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MATHEMATICAL MODELING OF A CONVECTIVE TEXTILE DRYING PROCESS

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Abstract - This study aims to develop a model that accurately represents the convective drying process of textile materials. The mathematical modeling was developed from energy and mass balances and, for the solution of the mathematical model, the technique of finite differences, in Cartesian coordinates, was used. It transforms the system of partial differential equations into a system of ordinary equations, with the unknowns, the temperature and humidity of both the air and the textile material. The simulation results were compared with experimental data obtained from the literature. In the statistical analysis the Shapiro-Wilk test was used to validate the model and, in all cases simulated, the results were p-values greater than 5 %, indicating normality of the data. The R-squared values were above 0.997 and the ratios $F_{calculated}/F_{simulated}$, at the 95 % confidence level, higher than five, indicating that the modeling was predictive in all simulations.
Keywords: Textile convective drying; Moisture distribution; Simulation.

INTRODUCTION

The textile industry is one of many industries that utilizes large volumes of water in the manufacturing process (Cerqueira *et al*., 2009). In the last three decades, the development of research related to drying has grown exponentially. In the textile industry, the drying process is one of the major cost elements among the textile finishing operations, directly affecting the specific energy consumption and the quality of the product; therefore, a proper understanding of drying is of great importance (Cay *et al*., 2010; Akyol *et al*., 2010a).

According to Efremov (2000), the finishing processes of textile material require repeated heat treatments, since more than 80% of all energy consumed in the production of textile material is due to the most common process and which consumes the most energy: the drying process. Drying is a complex process involving simultaneous coupled transient heat, mass and momentum transport (Haghi and Amanifard, 2008). During drying of the textile material, the water moves from areas of high humidity to low humidity areas, meaning that the exterior must be drier than the interior (Herrero and Watai Ponce, 1985).

The study of the finishing processes of textile material requires a more detailed investigation of the kinetics of such processes, due to the complexity of describing it due to the transient regime which is established (Efremov, 1999). Therefore, it is of great importance to develop models that best describe the drying process.

The convective drying process widely used in the textile industry consists of passing a hot air stream over the surface of the material to be dried. Air flow

*To whom correspondence should be addressed transfers heat to the material by forced convection and, at the same time, carries away evaporated water (Akyol *et al*., 2010b).

According to Cheng and Fan (2005), since 1980 various mathematical models have been proposed in this field. The simulation for drying textile material can be derived from the model solution for heat and mass transfer of textile material and air drying.

Many theoretical models have been proposed for drying of porous materials in agriculture, ceramics, food, pharmaceuticals, paper pulp, minerals, polymers and textile materials, and these models are based on conservation of mass, energy and momentum, consisting of two sets of differential equations with their respective initial and boundary conditions (Lee *et al*. 2002). According to Akyol *et al*. (2012), many studies are available in the literature on the investigation of drying or heat and mass transfer mechanisms in textile fibers. However, most of the research done the heat and mass transfer in textile materials is focused on simplified models, which usually neglect convection in the gas phase.

The aim of the present work was to model and mathematically simulate the convective drying process of textile material, validating the model obtained by comparison with experimental data obtained from the literature and statistical analysis.

MATERIALS AND METHODS

Modeling of the Convective Drying

The kinetics of drying can be described by drying curves and the drying rate, which differ according to the nature of the solids and drying conditions. These curves show periods of constant and decreasing drying rates. Obtaining generalized models which describe the drying of textile materials during these periods is of great interest since they can be used to predict the drying time, in different conditions of operation of the dryer, directly from the solution of the developed model. In tissue, the material thickness is small; therefore, in the modeling it will be considered that the diffusion process in tissue is fast and the controlling step in mass transfer will be diffusion in the outward film. In this case the drying rate, ψ, can be expressed in terms of a mass transfer coefficient, k_f, and of the difference in moisture in the gas, Y_{eq}, and within the fluid phase, Y:

$$\psi = k_f\left(Y_{eq}\left(T_S, X\right) - Y\right) \tag{1}$$

The coefficient k_f remains constant if the gas velocity and mass transfer area do not change during the drying process. The term Y_{eq} is the humidity of the air in equilibrium with the tissue moisture $\left(X\right)$ in the solid at temperature T_S.

For the mathematical formulation of the tissue drying model, the following considerations were made.

The air flows unidirectionally in the parallel direction of the textile material; the air behaves as an ideal gas in terms of modeling; the predominant step in mass transfer is diffusion in the outer film; conductive effects on the tissue are negligible compared to convective effects; the volume of the textile material remains constant; evaporation occurs only at the two faces of the tissue; the heat desorption of the textile-water system is equal to the vaporization heat of water; the humidity gradient varies only in the air flow direction; humidity of the tissue does not vary with thickness.

Based on these considerations, the following set of partial differential equations was obtained:

Mass balance of the water in air:

$$Y\frac{\partial \rho_G}{\partial t} + \rho_G\frac{\partial Y}{\partial t} = -v\rho_G\frac{\partial Y}{\partial z} - vY\frac{\partial \rho_G}{\partial z} + \frac{2\psi L_S}{A_{TG}} \tag{2}$$

where t is time, ρ_G is the density of dry gas, v is the velocity of the drying air, z is the space, $L_S = 0.15$ m is the side length of textile, $A_{TG} = 0.087\ \text{m}^2$ is the transversal area of the air feed box.

Mass balance of the dry air:

$$\frac{\partial \rho_G}{\partial t} = -v\frac{\partial \rho_G}{\partial z} \tag{3}$$

Mass balance of water in the textile material:

$$\frac{\partial X}{\partial t} = -\frac{2\psi}{\rho_S\varepsilon_S} \tag{4}$$

where ρ_S is the density of dry solid, $\varepsilon_S = 0.007m$ is the textile thickness.

Energy balance in air:

$$\frac{\partial}{\partial t}\left(\rho_G U_G\right) = -v\frac{\partial}{\partial z}\left(\rho_G H_G\right) - \frac{2h_c L_S}{A_{TG}}\left(T_G - T_S\right) + \frac{2L_S}{A_{TG}}\Delta H_s\psi \tag{5}$$

where U_G is the specific internal energy of the gas, H_G is the gas enthalpy, h_c is the convective heat transfer coefficient, T_G is the gas temperature, T_S is the solid temperature, ΔH_s is the desorption enthalpy.

Energy balance in the textile material:

$$\frac{\partial U_S}{\partial t} = \frac{2h_c}{\rho_S \varepsilon_S}\left(T_S - T_G\right) - \frac{2}{\rho_S \varepsilon_S}\Delta H_s \psi \qquad (6)$$

where U_S is the specific internal energy of the solid.

Equation (1) was used to represent the drying rate.

Resolution of the obtained model required some additional equations described below:

Heat of vaporization of pure water, as given by Silva (1995):

$$\Delta H_s = \Delta h_0 + (Cp_v - cp_{WS})T_S \qquad (7)$$

where Δh_0 is the water vaporization energy at 0 °C; Cp_v is the heat capacity at constant pressure of the steam in the solid; cp_{WS} is the heat capacity at constant pressure of water in the solid.

Desorption isotherm for the textile as analyzed by Sousa (2003):

$$X_e = \frac{\ln\left[1 - UR^{0.49}\right]}{-430.41 \;\; \exp\left(\dfrac{-936.26}{T_S}\right)} \qquad (8)$$

where X_e is the absolute equilibrium humidity content of the solid on a dry basis, UR is the air relative humidity on a dry basis.

The absolute humidity of water in air:

$$Y_{eq} = \frac{0.622 + Y}{1 - Y} \qquad (9)$$

The vapor pressure of water:

$$P_v(T_S) = v_8 \exp\left[\frac{(((v_5 T_S + v_4)T_S + v_3)T_S + v_2)T_S + v_1}{\left(v_6 - v_7 T_S\right)T_S}\right] \qquad (10)$$

where P_v is the vapor pressure of water, v_i are the equation coefficients, found in Brooker *et al.* (1974).

The following initial and boundary conditions were used:

$$\begin{cases} \text{ci1: } Y \;\; (t=0,z) = Y_0 \\ \text{ci2: } X \;\; (t=0,z) = X_0 \\ \text{ci3: } T_G\,(t=0,z) = T_{G\,0} \\ \text{ci4: } T_S\,(t=0,z) = T_{S\,0} \\ \text{ci5: } \rho_G\,(t=0,z) = \rho_{G\,0} \end{cases}$$

$$\begin{cases} \text{cc1: } Y \;\; (t,z=0) = Y_{feed} \\ \text{cc2: } T_G(t,z=0) = T_{G\,feed} \end{cases}$$

where Y_0 is the humidity, T_{G_0} is the initial temperature of ambient air, T_{S_0} is the initial temperature of the textile, ρ_{G_0} is the initial density of ambient air, Y_{feed} is the humidity of hot air used for drying.

Statistical Analysis

The statistical tests should be performed in order to validate the model obtained from the mass and energy balances, and even to test the considerations used in model simplification. The test for the coefficient of determination, known as a test of R-squared, is the square of the coefficient of Pearson correlation. It is a measure of the proportion of the variability in a variable which is explained by the variability of the other (Barros Neto *et al*., 1995). Considering the study of drying, the R-squared value indicates the proportion of the moisture of the textile material explained by the model when compared to experimental data.

Moreover, according to Barros Neto *et al*. (1995), the F-test statistical analysis can be used to determine if the model is predictive, or whether it can be used for prediction of the variables studied. According to the authors, for predictive purposes, the value of $F_{calculated}$ (average square error of regression / average square error of residuals) should be at least four to five times the value of $F_{tabelated}$.

The W test, developed by Shapiro and Wilk (1965), is essentially the square of the Pearson correlation coefficient, computed between the statistical order of the sample and that for a population that is Gaussian. If the value of W is close to 1.0 the sample behaves like a Normal sample, whereas if W is below 1.0 the sample is non-Gaussian. The original Shapiro-Wilk test is defined as:

$$W = \frac{\left[\sum_{i=1}^{k} a_i \left(x_{(n-i+1)} - x_{(i)}\right)\right]^2}{\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2} \qquad (11)$$

where n is the number of observations, k is approximately $n/2$, $a' = (a_1,...,a_n) = m'V^{-1}\left(m'V^{-2} - m\right)^{-1/2}$, m,V are the mean vector and covariance matrix of the order statistics of the standard normal distribution $X = \left(X_{1n},...,X_{nn}\right)'$ (Bai and Chen, 2003).

The p-value test of W is calculated by the following equation:

$$z = b_n + c_n\left[\ln\frac{W - d_n}{1 - W}\right] \quad (12)$$

where the coefficients are obtained in the second volume of the Biometrika Tables (Pearson and Hartley, 1976), by entering the value of n.

In statistics, the null hypothesis is a hypothesis that is presumed true until statistical evidence in the form of hypothesis testing indicates otherwise. It is a hypothesis that depends on confront with the facts. From the analysis of the results obtained with the drying model, it can be confirmed that the parameters or mathematical characteristics obtained from the model (population 1) are identical or not to the experimental results (population 2). The p-value is the probability that the sample could be taken from a population being tested assuming that the null hypothesis is true. A value of 0.05, for example, indicates that there is a probability of 5% that the test sample may have been removed from the assembly, assuming that the null hypothesis is true.

RESULTS AND DISCUSSION

The simulations were performed for velocities of 0.5, 1.0 and 1.5 m/s at temperatures of 50, 60 and 70 °C. These conditions were experimentally used by Sousa (2003) in order to compare the model results with those obtained in the laboratory.

To solve the system of differential equations from Eq. (2) to Eq. (6) the method of lines was used. In this method the spatial derivatives were approximated by finite differences. This method transforms the system of partial differential equations into a system of ordinary equations. To apply this method, the problem domain is divided into (n) discretization elements. In solving the mathematical model a computer program in FORTRAN language was developed, employing the subroutine DASSL (Differential/Algebric Equation system solver) developed by Petzold (1982) that uses backward differentiation formulas. The DASSL code requires a consistent set of values for the dependent variables (Y, X, T_G e T_S) and their respective derivatives with respect to time. In cases where the boundary condition is discontinuous it is necessary to modify it (Madras *et al*. 1994). In the drying problem investigated, the initial ambient temperature and humidity (T_{G_0}, Y_0) are suddenly raised to $T_{G\,feed}$ and Y_{feed} in dryer feeding ($z = 0$), and maintained at that value. This boundary condition was modified in order to make it continuous in the time variable according to the following expressions:

$$T_G = T_{G\,feed} + \left(T_{G_0} - T_{G\,feed}\right)\cdot e^{-s\cdot t} \quad (13)$$

$$Y = Y_{feed} + \left(Y_0 - Y_{feed}\right)\cdot e^{-s\cdot t} \quad (14)$$

with $s = 10^8$ min^{-1}

Table 1 shows the values of the heat capacities involved in the problem of crude cotton convective drying, material used by Sousa (2003) in the experimental analysis of textile drying process.

Table 1: Specific heat values of crude cotton.

Parameter	Value
cp_G	1.009 kJ / kgK
cp_S	1.26 kJ / kgK
cp_{WG}	4 kJ / kg K
cp_{WS}	4 kJ / kg K
Cp_v	2 kJ / kgK

The solution of the equation system occurred simultaneously with the optimization of the heat transfer coefficient, h_c. The initial estimates for values of h_c were calculated according to the method proposed by Xue (2004), using that from the experimental data, proposes the use of the following equation:

$$m_S\frac{dw}{dt} = \frac{h_c\left(T_G - T_S\right)}{\Delta H_S} \quad (15)$$

where m_S is the textile weight, dw/dt is the drying rate.

From the humidity data of the drying curve versus time, the drying rate (slope of the curve) was obtained, which was then substituted together with the other parameters in Equation (15), yielding the value of h_c.

According to Treybal (1980), when the flow direction and the air velocity are constant, the coefficients of mass and heat transfer are constant. One can then correlate the mass transfer coefficient with the coefficient of convective heat transfer by the Chilton-Colburn analogy, which is based on the analysis of the thermal and concentration boundary layers:

$$\frac{k_f}{hc} = \frac{1}{\rho_G cp_G}\left(\frac{\Pr}{Sc}\right)^{2/3} \tag{16}$$

where cp_G is the heat capacity at constant pressure of the dry gas, Pr is the Prandlt number, Sc is the Schmidt number.

Figure 1 shows the simulated and experimental results, obtained by Sousa (2003) from the moisture profile for a drying air temperature equal to 70 ºC, with flows of 0.5, 1.0 and 1.5 m/s. Figures 2 and 3 show the moisture profile at temperatures of 60 ºC and 50 ºC, respectively. From these three graphs, one can observe the influence of the air velocity on drying. In the case of a temperature of 70 °C, for a speed of 1.5 m/s, the moisture equilibrium is reached in about 15 minutes, for 60 °C in 20 min and for 50 °C in 30 min. As for drying at 1.0 m/s, it is observed that the equilibrium moisture content is reached in 20, 30 and 40 min at 70 ºC, 60 ºC and 50 ºC, respectively. In the case of low air velocity analyzed, 0.5 m/s, it was observed that the moisture equilibrium is only reached at operational times of 30, 40 and 60 minutes for air temperatures of 70 °C, 60 °C and 50 °C, respectively.

Therefore, when keeping the temperature constant and increasing the velocity of air different equilibration times are obtained. The higher the velocity, the shorter the time required to reach equilibrium between the moisture content of the textile material and air drying. This is the expected result, since the mass transfer is greater in air at higher speeds.

The statistical analysis was performed with the software Statistica 5.1®, with which the p-value of the Shapiro-Wilk test was calculated, besides the distribution profiles of waste. For all conditions tested, the R-squared value was higher than 0.997 and the p-value was higher than 0.05, indicating that the difference between the experimental values and those obtained by the model was not significant. The most normal behavior was 1.5 m/s and 60 °C and the lowest at 0.5 m/s and 70 °C.

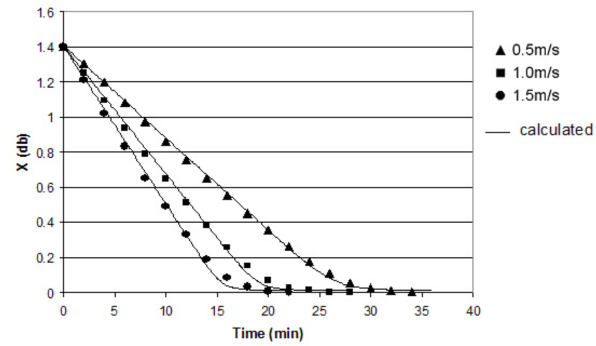

Figure 1: Effect of the air drying velocity at 70 ºC

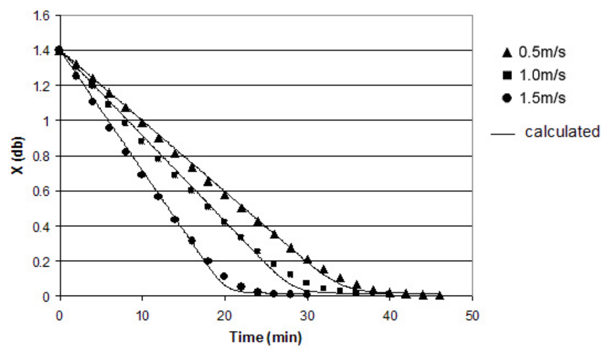

Figure 2: Effect of the air drying velocity at 60 °C.

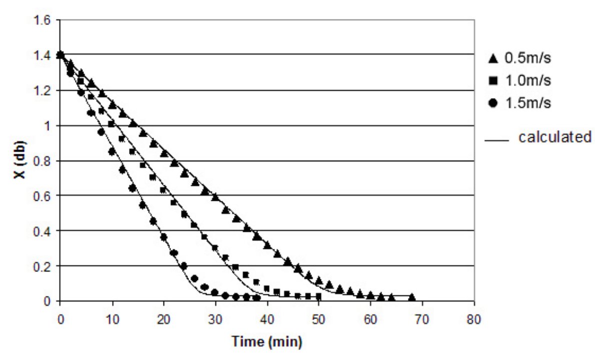

Figure 3: Effect of the air drying velocity at 50 °C.

The maximum and the average error between the experimental and calculated humidity was calculated. It is observed that the largest average error $(X_{exp}\text{-}X_{cal})/X_{exp}$, was 0.294 and the largest maximum error was 0.785.

To determine whether the obtained model was predictive, the F test was performed. For all simulated conditions, ratios $F_{calculated}/F_{simulated}$, at the 95% confidence level, greater than five were obtained, indicating that the model was predictive.

CONCLUSIONS

Based on the results it can be concluded that the model developed, together with the numerical method used for solving the equations system, was effective in describing the phenomenon of drying textile material by forced convection, in simulated operating conditions. The simulated moisture profiles are in accord with experiment and, in all cases, the statistical analyses used in the validation of the results presented Shapiro-Wilk tests with p-values greater than 5%, indicating normality between the literature data and the results obtained by the model solution of drying textiles materials, with R-squared values higher than 0.997. The $F_{calculated}/F_{simulated}$ ratios greater than five indicated that the modeling was predictive in all simulations.

NOMENCLATURE

a_i	Equation (11) parameter	-
A_{TG}	Transversal area of the air feed box	m^2
b	Equation (12) parameter	-
c	Equation (12) parameter	-
cp_G	Specific heat of dry air, at constant pressure	kJ/(kg air K)
cp_S	Specific heat of the dry solid, at constant pressure	kJ/(kg solid K)
cp_{WG}	Specific heat of steam in air, at constant pressure	kJ/(kg water K)
cp_{WS}	Specific heat of water in solid, at constant pressure	kJ/(kg water K)
Cp_v	Specific heat of steam in the solid, at constant pressure	kJ/(kg water K)
d	Equation (12) parameter	-
F_{obj}	Objective Function	-
h_c	Convective heat-transfer coefficient	kJ/(m^2 K min)
H_G	Air enthalpy	kJ/kg total
k_f	Mass transfer coefficient	kg water/ (m^2 min ΔY)
L_S	Length of textile	m
m_S	High basis weigh of textile	kg/m^2
n	Sample number	-
Nu	Nusselt number	-
Pr	Prandlt number	-
P_v	Steam pressure	Pa
Re	Reynolds number	-
s	Equation (11) parameter	-
t	Time	min
T_{G0}	Ambient temperature	K
T_G	Gas temperature	K
T_S	Solid temperature	K
T_{Gfeed}	Gas feed temperature	K
U_G	Internal energy of the gas	kJ/kg total air
U_S	Internal energy of the solid	kJ/kg solid
UR	Relative moisture of the air, dry basis	kg water/ kg dry air
v	Air velocity	m/min
v_i	Constant for steam pressure calculation	-
Y	Absolute moisture of air, dry basis	kg water/ kg dry solid
Y_{eq}	Absolute equilibrium moisture of air, dry basis	kg water/ kg dry solid
X	Absolute moisture of solid, dry basis	kg water/ kg dry solid
X_e	Absolute equilibrium moisture of solid, dry basis	kg water/ kg dry solid
X_{cal}	Calculated absolute moisture of solid, dry basis	kg water/ kg dry solid
X_{exp}	Experimental absolute moisture of solid, dry basis	kg water/ kg dry solid
W	Shapiro-Wilk statistics	-
z	Space	m

Greek Symbols

Ψ	Drying rate	kg water/m^3 s
Δh_0	Vaporization energy of water at 0 °C	kJ/kg K
ΔH_s	Desorption enthalpy	kJ/kg water
ρ_G	Dry air density	kg dry air/m^3
ρ_{G0}	Initial density of ambient air	kg dry air/m^3
ρ_S	Dry solid density	kg dry solid /m^3
ε_S	Solid thickness	m

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