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COMPUTATIONAL MODELLING OF MHD UNSTEADY FLOW AND HEAT TRANSFER TOWARD A FLAT PLATE WITH NAVIER SLIP AND NEWTONIAN HEATING

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Abstract - The combined effects of Navier slip and Newtonian heating on an unsteady hydromagnetic boundary layer stagnation point flow towards a flat plate in the presence of a magnetic field are studied. The self-similar equations are obtained using similarity transformations and solved numerically by a shooting algorithm with a Runge-Kutta Fehlberg integration scheme. The velocity profiles, temperature profiles, the local skin friction coefficient, and the local Nusselt number are computed and discussed in details for various values of the different parameters. Numerical results are presented both in tabular and graphical forms, illustrating the effects of these parameters on the thermal and concentration boundary layers. It is revealed that the thermal boundary layer thickens with a rise in the flow unsteadiness and as Newtonian heating intensifies, while the local skin friction and the rate of heat transfer at the plate surface change significantly due to the slip parameter.
Keywords: Unsteady flow; Flat plate; Navier slip; Newtonian heating; Magnetic field; Similarity solution.

INTRODUCTION

Analysis of unsteady hydromagnetic boundary layer flow and heat transfer of electrically conducting fluids is of great interest in many branches of engineering. Practical applications are found in the design of cooling systems for electronic devices, in the field of solar energy collection, geothermal reservoirs, heat exchangers, cooling of an infinite metallic plate in a cooling bath, magnetohydrodynamic (MHD) stirring of molten metal, magnetic-levitation casting, MHD marine propulsion, the boundary layer along a liquid film in condensation processes, and a polymer sheet or filament extruded continuously from a dye. Considerable reviews of this area have been made by many researchers such as Chamkha and Khaled (2000), Makinde and Ogulu (2008), Beg *et al*. (2009), Makinde (2010), etc. Vajravelu and Nayfeh (1992) discussed the hydromagnetic flow of a dusty fluid over a stretching sheet. The MHD heat and mass transfer in a flow of viscous incompressible fluid past an infinite vertical plate has been studied by Singh and Singh (2003). Ali and Magyari (2007) reported a numerical solution for unsteady boundary layer flow and heat transfer induced by a submerged stretching surface while its steady motion is slowed down gradually. An unsteady hydromagnetic free convection flow of elastico-viscous fluid past an infinite vertical plate taking into account the Hall effect has been investigated by Chaudhary and Jha (2008). A computational analysis of MHD boundary-layer flow and mass transfer past a vertical plate in a porous medium with constant heat flux was

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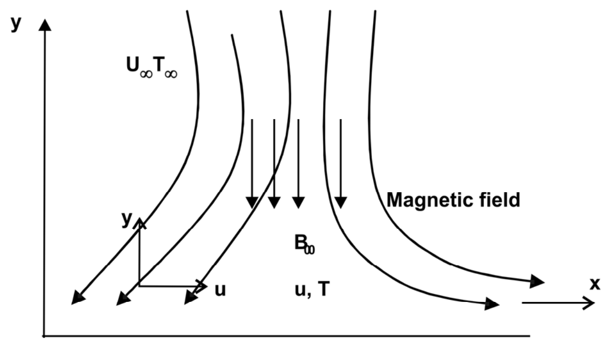
presented by Makinde (2009). Dulal and Hiremath (2009) investigated the heat transfer over an unsteady stretching surface embedded in a porous medium. In spite of the importance of these MHD-related studies on boundary layer flow problems, the possibility of fluid exhibiting apparent slip phenomenon on the solid surface has received little attention. The no-slip condition at the fluid–solid interface is a hypothesis rather than a condition deduced from any principle, and thus its validity has been continuously debated in the scientific literature (Choi *et al*., 2002). Meanwhile, many experimental results have provided evidence to support the slip condition (Pit *et al*., 2000; Huang and Breuer, 2007). In a pioneering work, Navier (1823) introduced a more general boundary condition, namely the fluid velocity component tangential to the solid surface, relative to the solid surface, is proportional to the shear stress on the fluid–solid interface. The proportionality is called the slip length, which describes the slipperiness of the surface. Matthews and Hills (2008) investigated numerically the effects of slip on momentum boundary layer thickness on a flat plate. Recently, Bhattacharyya *et al*. (2011) presented numerical results on the effects of velocity slip on hydromagnetic boundary layer flow and heat transfer over a flat plate. The combined effects of Hall current and wall slip on unsteady MHD flow of a viscoelastic fluid past an infinite vertical porous plate through a porous medium was investigated by Kumar and Chand (2011).

The objective of this study is to determine the combined effects of Navier slip and Newtonian heating on an unsteady hydromagnetic boundary layer flow over a flat surface. In the subsequent sections the classical similarity reductions of the boundary layer equations are derived and the resulting ordinary differential equations are solved numerically using shooting algorithm together with a Runge-Kutta Fehlberg integration scheme. Finally, we present a discussion of the results and we make some concluding remarks.

MATHEMATICAL MODEL

Consider an unsteady two-dimensional magnetohydrodynamic boundary layer stagnation point flow with heat transfer and Navier slip towards a flat plate. The lower surface of the plate is assumed to be heated by convection from a hot fluid at temperature T_f, which provides a heat transfer coefficient h_f, while the upper surface is subjected to a stream of an electrically conducting cold fluid at temperature T_∞ in the presence of magnetic field of strength B_0 imposed along the y-axis, as shown in Fig. 1. The induced magnetic field due to the motion of the electrically conducting fluid is negligible. It is also assumed that the external electrical field is zero and that the electric field due to the polarization of charges is negligible.



$$u = \beta \frac{\partial u}{\partial y}, \ \nu = 0, \ T_f$$

Figure 1: Flow configuration and coordinate system

Under the usual boundary layer approximation, the continuity, momentum, and energy equations describing the flow can be written as (Vajravelu and Nayfeh, 1992; Dulal and Hiremath, 2009):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_O^2 (u - U_\infty)}{\rho} + \frac{(\lambda + c)U_\infty}{1 - \lambda t} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where the free stream velocity is given as (Bhattacharyya *et al*., 2011):

$$U_\infty = \frac{xc}{1 - \lambda t}, \tag{4}$$

where c represents the straining parameter due to stagnation point flow of the fluid towards the plate surface, as shown in Fig.1 . The boundary conditions at the plate surface and far into the cold fluid may be written as:

$$u(t,x,0)=\beta\frac{\partial u}{\partial y}(t,x,0),\ v(t,x,0)=0$$

$$-k\frac{\partial T}{\partial y}(t,x,0,)=h_f[T_f-T(t,x,0)], \tag{5}$$

$$u(t,\ x,\infty)\ =\ U_\infty,\ T(t,\ x,\infty)\ =\ T_\infty.$$

The situation $\beta = 0$ corresponds to no-slip, while full lubrication is described in the limit $\beta\rightarrow\infty$. The stream function ψ, satisfies the continuity Eq. (1) automatically, with:

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ v=-\frac{\partial\psi}{\partial x}. \tag{6}$$

In order to simplify the mathematical analysis of the problem, we introduce the following dimensionless variables:

$$\eta=y\sqrt{\frac{c}{\upsilon(1-\lambda t)}},\quad \psi=x\sqrt{\frac{c\upsilon}{(1-\lambda t)}}f(\eta),\qquad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty} \tag{7}$$

Substituting Eq. (7) into Eqs. (1)-(6), we obtain:

$$f'''+ff''-f'^2-A\left(f'+\frac{\eta}{2}f''-1\right)-Ha(f'-1)+1=0, \tag{8}$$

$$\theta''+Pr\left(f\theta'-\frac{A}{2}\eta\theta'\right)=0, \tag{9}$$

$$f(0)=0,f'(0)=\delta f''(0),\ \ \theta'(0)=Bi[\theta(0)-1], \tag{10}$$

$$f'(\infty)=1,\quad \theta(\infty)=0, \tag{11}$$

where the prime symbol represents the derivative with respect to η and:

$$Ha=\frac{\sigma B_0^2}{\rho c}(1-\lambda t)$$

(the local magnetic field parameter),

$$Bi=\frac{h_f}{k}\sqrt{\frac{\upsilon(1-\lambda t)}{c}}$$

(the local Biot number),

$$A=\frac{\lambda}{c}$$

(Unsteadiness parameter),

$$Pr=\frac{\upsilon}{\alpha}$$

(the Prandtl number),

$$\delta=\beta\sqrt{\frac{c}{\upsilon(1-\lambda t)}}$$

(the local slip parameter).

The set of Equations (8)–(9) under the boundary conditions (10)-(11) have been solved numerically using a shooting algorithm with a Runge-Kutta Fehlberg integration scheme (Nachtsheim and Swigert, 1965). From the process of numerical computation, the plate surface temperature, the skin-friction coefficient and the Nusselt number, which are respectively proportional to $\theta(0)$, $f''(0)$ and $-\theta'(0)$, are also worked out and their numerical values are presented in a tabular form. The accuracy of this numerical method was validated by direct comparison with the numerical results reported by Dulal and Hiremath (2009) for the unsteady boundary layer flow over a moving plate in the absence of a magnetic field (Ha=0), Navier slip (δ =0) and Newtonian heating (Bi=0) modelled as:

$$f'''+ff''-f'^2-A\left(f'+\frac{\eta}{2}f''\right)=0\ \text{with}\qquad f(0)=0,f'(0)=1,\ \ f'(\infty)=0\ , \tag{12}$$

and a perfect agreement is observed, as demonstrated in Table 1 below;

Table 1: Computations showing comparison with Dulal and Hiremath (2009) results for δ = 0, Ha=0.

A	$f''(0)$ D-H (2009)	$f''(0)$ Present
0.5	-1.167221	-1.167221
1.0	-1.320540	-1.320540
1.5	-1.459687	-1.459687
2.0	-1.587403	-1.587403

Also, we conducted a direct comparison with the numerical results reported by Bhattacharyya *et al.* (2011) for the unsteady boundary layer stagnation-point flow towards a stretching sheet with slip in the absence of magnetic field (Ha=0) modelled as:

$$f''' + ff'' - f'^2 - A\left(f' + \frac{\eta}{2}f'' - R\right) + R^2 = 0$$
$$\text{with}\, f(0) = 0, f'(0) = 1 + \delta f'', \;\; f'(\infty) = R \;, \tag{13}$$

where R is the parameter representing the velocity ratio between the stretching sheet and that of the free stream. The results in Table 2 below agree almost perfectly with those of Bhattacharyya *et al*. (2011).

Table 2: Computations showing comparison with Bhattacharyya *et al*. (2011) results for Ha=A=δ=0.

R	$f''(0)$ Bhattacharyya *et al.* (2011)	$f''(0)$ Present
0.1	−0.969386	−0.9693871
0.2	−0.918107	−0.918110
0.5	−0.667263	−0.667263
2.0	2.017503	2.017510
3.0	4.729284	4.729283

RESULTS AND DISCUSSION

The numerical computation results are demonstrated in Table 3 and Figures 2-8. In order to have greater insight into the qualitative analysis of the results, we have taken the values of various parameters controlling the flow systems as 0.72 (Air) $\le Pr \le 7.1$ (Water), $0 \le A \le 1$, $0 \le \delta \le 5$, $0.1 \le Bi \le 1$, $0 \le Ha \le 5$. Table 3 illustrates the effects of thermophysical parameters on the local skin friction coefficient and local Nusselt number. It is noteworthy that the temperature gradient $\theta'(0)$ is negative for all parameter values considered in this study. This simply implies that the heat flow is from the hot fluid at the lower surface of the plate to the cold fluid on the upper surface of the plate. Moreover, it is interesting to note that the local heat transfer rate at the plate surface increases with increasing values of Bi, Ha, Pr, δ and decreases with increasing values of A. This can be attributed to the fact that the magnitude of the temperature gradient at the plate surface increases with an increase in local Biot number, magnetic field intensity, Prandtl number and slip length. Meanwhile, the effect of Navier slip parameter (δ) is to decrease the local skin friction. We also note that the local skin friction increases with an increase in the magnetic field intensity and flow unsteadiness.

Effects of Parameter Variation on Velocity Profiles

The illustrations of the velocity profiles with respect to the transverse distance are displayed in Figures 2-4. Generally, the fluid velocity is lowest at the plate surface and increases gradually to its free stream values satisfying the far field boundary condition. For the set of parameter values utilised, it is interesting to note that the far field conditions for velocity profiles are satisfied at a transverse distance $\eta = 3$. However, this is not the case for temperature profiles with the same set of parameter values. From Figure 2, we observe that the fluid velocity at the plate surface increases with an increase in the slip parameter (δ). This is consistent with the fact that higher δ means an increase in the lubrication and slipperiness of the surface. In Figure 3, a slight increase in the fluid velocity towards the plate surface is observed with an increase in the flow unsteadiness. An increase in the magnetic field intensity Ha causes an overshoot of the fluid velocity towards the plate surface.

Table 3: Computation showing $f''(0)$, $\theta(0)$ and $\theta'(0)$ for various values of key parameters

Bi	Ha	A	δ	Pr	$f''(0)$	$-\theta'(0)$	$\theta(0)$
0.1	0.1	0.1	0.1	0.72	1.186380878	0.08358870	0.16411293
0.5	0.1	0.1	0.1	0.72	1.186380878	0.25231253	0.49537492
1.0	0.1	0.1	0.1	0.72	1.186380878	0.33745722	0.66254277
0.1	0.5	0.1	0.1	0.72	1.296927194	0.08383121	0.16168785
0.1	1.0	0.1	0.1	0.72	1.418129883	0.08406929	0.15930705
0.1	0.1	0.5	0.1	0.72	1.267686514	0.08128091	0.18719082
0.1	0.1	1.0	0.1	0.72	1.361534490	0.07638725	0.23612748
0.1	0.1	0.1	1.0	0.72	0.603133607	0.08563567	0.14364329
0.1	0.1	0.1	3.0	0.72	0.275659634	0.08635366	0.13646331
0.1	0.1	0.1	0.1	1.00	1.186380878	0.08534327	0.14656723
0.1	0.1	0.1	0.1	3.00	1.186380878	0.09000268	0.09997318
0.1	0.1	0.1	0.1	7.10	1.186380878	0.09262444	0.07375555

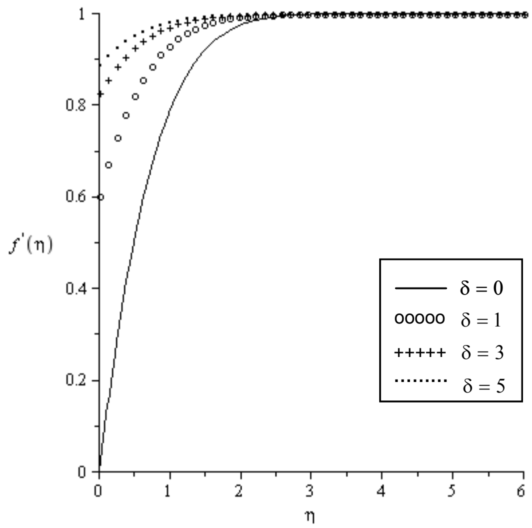


Figure 2: Velocity profiles for Pr = 0.72, A = 0.1, Bi=0.1, Ha = 0.1.

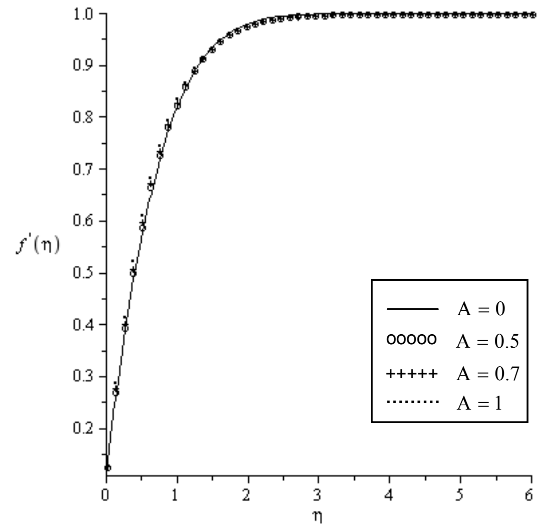


Figure 3: Velocity profiles for Pr = 0.72, δ = 0.1, Ha = 0.1, Bi=0.1

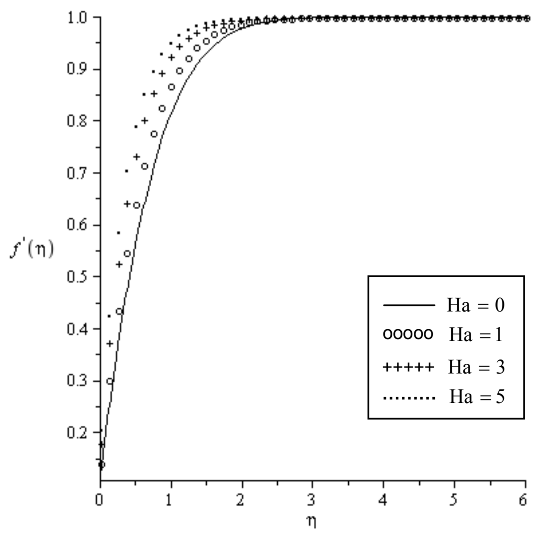


Figure 4: Velocity profiles for Pr = 0.72, δ = 0.1, A = 0.1, Bi=0.1

Effects of Parameter Variation on Temperature Profiles

Figures 5 and 6 show the effect of the Newtonian heating and the flow unsteadiness on the temperature profiles. The maximum value of fluid temperature is attained at the plate surface and decreases exponentially to the free stream zero value away from the plate, satisfying the boundary condition. It is seen that, in both cases, the thermal boundary layer thickness increases with an increase in the local Biot number (Bi) and the unsteadiness parameter (A). This can be attributed to the fact that, as Bi increases, the heat transfer rate from the hot fluid at the lower side of the plate to the cold fluid at the upper side increases. This results in an elevation of the fluid temperature at the upper side. The effects of the slip parameter and Prandtl number on the temperature profiles are plotted in Figures 7 and 8. It is evident from these figures that the presence of the surface slipperiness affects the temperature of the fluid inversely. This can be seen clearly from the temperature curves, which decrease as the slip parameter δ increases. A similar trend is observed with an increase in the Prandtl number Pr. The thermal boundary layer thickness decreases as the Prandtl number increases from 0.72 (Air) to 7.1 (Water) due to a decrease in the fluid thermal diffusivity. This is in agreement with the physical fact that, at higher Prandtl number, the fluid has a thinner thermal boundary layer and this increases the gradient of temperature.

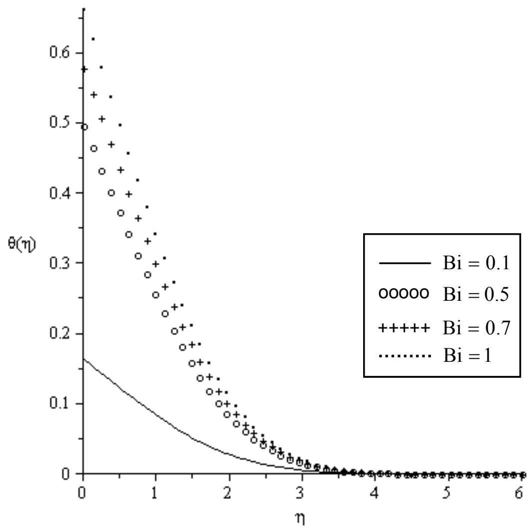


Figure 5: Temperature profiles for Pr = 0.72, A = 0.1, δ = 0.1, Ha = 0.1

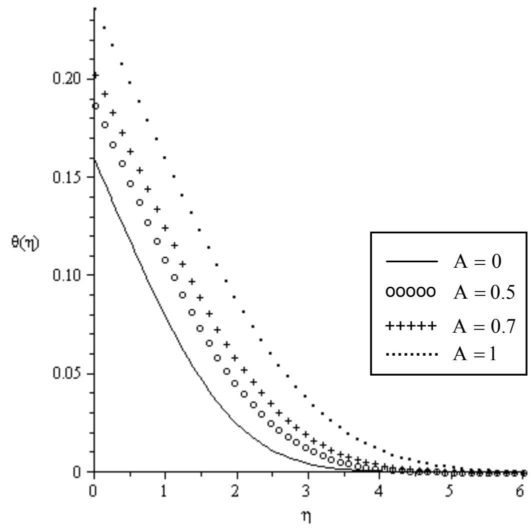


Figure 6: Temperature profiles for Pr = 0.72, δ = 0.1, Ha = 0.1, Bi=0.1

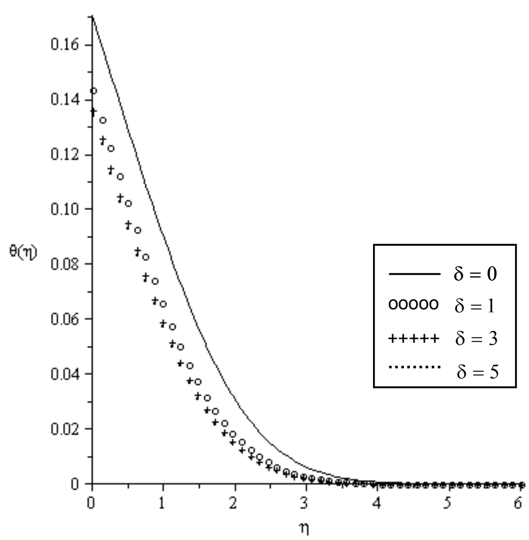


Figure 7: Temperature profiles for Pr = 0.72, A = 0.1, Bi=0.1, Ha = 0.1.

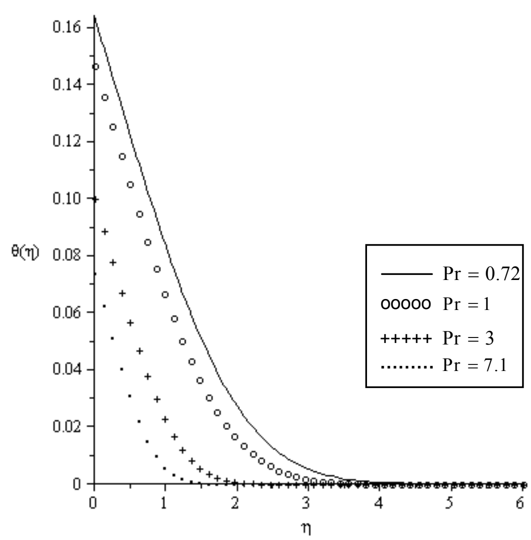


Figure 8: Temperature profiles for Ha = 0.1, δ = 0.1, A = 0.1, Bi=0.1

CONCLUSIONS

The problem of unsteady hydromagnetic boundary layer stagnation point flow towards a flat plate with Navier slip and Newtonian heating was studied. The governing equations were developed and transformed into a self-similar form and solved numerically by a shooting algorithm with a Runge-Kutta Fehlberg integration scheme. Our results revealed that the fluid velocity increases, while the local skin friction decreases, with an increase in the slip parameter (δ). The thermal boundary layer thickness is enhanced by increasing the intensity of Newtonian heating (Bi) and flow unsteadiness (A), while a decreases in the thermal boundary layer thickness is observed with an increase in the velocity slip (δ) and Prandtl number (Pr).

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NOMENCLATURE

(u, v)	velocity components	m/s
(x, y)	coordinates	m
B_0	magnetic field strength	
Pr	Prandtl number	
Bi	Biot number	
T_∞	free stream temperature	°C
f	dimensionless stream function	
t	time	s
T	temperature	°C
Ha	local magnetic field parameter	
c	free stream flow rate	
T_f	hot fluid temperature	°C
h_f	heat transfer coefficient	
A	unsteadiness parameter	
k	thermal conductivity coefficient	
U_∞	free stream temperature	m/s

Greek Symbols

Ψ	stream function
θ	dimensionless temperature
μ	dynamic viscosity
α	thermal diffusivity
η	similarity variable
λ	unsteadiness parameter
ρ	fluid density
υ	kinematic viscosity
σ	fluid electrical conductivity
δ	Navier slip parameter

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