/j.compstruct.2012.08.036

Soltani M, Asgarian B (2019) Finite element formulation for linear stability analysis of axially functionally graded nonprismatic Timoshenko beam. Int J Struct Stabil Dynam 19(2):1950002. https://doi.org/10.1142/S0219455419500020

Wadi KJ, Yadeem JM, Khazaal SM, Al-Ansari LS Abdulsamad HJ (2022) Static deflection calculation for axially FG cantilever beam under uniformly distributed and transverse tip loads. Res Eng 14:100395. https://doi.org/10.1016/j.rineng.2022.100395

Walaa MH (2021) Static and Dynamic Analysis of Axially FG Beam (doctoral dissertation). Kufa: University of Kufa.

Xu Q, Ma A, Li Y, Saleh B, Yuan Y, Jiang J, Ni, C (2019) Enhancement of mechanical properties and rolling formability in AZ91 alloy by RD-ECAP processing. Materials 12(21):3503. https://doi.org/10.3390/ma12213503.

Xu Q, Ma A, Saleh B, Li Y, Yuan Y, Jiang J, Ni C (2020) Enhancement of strength and ductility of SiC_p/AZ91 composites by RD-ECAP processing. Mater Sci Eng A 771:138579. https://doi.org/10.1016/j.msea.2019.138579

Zainy HZ, Al-Ansari LS, Al-Hajjar AM, Mahdi M, Shareef S (2018) Analytical and numerical approaches for calculating the static deflection of functionally graded beam under mechanical load. Int J Eng Technol 7(4):3889-3896.

Zhong Z, Yu T (2007) Analytical solution of a cantilever functionally graded beam. Compos Sci Technol 67(3-4):481-488. https://doi.org/10.1016/j.compscitech.2006.08.023

Zhu H, Sankar BV (2004) A combined Fourier series–Galerkin method for the analysis of functionally graded beams. J Appl Mech 71(3):421-424. https://doi.org/10.1115/1.1751184